Common Core Student Edition

SpringBoard™

Mathematics

Course 2

CollegeBoard

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ISBN: 1-4573-0149-X
ISBN: 0-978-1-4573-0149-0

1 2 3 4 5 6 7 8 13 14 15 16 17 18
Printed in the United States of America

Acknowledgments

The College Board gratefully acknowledges the outstanding work of the classroom teachers and writers who have been integral to the development of this revised program. The end product is testimony to their expertise, understanding of student learning needs, and dedication to rigorous but accessible mathematics instruction.

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We also wish to thank the members of our SpringBoard Advisory Council and the many educators who gave generously of their time and their ideas as we conducted research for both the print and online programs. Your suggestions and reactions to ideas helped immeasurably as we planned the revisions. We gratefully acknowledge the teachers and administrators in the following districts.

ABC Unified
Cerritos, California

Albuquerque Public Schools
Albuquerque, New Mexico

Amarillo School District
Amarillo, Texas

Baltimore County Public Schools
Baltimore, Maryland

Bellevue School District 405
Bellevue, Washington

Charlotte Mecklenburg Schools
Charlotte, North Carolina

Clark County School District
Las Vegas, Nevada

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Houston, Texas

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Collier County, Florida

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Denver, Colorado

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Frisco, Texas

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Gilbert, Arizona

Grand Prairie ISD
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Hillsborough County Public Schools
Tampa, Florida

Houston Independent School District
Houston, Texas

Hobbs Municipal Schools
Hobbs, New Mexico

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Volusia County Schools
DeLand, Florida

Contents

To the Student

Welcome to the SpringBoard program. We hope you will discover how SpringBoard can help you achieve high academic standards, reach your learning goals, and prepare for success in future mathematics studies.

The program has been created with you in mind: the content you need to learn, the tools to help you learn, and the critical thinking skills that help you build confidence in your own knowledge of mathematics. The College Board publishes the SpringBoard program. It also publishes the PSAT/NMSQT, the SAT, and the Advanced Placement exams—all exams that you are likely to encounter in your student years. Preparing you to perform well on those exams and to develop the mathematics skills needed for high school success is the primary purpose of this program.

Standards-Based Mathematics Learning

Knowledge of mathematics helps prepare you for future success in college, in work, and in your personal life. We all encounter some form of mathematics daily, from calculating the cost of groceries to determining the cost of materials and labor needed to build a new road. The SpringBoard program is based on learning standards that identify the mathematics skills and knowledge that you should master to succeed in high school and in future college-level work. In this course, the standards follow these broad areas of mathematics knowledge:

- Mathematical practices
- Number and operations
- Expressions, equations, and relationships
- Ratio and proportionality
- Geometry
- Statistics and probability

Mathematical practice standards guide your study of mathematics. They are actions you take to help you understand mathematical concepts rather than just mathematical procedures. For example, the mathematical practice standards suggest the following:

- Make sense of and connect mathematics concepts to everyday life and situations around you.
- Model with mathematics to solve problems, justify solutions and their reasonableness, and communicate mathematical ideas.
- Use appropriate tools, such as number lines, protractors, technology, or paper and pencil, strategically to help you solve problems.
- Communicate abstract and quantitative reasoning both orally and in writing through arguments and critiques.
- Analyze mathematical relationships through structure and repeated reasoning to connect ideas.
- Attend to precision in both written and oral communication of your mathematical ideas.

In the middle school years, your study of mathematics begins with a basic understanding of fractions and the operations performed with them. Your study continues with the development of a deep understanding of the rational numbers, their different representations, and the connections between these numbers and other number systems and operations. You will need a broad understanding of addition, subtraction, and multiplication with rational numbers, along with computational fluency with whole-number operations.

As you continue your studies, you will examine ratios and rates, which will allow you to make comparisons between numbers. Ratios and rates represent proportionality. Understanding the concepts of proportionality and linear equations are critical to future success in your study of algebra and the rest of the high school mathematics curriculum.

See pages xiii–xvi for a complete list of the Common Core State Standards for Mathematics for this course.

Strategies for Learning Mathematics

Some tools to help you learn are built into every activity. At the beginning of each activity, you will see suggested learning strategies. Each of these strategies is explained in full in the Resources section of your book. As you learn to use each strategy, you'll have the opportunity to decide which strategies work best for you. Suggested learning strategies include:

- Reading strategies, which help you learn to look at problem descriptions in different ways, from marking the text to highlight key information to turning problem information into questions that help you break the problem down into its separate parts.
- Writing strategies, which help you focus on your purpose for writing and what you're writing about.
- Problem-solving strategies, which give you multiple ways to approach the problem, from learning to identify the tasks within a problem to looking for patterns or working backward to see how the problem is set up.
- Collaborative strategies, which you'll use with your classmates to explore concepts and problems in group discussions and working with partners.

Building Mathematics Knowledge and Skills

Whether it is mathematics or sports or cooking, one way we learn something really well is by practice and repetition. To help you learn mathematics, the SpringBoard program is built around problem solving, reasoning and justification, communication, connections between concepts and ideas, and visual representation of mathematical concepts.

Problem Solving Many of the problems in this book are based on real-life situations that require you to ***analyze*** the situation and the information in the problem, ***make decisions, determine the strategies*** you'll use to solve the problem, and ***justify*** your solution. Having a real-world focus helps you see how mathematics is used in everyday life.

Reasoning and Justification One part of learning mathematics, or any subject, is learning not only how to solve problems but also why you solved them the way you did. You will have many opportunities to predict possible solutions and then to verify solutions. You will be asked to explain the reasoning behind how you solved the problem, the mathematics concepts involved, and why your approach was appropriate for solving the problem.

Communication When learning a language, saying words out loud helps you learn to pronounce the words and to remember them. Communicating about mathematics, orally and in writing, with your classmates and teachers helps you organize your learning and explain mathematics concepts and problem-solving strategies more precisely. Sharing your ideas and thoughts allows you and your classmates to build on each other's ideas and expand your own understanding.

Mathematics Connections As you study mathematics, you will learn many different concepts and ways of solving problems. Reading the problem descriptions will take you into the real-life applications of mathematics. As you develop your mathematics knowledge, you will see the many connections between mathematics concepts and between mathematics and your own life.

Representations Artists create representations through drawings and paintings. In mathematics, representations can take many forms, such as numeric, verbal, graphic, or symbolic. In this course, you are encouraged to use representations to organize problem information, present possible solutions, and communicate your reasoning. Creating representations is a tool you can use to gain understanding of concepts and communicate that understanding to others.

We hope you enjoy your study of mathematics using the SpringBoard program. We, the writers, are all classroom teachers, and we created this program because we love mathematics. We wanted to inspire you to learn mathematics *and* build confidence that you can be successful in your math studies and in using mathematics in daily life.

Common Core State Standards for Mathematics

Grade 7

Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

7.RP Ratios and Proportional Relationships

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks* $\frac{1}{2}$ *mile in each* $\frac{1}{4}$ *hour, compute the unit rate as the complex fraction* $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.RP.A.2c Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.

7.RP.A.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

7.NS The Number System

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.A.1a Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

7.NS.A.1b Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

7.NS.A.1d Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.A.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

7.NS.A.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.

7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

7.EE Expressions and Equations

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.B.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width*?

7.EE.B.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

7.G Geometry

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

7.SP Statistics and Probability

7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

> **7.SP.C.7a** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
>
> **7.SP.C.7b** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

> **7.SP.C.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
>
> **7.SP.C.8b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
>
> **7.SP.C.8c** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Number Systems

1

Unit Overview

In this unit you will extend your knowledge of numbers and expressions to the entire set of integers and develop an understanding of irrational numbers. You will apply your understanding of rational numbers as you solve problems.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- critique
- ascend
- descend

Math Terms

- absolute value
- subset
- rational number
- terminating decimal
- repeating decimal

ESSENTIAL QUESTIONS

? Why is it important to understand properties and operations involving integers and negative rational numbers?

? How can models be used to interpret solutions of real-world problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 2 and 4, will give you an opportunity to demonstrate how you can use your understanding of the number system to solve mathematical and real-world problems.

Embedded Assessment 1:

Positive Rational Numbers and Adding and Subtracting Integers p. 23

Embedded Assessment 2:

Rational Number Operations and Multiplying and Dividing Integers p. 47

UNIT 1

Getting Ready

1. Determine the value of each of the following expressions.

a. 32×21

b. $30{,}000 \div 10$

c. $478 + 593$

d. $101 - 68$

2. Determine the value of each of the following expressions.

a. 2.2×1.3

b. $39.5 + 8.74$

c. $33.4 - 2.11$

d. $470.4 \div 5.6$

3. Determine the value of each of the following expressions

a. $\frac{2}{5} + \frac{3}{10}$

b. $\frac{5}{6} - \frac{1}{3}$

c. $\frac{4}{5} \times \frac{7}{8}$

d. $\frac{6}{7} \div \frac{3}{4}$

4. Which property is illustrated by each example? Choose from the Associative, Commutative, and Distributive properties.

a. $6 + 8 = 8 + 6$

b. $(2 + 3) + 4 = 2 + (3 + 4)$

c. $2 \times 3 + 2 \times 5 = 2(3 + 5)$

5. Draw a number line like the one shown and graph the following points on the number line. Label each point with its letter.

a. 8 **b.** 3.5 **c.** $5\frac{1}{3}$

6. Order the following sets of numbers from least to greatest.

a. $\frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{7}{10}$

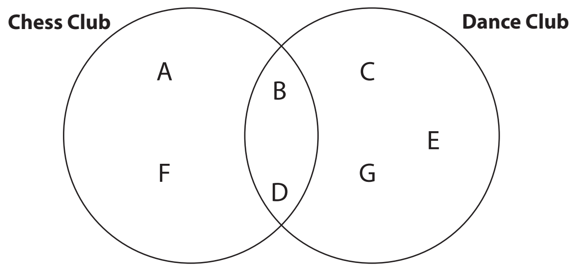
b. 32.51, 2.53, 514.37

7. Tell the value of each of the following expressions.

a. $|12|$ **b.** $|-13|$

c. $|-5| + |5|$ **d.** $|3 + 7| - |-7|$

8. This Venn diagram provides a visual representation of six students' memberships in after-school clubs. What does the diagram tell you about the club memberships of Student B and Student G? Explain.



Operations on Positive Rational Numbers

ACTIVITY 1

Paper Clips, Airplanes, and Spiders

Lesson 1-1 Adding and Subtracting Decimals

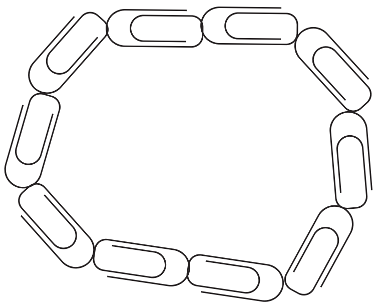
My Notes

Learning Targets:

- Solve problems with decimals, using addition and subtraction.
- Justify solutions with decimals, using addition and subtraction.
- Estimate decimal sums and differences.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Use Manipulatives, Think-Pair-Share, Discussion Groups

How long do you think it would take to make a paper clip chain that is 10 paper clips long? Last year, the student with the best time was able to do this in 26.25 seconds. Do you think you can do it in less time?



Work with your group. You will need

- Paper clips that are all the same size.
- A digital stopwatch that records time to the nearest hundredth of a second.

1. One at a time, each person makes a chain of 10 paper clips while the other students keep time with the stopwatch and record the amount of time. Fill in the times for your group in a chart like the one at the right.

Group Member	Time (in seconds)

2. Without computing an exact sum, estimate the total amount of time it took for your group to make their chains. How did you come up with this estimate?

3. Now compute the total time. Is your computed result reasonable? How can you justify your result?

4. Compare the fastest time in your group with last year's best time. Without computing an exact difference, estimate the difference in the times. How did you come up with this estimate?

DISCUSSION GROUP TIPS

If you do not understand something in group discussions, ask for help or raise your hand for help. Describe your questions as clearly as possible, using synonyms or other words when you do not know the precise words to use.

My Notes

5. Now compute the difference. Is your computed result reasonable? How can you justify your result?

```
  28.5
+29.75
   27
 32.87
```

6. **Critique the reasoning of others.** Julio's group did the paper clip chain activity and got the following times (in seconds): 28.5; 29.75; and 27. He wrote the numbers in a column and added, as shown at the left. What error did he make? Write your answer here and also explain the error to your group using clear descriptions and correct math language.

7. What is the correct sum of the times for Julio's group?

8. Write a rule for Julio to use when adding or subtracting decimals so that he does not make this type of error again.

ACADEMIC VOCABULARY

To **critique** is to analyze and discuss the details of something.

Check Your Understanding

Find each sum or difference. Justify your results.

9. 5.03 + 13.7 + 108 **10.** 3.084 − 1.7 **11.** 159 − 88.99

12. Ping is buying a sandwich for $5.95 and a bottle of juice for $1.75. He is going to pay with a $10 bill.

a. How can he estimate how much change he should receive?

b. What is his exact amount of change?

LESSON 1-1 PRACTICE

Find each sum. Justify your results.

13. 9.08 + 14.6 **14.** 12 + 1.12 **15.** 7.009 + 2.02

16. 0.66 + 6 **17.** 11.05 + 14.6 + 46 **18.** 59 + 5.9 + 0.59

Find each difference. Justify your results.

19. 8.644 − 3.7 **20.** 21.56 − 9.56

21. 36.8 − 36.55 **22.** 7 − 0.007

23. **Construct viable arguments.** Theo bought these items: Shoes: $19.99; socks: $4.19; T-shirt: $8.50; pants: $27.75. How he can estimate the total cost?

24. Find the actual total cost of Theo's items.

25. Ana took Ali out for lunch. Their lunches cost $13.28 and $14.25, including tax and tip. Ana paid with two $20 bills. How much change did Ana receive?

My Notes

Learning Targets:

- Estimate decimal products and quotients.
- Solve problems involving multiplication and division of decimals.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think-Pair-Share

Whitney did the paper clip chain activity but dropped some of the paper clips on the floor. The timekeeper in her group said that her time for completing the chain was 2.8 times as long as last year's best time of 26.25 seconds.

1. **Reason quantitatively.** Estimate the amount of time it took Whitney to complete her chain. How did you determine your estimate?

2. Explain what you already know about multiplying decimals.

You do not have to vertically align the decimal points when you multiply, but you do have to keep track of the number of decimal points in each of the numbers you multiply.

Example A

Find the exact amount of time it took Whitney to complete her chain.

Step 1: Set up the multiplication.

$$\begin{array}{r} 26.25 \\ \times\ 2.8 \end{array}$$

Step 2: Multiply. Locate the decimal point in the product.

$$\begin{array}{rl} 26.25 & \leftarrow 2 \text{ decimal places} \\ \times\ 2.8 & \leftarrow 1 \text{ decimal place} \\ \hline 21000 & \\ 5250 & \\ \hline 73.500 & \leftarrow 2 + 1 = 3 \text{ decimal places} \end{array}$$

Solution: It took Whitney 73.5 seconds to complete her chain. This should be close to your estimate in item 1 and therefore reasonable.

Try These A

Find each product. Justify your results.

a. 8.5×2.3 **b.** 0.03×14 **c.** 1.08×2.014

Check Your Understanding

3. Jerry multiplied 3.04×7.091 and got the product 2.155664. Is his answer reasonable? Why or why not?
4. Joanie multiplied 0.78×0.34 and got the product 26.52. What error did she make?

My Notes

MATH TIP

$0.045\overline{)1.8}$ is the same as the fraction $\frac{1.8}{0.045}$. When you multiply the numerator and denominator by the same number, the value does not change:

$$\frac{1.8 \times 1000}{0.045 \times 1000} = \frac{1800}{45}$$

You must also keep track of decimal points when dividing.

Example B

A new road is 1.8 km long. Each lot along the road will be 0.045 km long. How many lots will there be along the road?

Step 1: Set up the division. $0.045\overline{)1.8}$

Step 2: Multiply the divisor by 1000 to make 0.045 a whole number. You must also multiply the dividend, 1.8, by 1000. Then divide. Make sure to place the decimal point in the quotient above the decimal point in the dividend.

$$\begin{array}{r} 40. \\ 45\overline{)1800.} \\ \underline{180} \\ 00 \\ \underline{0} \\ 0 \end{array}$$

Solution: There will be 40 lots along the road.

Try These B

Find each quotient.

a. $300.6 \div 18$ **b.** $3.24 \div 3.6$ **c.** $28.8 \div 0.24$

Check Your Understanding

5. Curtis divided 27.16 by 2.8 and got 0.97. Is his answer reasonable? Why or why not?
6. Write a set of directions for dividing 3.6 by 0.25. Then find the quotient.

LESSON 1-2 PRACTICE

Find each quotient.

7. $601.2 \div 18$
8. $3.24 \div 7.2$
9. $80 \div 32$
10. $7.2 \div 0.12$
11. Josiah paid \$19.75 for 2.5 pounds of coffee beans. What was the cost of the beans per pound?
12 Keisha bought 1.2 pounds of Swiss cheese that was selling for \$5.95 per pound. How much did Keisha pay for the Swiss cheese?
13. **Make sense of problems.** Ralph has a spool with 9.8 meters of wire. How many 0.14 meter pieces of wire can he cut from the spool?

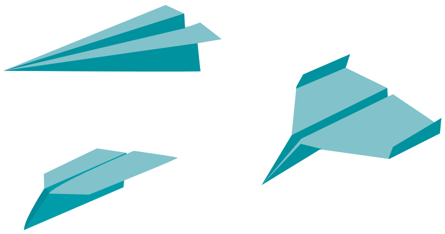
My Notes

Learning Targets:

- Solve problems with fractions using addition, subtraction, multiplication, and division.
- Estimate with fractions.

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations

How far can you throw a paper airplane? According to a recent entry in Guinness Book of World Records, the record holder threw a paper airplane a distance of $207\frac{1}{3}$ feet.



Work with your group to make a paper airplane. Listen to group members' ideas and share your own. Ask and respond to questions to help the group accomplish this task. Your teacher will give you a set of directions on how to make an airplane if you need one.

Test your airplane. Mark a starting line on the classroom floor, and then measure the distance the plane flies to the nearest $\frac{1}{12}$ of a foot. Record the three best distances in the table in the My Notes space.

Best Distances (ft)

1. Write, but do not evaluate, expressions that could be used to answer each question.

a. What was the distance between the record and your best distance?

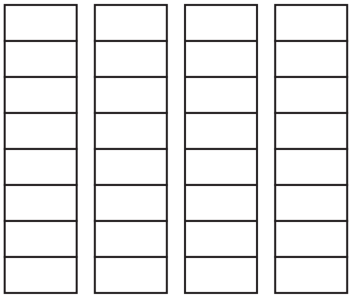
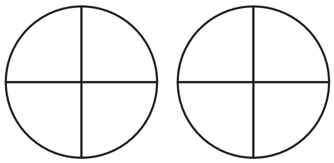
b. If another group had a best distance that was $1\frac{1}{4}$ times your group's best distance, what would that distance be?

c. How many times your group's best distance is the world record?

d. What is the average of your three best distances?

My Notes

2. In the table below, represent the processes for operations on fractions with models, numbers, and words. Shade or mark the models to show each operation. Then use words to explain the process. Finally, find the answer to the operation.

Operation	With Model	Explanation in words	Answer
a. $\frac{17}{8}+\frac{11}{8}$			
b. $1\frac{1}{4}-\frac{3}{4}$			

3. How is the process of adding $\frac{3}{8}+\frac{1}{4}$ different from the addition shown in 2a?

Check Your Understanding

On a middle school track team, the record time for a completing an obstacle course is $8\frac{1}{3}$ minutes.

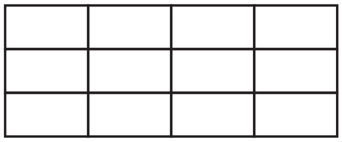
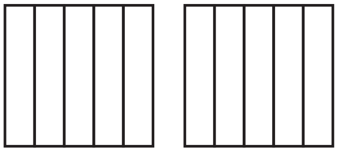
4. Alana's time for completing the obstacle course was $13\frac{1}{3}$ minutes. What is the difference between Alana's time and the record time?

5. Leesa's best time for completing the obstacle course was $10\frac{1}{2}$ min, and Sandy's best time was $9\frac{1}{5}$ min. What was the total of their best times?

6. How much less is the team record time than the sum of Leesa's time and Sandy's time?

My Notes

7. In the table below, represent the processes for operations on fractions with models, numbers, and words. Shade the models to show each operation. Then use words to explain the process. Finally, find the answer to the operation.

Operation	With Model	Explanation in words	Answer
a. $\frac{1}{4} \cdot \frac{2}{3}$			
b. $1\frac{3}{5} \div \frac{2}{5}$			

8. Yanni threw his paper airplane $15\frac{1}{2}$ feet. Adrian threw his paper airplane $\frac{3}{4}$ of Yanni's distance. What is the distance Adrian threw his paper airplane? Explain how you found your answer.

9. Mr. Adams has poster paper that is $10\frac{3}{4}$ feet long. He wants to make sheets that are $\frac{1}{4}$ of a foot long to make paper airplanes. How many sheets can he make?

My Notes

Check Your Understanding

Evaluate each expression you wrote for item 1 of this lesson to determine how close you are to setting a new Guinness World Record for throwing a paper airplane. Show your work.

10. What is the difference between the record and your best distance?

11. If another student threw a paper airplane $1\frac{1}{4}$ times farther than you did, what would that distance be?

12. How many times farther than your distance was the record holder able to throw the paper airplane?

13. Find your average distance by calculating the mean of the three trials.

LESSON 1-3 PRACTICE

Find each sum or difference.

14. $35\frac{3}{4}+18\frac{3}{4}$

15. $\frac{5}{6}+\frac{1}{3}$

16. $11\frac{1}{12}+\frac{7}{8}$

17. $\frac{13}{17}-\frac{5}{17}$

18. $12\frac{2}{9}-\frac{5}{6}$

19. $41\frac{9}{11}-27\frac{1}{3}$

20. The recipe for a cake calls for the following dry ingredients: $\frac{1}{3}$ cup of sugar; $\frac{3}{4}$ cup of cornmeal; and $\frac{1}{2}$ cup of flour. What is the total number of cups of dry ingredients called for?

21. In 1935, American athlete Jesse Owens set a world record for the long jump by jumping 26 ft $8\frac{1}{16}$ in. In 1960, Ralph Boston set a new world record by jumping 26 ft $11\frac{1}{4}$ in. How much longer was Ralph Boston's jump?

Find each product or quotient.

22. $\frac{1}{8}\cdot 5$

23. $\frac{7}{10}\cdot 3\frac{3}{7}$

24. $9\frac{2}{7}\cdot 2\frac{2}{13}$

25. $\frac{3}{8}\div 2$

26. $\frac{2}{3}\div\frac{4}{7}$

27. $1\frac{5}{6}\div 5\frac{2}{5}$

28. Lilly jogged $3\frac{1}{4}$ miles each day for 24 days last month. How many miles did she jog in all?

29. Lester jogs $5\frac{3}{4}$ miles on each day that he jogs. Last month, he jogged a total of 115 miles. How many days did he jog last month?

30. **Reason quantitatively.** Parmesan cheese was on sale for $13.60 per pound. Wesley bought a piece of the Parmesan cheese that weighed $1\frac{1}{8}$ pounds. How much did he pay?

My Notes

Learning Targets:

- Convert a fraction to a decimal.
- Understand the difference between terminating and repeating decimals.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think-Pair-Share

Sarai is researching spiders. She read that outside the United States, it is not unusual to find a camel spider that is $6\frac{3}{8}$ inches long. Her classmate Akeem is researching insects. He read an article about an insect known as a titan beetle that was $6\frac{1}{3}$ inches long.

It can sometimes be helpful to compare numbers expressed in fraction form by converting the fractions to decimals. Some decimal forms of fractions *terminate*, and some decimal forms *repeat*.

Example A

Express $6\frac{3}{8}$, the length in inches of the camel spider Sarai researched, as a decimal.

Step 1: Write the mixed number $6\frac{3}{8}$ as an improper fraction.

$$6\frac{3}{8} = 6 + \frac{3}{8} = \frac{48}{8} + \frac{3}{8} = \frac{51}{8}$$

Step 2: Divide the numerator by the denominator.

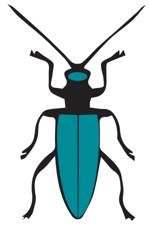
$$\begin{array}{r} 6.375 \\ 8\overline{)51.000} \\ -48 \\ \hline 3\,0 \\ -2\,4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

The remainder is 0, so the decimal form of $6\frac{3}{8}$ is a **terminating decimal**.

Solution: The decimal form of $6\frac{3}{8}$ is 6.375.

CONNECT TO SCIENCE

Spiders belong to the class Arachnida and are commonly mistaken for insects. One major difference between arachnids and insects is that arachnids have eight legs and insects have six legs.



MATH TERMS

A **terminating decimal** has a finite or limited number of digits following the decimal point.

My Notes

Example B

Express $6\frac{1}{3}$, the length in inches of the titan beetle, as a decimal.

Step 1: Write the mixed number as an improper fraction.

$$6\frac{1}{3} = 6 + \frac{1}{3} = \frac{18}{3} + \frac{1}{3} = \frac{19}{3}$$

Step 2: Divide the numerator by the denominator.

```
    6.333
3)19.000
 -18
   10
   -9
    10
    -9
     10
     -9
      1
```

The remainder repeats so the digits in the quotient repeat. The quotient is a **repeating decimal**.

The *bar notation* indicates which digits repeat.

Solution: The decimal form of $6\frac{1}{3}$ is $6.\overline{3}$.

MATH TERMS

A **repeating decimal** has one or more digits following the decimal point that repeat endlessly.

Try These A–B

Express each mixed number as a decimal. Indicate whether the decimal is terminating or repeating.

a. $7\frac{7}{8}$ **b.** $2\frac{1}{6}$ **c.** $5\frac{3}{4}$ **d.** $12\frac{5}{9}$

CONNECT TO AP

In Calculus, answers are rounded to three decimal places.

Check Your Understanding

1. Compare the answers of Examples A and B. Which answer is greater? How do you know?
2. **Critique the reasoning of others.** Nathan converted $\frac{5}{11}$ to a repeating decimal and wrote $0.4\overline{5}$ as the answer. What error did he make?

LESSON 1-4 PRACTICE

Express each fraction or mixed number as a decimal. Identify the repeating decimals.

3. **a.** $\frac{2}{3}$ **b.** $\frac{5}{8}$ **c.** $\frac{4}{5}$
4. **a.** $3\frac{3}{16}$ **b.** $8\frac{2}{9}$ **c.** $11\frac{7}{11}$
5. Which is greater, 0.32 or $0.\overline{3}$? How do you know?
6. Philip takes $2\frac{3}{5}$ hours to clean his room. Ashton takes $2\frac{5}{8}$ hours to clean his room. Who took less time to clean up his room?
7. **Look for and make use of structures.** What kinds of denominators generate repeating decimals?

ACTIVITY 1 PRACTICE

Lesson 1-1

In items 1–4, estimate each sum or difference. Explain how you determined your estimate.

1. 3.77 + 1.39
2. 4.35 + 3.8 + 4.129 + 3.672
3. 17.129 − 9.7
4. 38.8 − 12.2

Evaluate each expression in items 5–8.

5. 2.9 + 0.29
6. 0.34 + 495.5 + 99.008
7. 87.6 − 53.909
8. 48 − 0.48
9. At one time, the world record for running 100 yd backward was 13.5 seconds. If the record is now 12.7 seconds, how many seconds faster is the new record?
10. In 1985, American swimmer Tom Jager completed a 50-meter freestyle swim in 22.40 seconds. In 1990, he was able to complete the swim in 21.81 seconds. How many seconds slower was his 1985 swim?
11. Linda is running in a marathon, which is 26.2 miles long. Checkpoint 1 is 3 miles past the start; checkpoint 2 is 2.5 miles after checkpoint 1; and checkpoint 3 is 3.75 miles after checkpoint 2. When Linda makes it to checkpoint 3, how many miles does she have to run to complete the marathon?

Lesson 1-2

Evaluate each expression in items 12–15.

12. 1.4 • 27
13. 0.17 • 0.6
14. 14.127 ÷ 5.1
15. 6.58 ÷ 9.4
16. Without doing the computation, explain why or why not 12.702 is a reasonable value for the expression 5.8 • 2.19.
17. Without doing the computation, explain why or why not 14.766 is a reasonable value for the expression 3.21 • 0.46.
18. Without doing the computation, explain why or why not 19.7 is a reasonable value for the expression 122.14 ÷ 6.2.
19. Three people bought books for a total of $12.42. If they shared the cost equally, how much did each person pay?
    A. $6.21 **B.** $4.14
    C. $4.00 **D.** $4.52
20. Cheryl makes $8.40 an hour. If she works 10.75 hours in a week, how much will she earn for the week?
    A. $9.30 **B.** $90.30
    C. $900.30 **D.** $9000.30
21. Daniel is buying a video game that costs $52.99. The sales tax is found by multiplying the cost of the video game by 0.07. How much is the sales tax for the video game? What is the total cost, including tax?

22. Cory earns $9.50 per hour for the first 40 hours he works in a week. For any hours over 40 hours per week, his hourly rate is multiplied by 1.5. How much does he earn if he works 43.5 hours in one week?

Lesson 1–3

Evaluate each expression in items 23–26.

23. $4\frac{1}{2}+1\frac{2}{7}+3\frac{1}{3}$

24. $132\frac{1}{6}-99\frac{5}{6}$

25. $\frac{1}{10}\cdot\frac{3}{11}$

26. $21\div3\frac{1}{2}$

27. A machine can make a box in $1\frac{3}{10}$ seconds. How many boxes can the machine make in 1 hour?

28. Carrie has a 10-ft plank of wood. She wants to cut 3 pieces that are each $2\frac{2}{3}$ feet long from the plank. How long will the plank be after she cuts off the three pieces?

29. A large carton of juice holds 12 cups. How many $\frac{3}{4}$-cup servings does the carton hold?

30. Gary is $61\frac{1}{8}$ inches tall. His friends Gino and Gilbert are $56\frac{1}{2}$ inches tall and $63\frac{1}{8}$ inches tall. What is the average height of the three friends?

31. Can you think of situations in which it might be preferable to compute with decimals rather than fractions or to compute with fractions rather than decimals? Give examples of each situation and tell why you think that number form is preferable.

Lesson 1–4

For items 32–37, write the fraction as a decimal. Then identify the decimal as terminating or repeating.

32. $\frac{3}{5}$

33. $\frac{1}{6}$

34. $\frac{5}{9}$

35. $\frac{9}{20}$

36. $\frac{13}{25}$

37. $\frac{10}{11}$

38. Which fraction is equivalent to a repeating decimal?

A. $\frac{3}{12}$

B. $\frac{6}{12}$

C. $\frac{8}{12}$

D. $\frac{9}{12}$

39. Order the numbers from least to greatest:

$1\frac{4}{5}, 1.78, 1\frac{5}{6}, \frac{7}{4}, 1.\overline{7}, 1\frac{8}{11}$

40. Two turtles are competing in a race. Turtle A reaches the finish line in $1\frac{3}{7}$ hours. Turtle B finished in $1\frac{2}{5}$ hours. Which turtle had the faster time?

41. Emily says that she can convert $\frac{18}{25}$ to a decimal by using equivalent fractions instead of dividing 18 by 25. Use Emily's method to convert $\frac{18}{25}$ to a decimal.

MATHEMATICAL PRACTICES

Critique the Reasoning of Others

42. Nilsa converted $\frac{1}{12}$ to a repeating decimal and wrote $0.0\overline{83}$ as the answer. What error did she make?

Addition and Subtraction of Integers

ACTIVITY 2

Elevation Ups and Downs

Lesson 2-1 Adding Integers

Learning Targets:

- Add two or more integers.
- Identify and combine opposites.
- Solve real-world problems by adding integers.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Create Representations, Quickwrite

A passenger jet that ascends $+5$ miles and then descends -3 miles will end at an elevation 2 miles above where it began.

$$+5 + (-3) = +2$$

A similar relationship holds in chemistry. An ionic bond is formed by an attraction between two oppositely charged ions. *Cations* are positively charged ions, and *anions* are negatively charged ions. Sodium (Na) has one cation with a $+1$ charge, and chlorine (Cl) has one anion with a -1 charge. When put together, sodium chloride (NaCl), table salt, is formed, and it has a charge of 0.

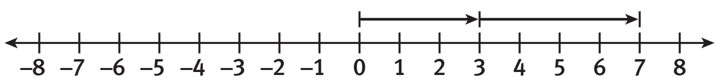
$$+1 + (-1) = 0$$

1. Write an equation to represent the resulting charge when each of the following ionic bonds of cations and anions are formed.

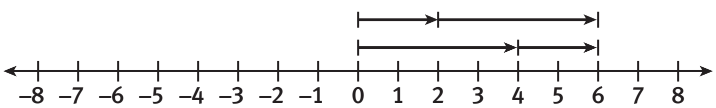
a. 5 cations and 3 anions **b.** 2 cations and 7 anions

The equations you wrote are examples of integers being added. One way to visualize integer addition is to use number lines. You can then connect the number line representations to equations and develop rules for adding integers.

2. Explain how the number line shows the sum of 3 and 4. What is the sum? Write the equation.



3. What property of addition is shown by the number line? Explain your reasoning.



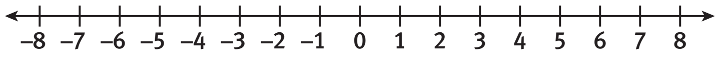
My Notes

CONNECT TO SCIENCE

Cations and anions are the building blocks of molecules, which are the building blocks of all matter in the universe.

My Notes

4. Use the number line to find the sum $(-3) + (-5)$.



$(-3) + (-5) =$ ______

Your results can be summarized with this rule:

- To add two integers with the same sign, add the **absolute values** of the integers. The sum has the same sign as the addends.

MATH TERMS

The **absolute value** of a number is its distance from zero on a number line. Distance, or absolute value, is always positive, so $|-6| = 6$ and $|6| = 6$.

Example A

Add: $15 + 23$

The signs are the same, so add the absolute values.

$$|15| + |23| = 15 + 23 = 38$$

Since both addends are positive, the sum is positive.

Solution: $15 + 23 = +38$

Example B

Add: $(-12) + (-7)$

The signs are the same, so add the absolute values.

$$|-12| + |-7| = 12 + 7 = 19$$

Since both addends are negative, the sum is negative.

Solution: $(-12) + (-7) = -19$

Try These A–B

Add.

a. $(-14) + (-36)$

b. $19 + 16$

c. $26 + 45$

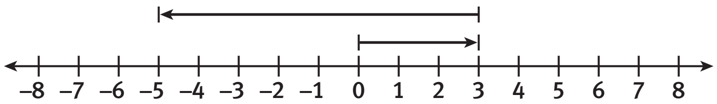
d. $(-28) + (-28)$

5. A scuba diver descended to an elevation of -43 feet, stopped descending, and then descended 17 feet more. What was the diver's final elevation?

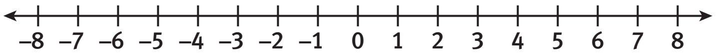
My Notes

You can also use a number line to add two integers with different signs.

6. **Model with mathematics.** Explain how the number line shows the sum of 3 and -8. What is the sum?



7. Use the number line below to find the sum $(-6) + 9$.



$(-6) + 9 =$ ______

Your results can be summarized with this rule:

- To add two integers with different signs, find the difference of the absolute values of the integers. The sum has the sign of the integer with the greater absolute value.

Example C

Add: $-13 + 8$

The signs of the addends are different. Find the difference of the absolute values: $|-13| - |8| = 5$

Use the sign of the integer with the greater absolute value. The integer with the greater absolute value is -13, so the sum is negative.

Solution: $-13 + 8 = -5$

Try These C

Add.

a. $21 + (-14)$ **b.** $11 + (-17)$ **c.** $(-32) + 19$

8. Why are 89 and -89 called opposites? Use a number line to explain.

9. Find the sum of 89 and -89.

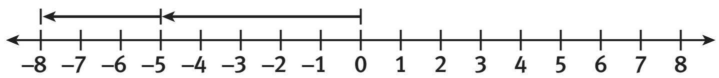
My Notes

10. Why do you think that 89 and −89 are called additive inverses?

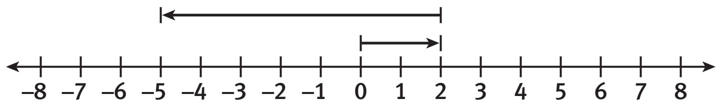
Check Your Understanding

Write the sum shown by the arrows.

11.



12.



13. What is the sum of any integer and its opposite?

14. Identify a real-life situation where opposite quantities combine to make 0.

LESSON 2-1 PRACTICE

Find each sum.

15. Add.

a. $-21 + 25$ **b.** $(-13) + (-21)$

c. $46 + (-58)$ **d.** $(-39) + 16$

e. $28 + (-24) + (-3)$ **f.** $15 + (-42) + (-5)$

16. A mountain climber camped at an elevation of 18,492 feet. The following day the climber descended 2,516 feet to another campsite. Write a numerical expression you can evaluate to find the elevation of the second campsite. Then find the elevation.

17. Explain how to determine if the sum of two integers with different signs is positive or negative.

18. **Reason quantitatively.** If you stood at sea level, the base of the Hawaiian volcano Mauna Kea would be at the bottom of the ocean, at 19,680 feet below you. The top would be 33,476 feet above the base. Write a numerical expression you can evaluate to find the elevation of the top of Mauna Kea above sea level. Then find the elevation.

19. Justify Steps 1 and 2 in the evaluation of the expression $5 + ((-7) + 3) + (-6)$.

Step 1 $5 + ((-7) + 3) + (-6) = 5 + (3 + (-7)) + (-6)$

Step 2 $= (5 + 3) + (-7) + (-6)$

$= 8 + (-13)$

$= -5$

My Notes

Learning Targets:

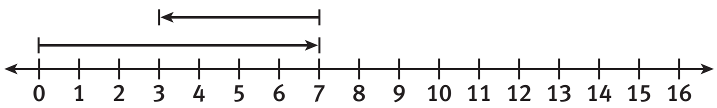
- Subtract integers.
- Find distances using absolute value.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Quickwrite

Long before you knew anything about integers, you were able to subtract whole numbers.

$$7 - 4 = 3$$

You may have used a number line to show subtraction.



1. Compare the above graph with the one you would draw to find the sum $7 + (-4)$.

2. Compare the graph you would draw to find $12 - 5$ (the difference between the whole numbers 12 and 5) and the one you would draw to find the sum $12 + (-5)$.

These examples show that you can convert a subtraction problem to an addition problem: $9 - 2 = 9 + (-2)$. This leads to the rule:

- To subtract an integer, add its opposite.

Example A

Subtract: $-12 - (-5)$

Step 1: To -12, add the opposite of -5. $\quad -12 - (-5) = -12 + 5$

Step 2: Find the difference of the absolute values. $\quad |-12| - |5| = 12 - 5 = 7$

Step 3: Use the sign of the integer with the greater absolute value. $\quad |-12| > |5|$, so $= -12 + 5 = -7$

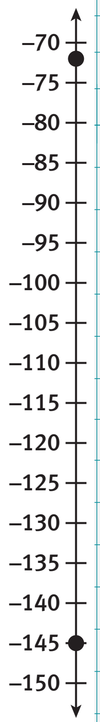
Solution: $-12 - (-5) = -7$.

Try These A

Subtract.

a. $16 - (-4)$ **b.** $7 - 12$ **c.** $-9 - 9$

My Notes



Check Your Understanding

3. Write the subtraction problem as an addition problem.

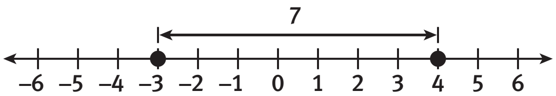
a. $19 - 6$ **b.** $-4 - (-8)$ **c.** $-3 - 5$
d. $0 - 12$ **e.** $13 - 14$ **f.** $-2 - (-2)$

4. Subtract.

a. $14 - 7$ **b.** $-11 - (-7)$ **c.** $-12 - 8$
d. $6 - (-6)$ **e.** $21 - 30$ **f.** $-17 - (-20)$

5. Tristan rewrote the expression $6 - (-8)$ as $6 - (+8)$. Was he correct? Why or why not?

You can find the *distance* between −3 and 4 by counting the number of units from −3 to 4 on a number line. The distance is 7 units.



Another way to find the distance is to find the absolute value of the difference of −3 and 4.

$$|-3 - 4| = |-7| = 7$$

The order of the subtraction does not matter. The result will be the same:

$$|4 - (-3)| = |4 + (+3)| = |7| = 7$$

Example B

A team of divers was at an elevation of 145 feet below the surface of the water, or −145 ft. Another team was directly above the first team at an elevation of −72 ft. What was the distance between the teams?

Step 1: Visualize the problem.

Think of a vertical number line with points at −145 and −72.

Step 2: Write and evaluate an absolute value expression to find the distance.

$$|-145 - (-72)| = |-145 + (72)| = |-73| = 73$$

Solution: The distance between the teams is 73 feet.

Try These B

Find the distance between each pair of numbers.

a. −34 and 7 **b.** −42 and −78 **c.** 29 and 4

My Notes

Check Your Understanding

Write an absolute value expression you can use to find the distance between each pair of numbers. Then find the distance.

6. 15 and −15 **7.** −47 and 53 **8.** −24 and −42

9. Howard needs to find the distance between 178 and −395 on a number line. Write two absolute value expressions Howard can write to find the distance.

LESSON 2-2 PRACTICE

10. At noon, a hot-air balloon was at an elevation of 2,400 feet. One hour later it was at an elevation of 1,700 feet.

a. Write a numerical expression you can use to find the change in altitude from noon to 1 P.M.

b. Evaluate your expression and explain what it means.

11. Yesterday's high temperature was −8°F. Today's high temperature is −3°F.

a. Write a numerical expression you can use to find the change in temperature from yesterday to today.

b. Evaluate your expression and explain what it means.

12. A submarine is at −750 feet, or 750 feet below sea level. It descended 300 feet, then ascended 550 feet, and then descended 425 feet.

a. Write and evaluate a numerical expression to find the submarine's final elevation.

b. The submarine next descended to the ocean floor, which was at an elevation of −2,250 feet. How far did the submarine descend?

13. Reason abstractly. If you subtract a negative number from another number, will the other number increase or decrease? Explain.

14. Justify Steps 1 and 2 in the evaluation of the expression 23 − 48 + 7.

Step 1: $23 - 48 + 7 = 23 + (-48) + 7$

Step 2: $= 23 + 7 + (-48)$

$= 30 + (-48)$

$= -18$

Classify each statement in Items 15–16 as *true* or *false*. If false, explain why.

15. The difference of two integers can never be 0.

16. The difference of two negative integers is always a negative integer.

17. Which expression can you use to find the distance between 19 and −31?

A. |19 − 31| **B.** |19| − |31|

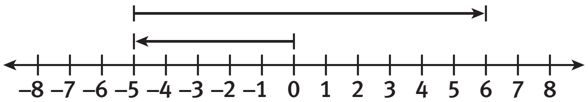
C. |−31 − 19| **D.** |−31| − |19|

ACTIVITY 2 PRACTICE

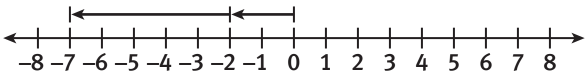
Lesson 2-1

For Items 1–2, write the sum shown by the arrows.

1.



2.



For Item 3–6, draw a number line from −8 to 8. Illustrate the move along the number line to find each sum.

3. $5 + (-7)$

4. $-5 + 3$

5. $-6 + 10$

6. $-2 + (-5)$

For Item 7 and 8, write an addition expression to represent each problem situation. Then solve the problem by finding the sum.

7. At 8:00 A.M., the temperature was −6°F. By noon, the temperature had risen by 9°F. What was the temperature at noon?

8. Jamal reached into a bag and pulled out a handful of counters. He pulled out 16 negative counters and 27 positive counters. What was the combined value of the counters?

Classify each statement in Item 9–10 as *true* or *false*. If false, explain why.

9. The sum of two integers cannot be 0.

10. The sum of two negative integers is always a negative integer.

11. What number must you add to −6 to get a sum of zero? Explain.

In Items 12–16, find each sum.

12. $56 + (-48) + (-30)$

13. $-45 + (-45) + (-45)$

14. $97 + (-112) + 15$

15. $-38 + 7 + 59$

16. $-154 + (-89) + 226$

Lesson 2-2

Write each subtraction problem as an addition problem. Then find the difference.

17. $5 - 7$

18. $4 - (-3)$

19. $-6 - 1$

20. $-2 - (-5)$

For Items 21–22, write a subtraction expression to represent each problem situation. Then solve the problem by finding the difference.

21. At 8:00 P.M., the temperature was 16°F. By midnight, the temperature had fallen by 19°F. What was the temperature at midnight?

22. Gina was touring New Orleans, which has an elevation of 5 feet below sea level, or −5 ft. A helicopter flew over her at an elevation of 186 ft. How far above the ground where Gina was standing was the helicopter?

In Item 23–24, evaluate each expression.

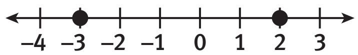
23. $132 - 178 + 59$

24. $-6.75 + 8 - 2.2$

25. What number must you subtract from −13 to get a difference of 0? Explain.

26. Identify a situation involving money where opposites combine to make 0.

27. Write and evaluate an absolute value expression to find the distance between the two points graphed below.



MATHEMATICAL PRACTICES
Model with Mathematics

28. Which expression can you use to find the distance between 28 and −53?

A. $|28 - 53|$
B. $|28| - |53|$
C. $|-53 - 28|$
D. $|-53| - |28|$

Positive Rational Numbers and Adding and Subtracting Integers
OFF TO THE RACES

Write your answers on notebook paper. Show your work.

The Middle School Track and Field Championships are held every year on the last day of school. The table gives the best times and distances in three events from previous years.

Event	Record
100-Meter Run	13.76 sec
400-Meter Run	1 min, 5.21 sec
High Jump	$5\frac{7}{12}$ ft

1. In his three high jumps, Kevin jumped $4\frac{3}{4}$ feet, $4\frac{5}{6}$ feet, and $4\frac{2}{3}$ feet.

a. Find the mean of the heights. Explain how you found the answer.

b. Estimate how much higher than his best jump Kevin would have had to jump to tie the record. Explain how you made your estimate.

c. How much higher than his best jump would Kevin have to jump to tie the record? Find the exact answer.

d. Consider only the fractional parts of the three mixed numbers that make up Kevin's three heights. Find the fractions which, written as decimals, would be repeating decimals, and write them as repeating decimals.

2. Elena completed the 100-meter run in 15.58 seconds.

a. How much faster would she have had to run to tie the record?

b. If she could have run 400 meters at the same rate as she ran 100 meters, would she have broken the record? Find the difference between her time for 400 meters and the record time.

c. The 400-meter run consists of four laps around a 100-meter track. What was the record holder's average time per lap?

Times and distances are sometimes given by comparing them with the record for the event. A negative number indicates the amount by which a record has been broken. A positive number indicates the amount by which the record has failed to be broken.

3. In the discus throw, Devan scored 7, Joel scored +15, and Greg scored the opposite of Devan.

a. By how much did Greg's distance exceed Joel's?

b. Leo's score was 4 less than Greg's. What was Leo's score?

c. Order the scores from greatest to least.

4. Explain how you can use absolute value to compare a score with the record for the event when scores are given as integers.

Positive Rational Numbers and Adding and Subtracting Integers

OFF TO THE RACES

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-d, 2a-c, 3a-c, 4)	• Clear and accurate understanding of operations with fractions, decimals, and integers. • Effective understanding and accuracy in ordering and comparing integers.	• Operations with fractions, decimals, and integers that are usually correct. • Correct comparison of integers by ordering a set or using absolute value.	• Operations with fractions, decimals, and integers that are sometimes correct. • Partially correct comparison or ordering of integers; incorrect use of absolute value.	• Incorrect or incomplete computation in operations with fractions, decimals, and integers. • No comparison or ordering of integers.
Problem Solving (Items 1d, 2a-c, 3a-b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-d, 2a-c, 3a-c, 4)	• Clear and accurately written expressions involving operations with fractions, decimals, and integers. • Clear and correct ordering and comparison of integers. • Correct use of absolute value to compare scores.	• Some difficulty in writing the best expression for a problem situation, but can get correct answers. • Correct conversion of fractions to decimals. • An understanding of ordering integers. • An understanding of absolute value.	• Errors in writing expressions for a given problem situation. • Errors in ordering rational numbers (for example, orders least to greatest instead of greatest to least). • Incorrect use of absolute value to compare scores.	• Inaccurately written expressions. • Inaccurate conversion of fractions to decimals. • Incorrect ordering of rational numbers. • Little or no understanding of absolute value.
Reasoning and Communication (Items 1a-b, 4)	• Precise use of appropriate math terms and language to explain finding a mean and estimating a difference. • A thorough understanding of using absolute value to compare scores.	• An adequate explanation of finding a mean and estimating a difference. • An adequate explanation of how to use absolute value to compare scores.	• A misleading or confusing explanation of finding a mean or estimating a difference. • Partial understanding of absolute value.	• An incomplete or inaccurate description of finding a mean or estimating a difference. • Little or no understanding of absolute value.

Multiplication and Division of Integers

ACTIVITY 3

What's the Sign?

Lesson 3-1 Multiplying Integers

Learning Targets:

- Multiply two or more integers.
- Apply properties of operations to multiply integers.
- Solve real-world problems by multiplying, adding, and subtracting integers.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Summarize/Paraphrase/Retell, Create Representations

Kaleena's brother is a helicopter pilot who performs rescue operations for the Coast Guard. Kaleena is doing research to learn how a helicopter moves up and down. She learns that the helicopter her brother flies takes about 3 minutes to ascend to an altitude of 900 feet from ground level.

1. What is the vertical rate of ascent, in feet per second, when a helicopter ascends 900 feet in 3 minutes?

2. Would it be more appropriate to represent this rate of ascent as a positive integer or a negative integer? Explain your reasoning.

3. What is the vertical rate of descent, in feet per second, when a helicopter descends 900 feet in 5 minutes?

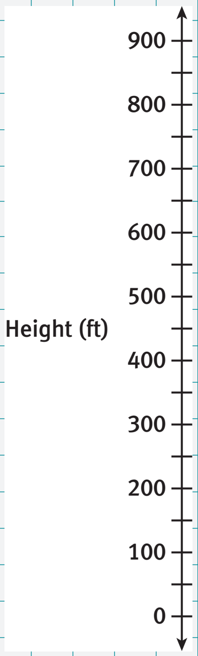
4. Would it be more appropriate to represent this rate of descent as a positive integer or a negative integer? Explain your reasoning.

My Notes

ACADEMIC VOCABULARY

Ascend means to "move upward."
Descend means "to move downward."

My Notes



Kaleena's brother sent her a video of him taking off and ascending. Kaleena decides to represent the ascent of the helicopter using a vertical number line.

5. Use the rate of ascent you found in Item 1. On the number line at the left, mark the height of the helicopter at 10-second intervals, from 0 to 3 minutes.

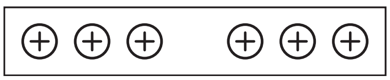
6. Use  to represent the helicopter ascending for 10 seconds at the rate you found in Item 1. Draw a diagram to represent the total ascent of the helicopter.

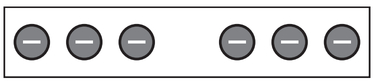
7. Use  to represent the helicopter descending for 10 seconds at the rate you found in item 3. Draw a diagram to represent the total descent of the helicopter.

In Items 6 and 7, you represented multiplication of positive and negative numbers using triangle symbols. You can also use counters to represent multiplication problems.

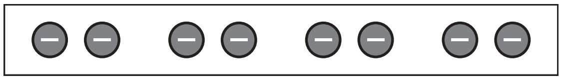
8. If ⊖ represents -10, what does ⊖⊖⊖ represent?

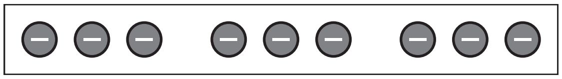
9. Use multiplication to write an equation illustrated by each diagram. Each counter stands for 10.

a. 

b. 

My Notes

c. 

d. 

10. Use your results from Item 9 to answer the following.

a. What is the sign of the product of a positive integer and a positive integer?

b. What is the sign of the product of a positive integer and a negative integer?

11. a. To find the sign of the product of two negative integers, start by filling in all the squares in the multiplication table below except for the 9 shaded squares in the lower right corner of the table.

·	3	2	1	0	−1	−2	−3
3							
2							
1							
0							
−1							
−2							
−3							

b. Now use patterns in the rows and columns you completed to fill in the 9 shaded squares.

12. What patterns did you see in the table that helped you fill in the shaded squares?

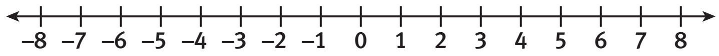
13. What rule can you use to multiply two negative integers?

My Notes

Check Your Understanding

14. Find each product.
a. $-5(7)$ **b.** $9 \cdot 5$ **c.** $-8(-8)$ **d.** $12(-4)$

15. Show how to find the product $4(-2)$ using the number line.



What is the product $4(-2)$?

16. Evaluate.
a. $(-1)^2$ **b.** $(-1)^3$ **c.** $(-1)^4$ **d.** $(-1)^5$
e. Write a rule you can use to evaluate -1 to any power.

LESSON 3-1 PRACTICE

17. A Navy submarine descended from sea level at a rate of 7 feet per second.
a. Write the descent rate as an integer.
b. Find the submarine's elevation after 10 minutes.

18. In golf, *par* is a score of zero. One golfer scored 3 under par each day of a 4-day tournament.
a. Write the golfer's daily score as an integer.
b. Find the golfer's final score for the entire tournament.

19. Frances has no money in her checking account. She writes 3 checks for \$35 each. The bank imposes a \$15 penalty because she has overdrawn her account. How much money is in her account now?

20. A hot-air balloon leaves the ground and ascends at a rate of 6 feet per second for 3 minutes. Then it descends at a rate of 3 feet per second for 2 minutes. Finally, it ascends at a rate of 4 feet per second for 5 minutes. How far above the ground is the balloon now?

21. **Construct viable arguments.** You used a multiplication table to show that the product of two negative integers is positive. The following proof uses a different approach to show that $(-1)(-1) = 1$. Justify each step. You can use the step "Simplify" if necessary.

Step 1: $-1(1 + (-1)) = -1(1) + (-1)(-1)$

Step 2: $-1(1 + (-1)) = -1 + (-1)(-1)$

Step 3: $-1(0) = -1 + (-1)(-1)$

Step 4: $0 = -1 + (-1)(-1)$

Step 5: $1 = (-1)(-1)$

My Notes

Learning Targets:

- Divide integers.
- Solve real-world problems by dividing integers and possibly adding, subtracting, or multiplying integers as well.

SUGGESTED LEARNING STRATEGIES: Think Aloud, Think-Pair-Share, Look for a Pattern

The table gives the elevations of four neighboring California towns. A surveyor wanted to calculate the average elevation of the towns. To do so, the surveyor needed not only to add integers but also to divide them.

City	Elevation (ft)
Coachella	−71
Indio	−9
La Quinta	120
Mecca	−180

Because division is the inverse operation of multiplication, you can use that relationship to find the rules for dividing positive and negative integers.

1. **Make use of structure.** The equation $3 \cdot 4 = 12$ shows that the numbers 3, 4, and 12 are related by multiplication. Write two equations to show that 3, 4, and 12 are related by division.

2. Use the fact that $5(-2) = -10$ to write two equations showing that 5, −2, and −10 are related by division.

3. Use the fact that $(-7)(-3) = 21$ to write two equations showing that −7, −3, and 21 are related by division.

4. Use your results above to complete these statements:

   The quotient of two integers with the same sign is __________.

   The quotient of two integers with different signs is __________.

5. What is the average elevation of the four California towns?

MATH TIP

Remember that you can express division in three ways. For example, the following all mean 12 divided by 3.

$$12 \div 3$$

$$\frac{12}{3}$$

$$3\overline{)12}$$

My Notes

Check Your Understanding

6. Find each quotient.
a. $24 \div (-6)$ **b.** $40 \div 8$ **c.** $-49 \div (-7)$ **d.** $-36 \div 4$

7. Simplify each fraction.
a. $-\frac{25}{5}$ **b.** $\frac{-25}{5}$ **c.** $\frac{25}{-5}$

d. What conclusion can you draw about the placement of the negative sign in expressions like those above?

LESSON 3-2 PRACTICE

8. Find the number that goes in each blank.
a. $14 \times$ ______ $= -266$ **b.** $-23 \times$ ______ $= 345$
c. $18 \times$ ______ $= 306$ **d.** $-11 \times$ ______ $= -341$

9. Evaluate each expression.
a. $-4 \times (-3) \div (-6)$ **b.** $30 \div (-2) \div (-5)$
c. $|4 \times (-15)| \div (-12)$ **d.** $[13 + (-19)] \times (-7) \div (-3)$

10. The temperature of a pot of water fell from 72°F to 36°F in 4 minutes. Find the average change in temperature per minute.

11. The price of one share of stock in BadInvestment.com plunged 14 points in 4 weeks. Find the average change in the stock price per day.

12. The low temperatures in Colton for 5 consecutive days were −8°F, −13°F, −4°F, −9°F, and −16°F. What was the average low temperature for the 5 days?

13. **Reason quantitatively.** Find two integers with a sum of 16 and a quotient of −9.

14. Use a related multiplication equation to show why the equation $\frac{5}{n} = 0$ has no solution.

15. The product of two integers, $m \times n$, is negative. Is $m \div n$, the quotient of the same integers, positive, negative, or impossible to find without knowing the values of m and n? Explain.

ACTIVITY 3 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 3-1

1. $3(-5)$
2. $-12(4)$
3. $0(-6)$
4. $-8(-10)$
5. $13(3)$
6. $7(-1)$

Evaluate.

7. $-8 \cdot |-8|$
8. $|-3| \cdot |-11|$
9. $-|7-13| \cdot (-|13-7|)$
10. $-14 \cdot (-|-5|)$
11. $-5 \cdot |-9| + 3 \cdot |4|$
12. $-|6(-4)|-7|(-3)(-2)|$

Find the number that goes in the blank.

13. $-10 \times$ ______ $= -20$
14. $5 \times$ ______ $=-45$
15. $-12 \times$ ______ $= 84$
16. $9 \times$ ______ $= 99$
17. $90 = -15 \times 3 \times$ ______
18. $-84 = -2 \times (-3) \times$ ______

Write $<$ or $>$ in the box.

19. $-3(-5)$ ☐ $4(-4)$
20. $-5(5)$ ☐ $6(-4)$
21. $8(5)$ ☐ $13(3)$
22. $-7(8)$ ☐ $-11(-5)$

23. In the 3×3 array below, the product of the integers in each row and each column is the same number. The numbers in four of the squares are given. Find the remaining five numbers.

−18	−4	3
		2

24. An airplane descends at a rate of 500 feet per minute. Write and evaluate an expression to show how far the plane will descend in 6 minutes.
25. Starting at sea level, a diver descends into the ocean at a rate of 12 feet per minute. Write and evaluate an expression to show how far the diver will descend in 7 minutes.
26. Between low tide and high tide, the width of a beach changes by −17 feet per hour. Write and evaluate an expression to show how much the width of the beach changes in 3 hours.

State whether the product is positive or negative.

27. $(-3)5$
28. $(-2)10$
29. $(-6)3$
30. $(-11)20$
31. Two numbers, m and n, are integers, with $m < n$. Is it always true that $m^2 < n^2$. Explain your reasoning

Lesson 3-2

Complete the table.

Product	Related Quotients
$3 \cdot 7 = 21$	$21 \div 3 = 7$
	$21 \div 7 = 3$
32. $10(-4) =$ ______	
33. $-5(-9) =$ ______	
34. $-20(6) =$ ______	

Simplify.

35. $\frac{33}{-11}$

36. $-\frac{54}{9}$

37. $\frac{72}{8}$

38. $\frac{-32}{-2}$

39. Which of the following expressions is not equivalent to the others?

A. $\frac{-2}{-3}$ **B.** $-\frac{2}{3}$

C. $\frac{-2}{3}$ **D.** $\frac{2}{-3}$

40. Which expression gives the least product or quotient?

A. $-4(-2)$ **B.** $-3 \cdot 3$

C. $-15 \div 5$ **D.** $-36 \div (-4)$

Evaluate.

41. $64 \div [-8 \div (-2)]$

42. $[64 \div (-8)] \div (-2)$

43. $\frac{-45}{9} \cdot \frac{-15}{-5}$

44. $\frac{100}{-20} \cdot \frac{-15}{-5}$

Find the number that goes in the blank.

45. $-30 \div$ ______ $= -5$

46. $56 \div$ ______ $= -8$

47. $48 \div$ ______ $= 16$

48. $-76 \div$ ______ $= 19$

49. $3 = 48 \div (-4) \div$ ______

50. $-2 = -100 \div 10 \div$ ______

Write $<$ or $>$ in the box.

51. $32 \div (-8)$ ☐ $-5 \div (-1)$

52. $-60 \div 4$ ☐ $32 \div (-2)$

53. $0 \div (-49)$ ☐ $49 \div (-1)$

54. $33 \div (-33)$ ☐ $-32 \div 16$

55. Explain how multiplication and division are related.

56. Over the past five weeks, the average daily temperature in Wellington has dropped 40 degrees Fahrenheit. Write and evaluate an expression to show the average temperature change per week.

57. The high temperatures in Weston for 7 consecutive days were −14°C, −10° C, −3°C, 6°C, 8°C, −4°C, and −11°C. What was the average high temperature for the 7 days?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

58. Is there a greatest integer value for x that makes the inequality $\frac{x}{-5} > 4$ true? If so, what is it? Explain your reasoning.

Operations on Rational Numbers

ACTIVITY 4

Let's Be Rational!
Lesson 4-1 Sets of Rational Numbers

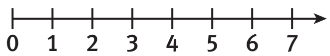
Learning Targets:

- Given a rational number, determine whether the number is a whole number, an integer, or a rational number that is not an integer.
- Describe relationships between sets of rational numbers.

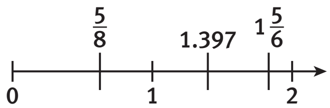
SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Think-Pair-Share, Create Representations

The history of numbers is the story of the gradual filling in of the number line. Ancient peoples had no concept of zero and needed numbers only to count items, such as cattle. Their number line consisted of the *natural numbers* 1, 2, 3, . . .

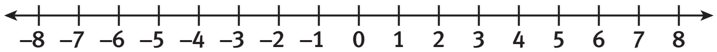
The idea of zero occurred to the ancient Babylonians as well as to the Mayans of Mesoamerica. Adding zero to the natural numbers on the number line creates the set of *whole numbers.*



Points between the whole numbers were known to the ancient Greeks. They comprise *fractions, decimals* and *mixed numbers.*



Negative numbers have been used in China and India for more than a thousand years. They did not come into wide use in Europe until the 17th century. The whole numbers and their negative-number opposites form the set of *integers*.



These three sets of numbers are **subsets** of the set of **rational numbers**. A rational number is a number that can be expressed as a ratio $\frac{a}{b}$, where both a and b are integers and $b \neq 0$. The number -5, for example, can be expressed as the ratio $\frac{-15}{3}$.

My Notes

WRITING MATH

Use *ellipses*—three periods in a row—to represent all the numbers in an infinite sequence. For example, 0, 1, 2, 3, 4, 5, … represents the unending sequence of whole numbers.

MATH TERMS

A **subset** is a set whose elements are all in the original set. Every set is a subset of itself.

A **rational number** is a number that can be expressed as a ratio $\frac{a}{b}$, where both a and b are integers and $b \neq 0$.

My Notes

1. Show that each number is a rational number by expressing it as a ratio of two integers.

a. 27 **b.** 0.75 **c.** $4\frac{2}{3}$

d. -9 **e.** -0.43 **f.** -1.8

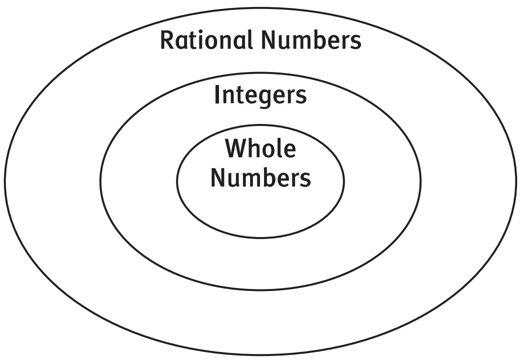
2. Classify each rational number as a whole number, as an integer, or as a rational number that is not an integer.

a. -34 **b.** 1.57

c. 0 **d.** $\frac{13}{14}$

3. The Venn diagram shows the relationships among whole numbers, integers, and rational numbers. Write the following numbers in their correct places in the diagram:

$-13, 11\frac{9}{10}, 4.78, -803, -7\frac{5}{6}, 0, \frac{17}{3}, -91.55, -45$



4. **Reason abstractly.** Tell whether each statement is *true* or *false*. Explain why any false statements are false.

a. If *n* is an integer, then *n* is a whole number.

b. There are no rational numbers that are also whole numbers.

c. All rational numbers are integers.

d. A number cannot be both a whole number and a rational number.

My Notes

Check Your Understanding

5. Place a checkmark in the box for any set of which the given number is a member.

Number	Whole Number	Integer	Rational Number
0.25			
3.14159			
−12			
0			
−0.333 . . .			
$5\frac{9}{10}$			
29,116			
$-2\frac{1}{89}$			

6. Tell whether each statement is *never*, *sometimes* or *always* true.
a. An integer is a whole number.
b. A whole number is a rational number.
c. A rational number is a whole number.

LESSON 4-1 PRACTICE

7. Name all the sets of which the given set is a subset.
a. the set of whole numbers
b. the set of positive integers
c. the set of negative rational numbers
d. the set of natural numbers

8. Explain why 2 is a rational number.

9. Reason abstractly. Why does the definition of rational number state that b, the denominator of the rational number $\frac{a}{b}$, cannot equal 0?

10. Construct viable arguments. A rational number is defined as a ratio of two integers. Given that a ratio is a fraction, how can a decimal be a rational number?

11. Explain why the set of mixed numbers is not a subset of the set of integers.

My Notes

Learning Targets:

- Add two or more rational numbers.
- Use properties of addition to add rational numbers.
- Solve real-world problems by adding two or more rational numbers.

SUGGESTED LEARNING STRATEGIES: KWL Chart, Think Aloud, Create Representations

When you add rational numbers, use the same rules for determining signs as you used to add integers.

MATH TERMS

A **common denominator** is a common multiple of two or more denominators.

Example A

Julia needed to do some repainting around her pool so she drained $4\frac{1}{2}$ feet of water. After painting, she added $1\frac{2}{3}$ feet of water. How far below its original level did she leave the water in order to let the paint dry?

Step 1: $-4\frac{1}{2}+1\frac{2}{3}=-\frac{9}{2}+\frac{5}{3}$ Write the mixed numbers as improper fractions.

Step 2: $=-\frac{27}{6}+\frac{10}{6}$ Write the fractions with a common denominator.

Step 3: $=-\frac{17}{6}$ Add using the rules for adding integers.

Step 4: $=-2\frac{5}{6}$ Write the improper fraction as mixed number.

Solution: Julia left the water $2\frac{5}{6}$ feet below its original level.

Try These A

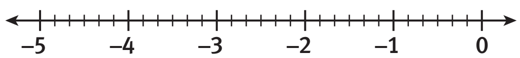
Find each sum.

a. $-5\frac{5}{6}+2\frac{1}{4}$

b. 14.62 + (–19.3)

c. Make sense of problems. Explain how the sum was found in Example A, Step 3.

1. Model with mathematics. Show how the final water level can be found using a number line.



My Notes

Example B

The water level in the Blue River was already 1.75 meters below normal when a drought caused the level to fall an additional 2.5 meters. What was the water level after the drought?

Step 1: $-1.75 - 2.5 = |-1.75| + |-2.5|$ Add using the rules for adding integers.

Step 2: $= 1.75 + 2.5$ Write the absolute values.

Step 3: $= 4.25$ Add.

Step 4: $= -4.25$ Use the sign of the addends.

Solution: The water was 4.25 meters below normal after the drought.

Try These B

a. $-420.5 - 98.6$

b. $-\frac{4}{15} + \left(-\frac{3}{10}\right)$

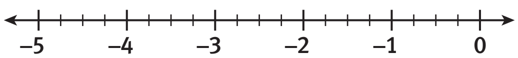
CONNECT TO SCIENCE

A *drought* is a long period of unusually low rainfall, especially one that causes extensive damage to crops.

2. Explain Step 1 of Example B.

3. How do you know that the final water level was below normal rather than above normal?

4. **Model with mathematics.** Show how the final water level can be found using a number line.



My Notes

Check Your Understanding

5. Find each sum.

a. $\frac{5}{12}+\left(-\frac{11}{12}\right)$

b. $3\frac{3}{8}+2\frac{1}{4}$

c. $-\frac{7}{15}+\left(-\frac{4}{5}\right)$

d. $-3.49+7.22$

e. $12.5+(-21.32)$

f. $-36.91+(-16.7)$

g. $\frac{1}{6}+\left(-\frac{11}{12}\right)+\frac{2}{3}$

h. $29+(-15.7)+(-31.05)$

i. Describe a possible real-world context for the expression in Item 5b.

j. Describe a possible real-world context for the expression in Item 5d.

6. a. Describe how to use the Commutative Property of Addition to simplify finding this sum:

$$\frac{9}{20}+(-2.45)+\left(-\frac{3}{5}\right)+6.7$$

b. Use the Commutative Property to find the sum.

LESSON 4-2 PRACTICE

7. Olympic swimming pools are rectangles measuring 164.042 feet in length and 82.021 feet in width. What is the perimeter of an Olympic pool?

8. Starting at sea level, a kingfisher flew to an elevation of $37\frac{1}{4}$ feet. Spotting a fish below, the bird descended $41\frac{5}{6}$ feet and caught the fish.

a. Write a numerical expression involving addition that you can use to find the elevation of the fish.

b. What was the elevation of the fish?

9. The lowest temperature ever recorded on Earth's surface was −128.5°F. The highest temperature was 262.5°F higher than the lowest.

a. Write a numerical expression involving addition that you can use to find the highest temperature.

b. What was the highest temperature ever recorded?

10. Make sense of problems. Justify Step 1 in the following evaluation:

Step 1: $-2.79+\left((-3.91)-5\frac{1}{2}\right)=\left(-2.79+(-3.91)\right)-5\frac{1}{2}$

Step 2: $=-6.7-5.5$

Step 3: $=-12.2$

My Notes

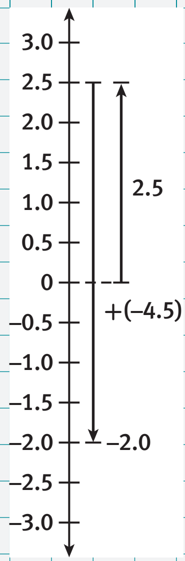
Learning Targets:

- Subtract rational numbers.
- Apply the fact that for all rational numbers a and b, $a - b = a + (-b)$, to add and subtract rational numbers.
- Solve real-world problems by subtracting rational numbers and possibly by adding rational numbers as well.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share

Recall that you can subtract an integer by adding its opposite. The number line at the right illustrates $2.5 + (-4.5)$ and shows that the same rule applies to subtracting rational numbers: $2.5 - 4.5 = -2$.

- To subtract a rational number, add its opposite.



Example

As the Yellowstone River flows through Yellowstone National Park, it breaks into two waterfalls. At the Upper Falls, the river drops 33.22 meters. At the Lower Falls, it drops 93.88 meters. Find the river's total change in elevation as it passes the two falls.

Subtract: $-33.22 - 93.8$

Step 1:	To -33.22, add the opposite of 93.8	$-33.22 - 93.88 = -33.22 + (-93.88)$
Step 2:	The signs are the same so find the sum of the absolute values.	$\lvert -33.22 \rvert + \lvert -93.88 \rvert = 33.22 + 93.88 = 127.1$
Step 3:	Use the sign of the addends:	-127.1

Solution: The river's total change of elevation is -127.1 meters.

Try These

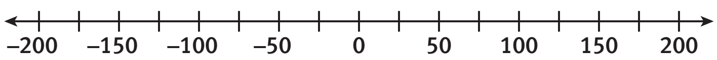
a. $-4.13 - (-5.46)$

b. $\frac{5}{12} - \frac{7}{8}$

CONNECT TO HISTORY

Established in northwestern Wyoming in 1872, Yellowstone National Park was America's first national park.

1. **Model with mathematics.** Draw arrows on the number line below to show the changes in the river's elevation at the Upper Falls and Lower Falls.



My Notes

Check Your Understanding

2. Write each subtraction problem as an addition problem.

a. $\frac{2}{3} - \frac{4}{5}$

b. $-0.08 - 3.62$

c. $-7\frac{3}{8} - \left(-2\frac{1}{4}\right)$

d. $527.4 - (-748.62)$

3. Subtract.

a. $\frac{9}{13} - \frac{11}{13}$

b. $-57.49 - (-35.7)$

c. $7\frac{3}{10} - \left(-4\frac{1}{5}\right)$

d. $-319.12 - 88.16$

LESSON 4-3 PRACTICE

4. Yosemite Falls in Yosemite National Park drops in three separate sections.

Name	Height (m)
Upper Falls	435.86
Middle Cascades	205.74
Lower Falls	97.54

a. Write a numerical expression you can use to find the total change in elevation.

b. What order of operation rule must you use to evaluate the expression?

c. What is the total change in elevation from the top of the falls to the bottom?

5. On January 22, 1943, the temperature in Spearfish, South Dakota, fell from 53.6°F to −4°F in just 27 minutes.

a. Write a numerical expression you can use to find the change in temperature.

b. Evaluate your expression.

c. What was the mean change in temperature per minute? Write your answer using bar notation.

6. **a.** Describe two ways to find the difference $\frac{13}{50} - 4.93$.

b. Which way is better? Explain your reasoning.

7. **Construct viable arguments.** Is the difference between two rational numbers always as rational number? Why or why not?

8. Jodi is finding the sum $4 + (-6.5)$ on a number line.

a. What is the distance from 4 to the sum?

b. Is the sum to the left or to the right of 4 on the number line? How do you know?

c. What is Jodi's sum?

My Notes

Learning Targets:

- Multiply and divide rational numbers.
- Apply properties of operations to multiply and divide rational numbers.
- Solve real-world problems involving the four operations with rational numbers.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share

Engineers at an underwater oil drilling operation drilled beneath the ocean floor an average of −734.8 meters per day for four consecutive days. What was the total change in elevation from the ocean floor to the deepest point drilled?

To answer this question, you can use addition of rational numbers.

$$(-734.8) + (-734.8) + (-734.8) + (-734.8) = -2{,}939.2$$

You could also multiply 4(−734.8).

1. The above methods work if all the numbers are the same, but suppose you want to find the product 7.9(−3.5). One way would be to use the following method. Write a reason for each step.

$0 = 7.9(0)$	**a.** ______________
$0 = 7.9(3.5 + (-3.5))$	**b.** ______________
$0 = 7.9(3.5) + 7.9(-3.5)$	**c.** ______________
$0 = 27.65 + 7.9(-3.5)$	**d.** ______________
$-27.65 = 7.9(-3.5)$	**e.** ______________

CONNECT TO HISTORY

The first underwater oil wells were drilled in 1891, in Ohio's Grand Lake St. Mary's, a freshwater lake. The first saltwater wells were drilled five years later, in California's Santa Barbara

The last step shows that 7.9(−3.5) = −27.65. Not only does this give the product, but it establishes an important fact about multiplication of rational numbers:

- The product of two rational numbers having *different* signs is negative.

2. Find each product.

a. $-2.5(6.7)$

b. $\frac{2}{3}\left(-\frac{9}{10}\right)$

c. $-\frac{11}{12}\left(\frac{4}{7}\right)$

d. $9.02(-3.4)$

My Notes

You already know that the product of two positive rational numbers is positive. What about the product of two negative rational numbers? You can find out using a method like the one used in Item 1 above for numbers with different signs.

3. **Make sense of problems.** Find the product $-\frac{2}{3}\left(-\frac{5}{7}\right)$. Write a reason for each step.

$0 = -\frac{2}{3}(0)$ **a.** ______________________

$0 = -\frac{2}{3}\left(\frac{5}{7} + \left(-\frac{5}{7}\right)\right)$ **b.** ______________________

$0 = -\frac{2}{3}\left(\frac{5}{7}\right) + \left(-\frac{2}{3}\left(-\frac{5}{7}\right)\right)$ **c.** ______________________

$0 = -\frac{10}{21} + \left(-\frac{2}{3}\left(-\frac{5}{7}\right)\right)$ **d.** ______________________

$\frac{10}{21} = -\frac{2}{3}\left(-\frac{5}{7}\right)$ **e.** ______________________

The last step shows that $-\frac{2}{3}\left(-\frac{5}{7}\right) = \frac{10}{21}$. You already know that the product of two positive rational numbers is positive.

The multiplication of two negative rational numbers, shown above, establishes this important fact:

- The product of two rational numbers having the *same* sign is positive.

4. **Make use of structure.** Why must the rules for finding the signs when you multiply two integers be the same as the rules for finding the signs when you multiply two rational numbers?

5. State the sign of each product.
   a. three negative rational numbers
   b. four positive rational numbers and one negative rational number
   c. three positive rational numbers
   d. one positive and two negative rational numbers
   e. thirteen negative rational numbers
   f. five positive and four negative rational numbers

My Notes

6. Find each product.

a. $-\frac{11}{12}\left(\frac{4}{7}\right)$ **b.** $9.02(-3.4)$

c. $-2.5(6.7)$ **d.** $\frac{2}{3}\left(-\frac{9}{10}\right)$

You can use inverse operations to find the sign of the quotient of two rational numbers.

7. Use the facts that $7.2(-3.5) = -25.2$ and that $-7.2(-3.5) = 25.2$ to complete these equations:

a. $\frac{-25.2}{7.2} =$ **b.** $\frac{25.2}{-7.2} =$

8. Use the results to complete this statement:

The quotient of two rational numbers with different signs is ________.

9. Use the facts from Item 7 to complete this equation:

$$\frac{-25.2}{-7.2} =$$

10. Use your results and your knowledge of the quotient of two positive numbers to complete this statement:

The quotient of two rational numbers with the same sign is ________.

11. Compare the rules for finding the signs of the products and the signs of the quotients of two rational numbers.

12. A well-drilling crew drilled these distances into Earth's crust on four successive days, beginning at the bottom of the ocean:

$-1{,}574\frac{1}{4}$ feet, $-1{,}289\frac{1}{2}$ feet, $-1{,}719\frac{3}{4}$ feet, $-1{,}400\frac{1}{2}$ feet

What was the mean daily change in elevation of the bottom of the well?

13. Find each quotient.

a. $-60.48 \div 4.8$ **b.** $-\frac{1}{5}\left(-\frac{3}{10}\right)$

c. $\frac{11}{24} \div \left(-\frac{5}{8}\right)$ **d.** $1.376 \div 0.8$

MATH TIP

You can use the rules for finding the signs of the sums, differences, products, and quotients of two integers to find the sign of the sum, difference, product, or quotient of any two rational numbers.

My Notes

Check Your Understanding

14. *m* and *n* are positive rational numbers.
  a. What is the sign of their product?
  b. What is the sign of their quotient?

15. *m* and *n* are negative rational numbers.
  a. What is the sign of their product?
  b. What is the sign of their quotient?

16. *m* and *n* are rational numbers with different signs.
  a. What is the sign of their product?
  b. What is the sign of their quotient?

LESSON 4-4 PRACTICE

17. Make use of structure. Which of the following fractions are equal to −7?

$\frac{-7}{1}, \frac{7}{1}, \frac{-7}{-1}, -\frac{7}{1}, \frac{-7}{-1}, -\frac{-7}{1}, -\frac{7}{-1}, \frac{7}{-1}, -\frac{-7}{-1},$

18. The low temperatures for one week in Scottsburg, IN are given below. What was the mean daily low temperature for the week?

−7.9°F, −10.3°F, −3.4°F, 2.6°F, 4.9°F, 11.0°F, −2.5°F

19. Margo's grade average in math was 92. Then for seven months, her average dropped an average of five-eighths of a point per month.
  a. Write a rational number expression involving addition that you can evaluate to find her average at the end of seven months.
  b. What was her final average?

20. Given a temperature in degrees Fahrenheit, the formula $C = \frac{5}{9}(F - 32)$ can be used to find the corresponding Celsius temperature. Find the Celsius temperatures corresponding to the following Fahrenheit temperatures.
  a. 113°F **b.** 32°F
  c. −25°F **d.** −40°F

21. Construct viable arguments. Two rational numbers are each less than 1. Is their product less than 1? Why or why not? Give examples to support your answer.

ACTIVITY 4 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 4-1

1. Place a checkmark in the box for any set of which the given number is a member.

Number	Whole Number	Integer	Rational Number
-2			
10.5			
0			
9			
0.9812			
$2\frac{15}{17}$			
-68.555			
$-0.787878\ldots$			

2. Which statement is false?

A. A whole number is always a rational number.
B. An integer is always a whole number.
C. A number that can be expressed as a ratio $\frac{a}{b}$, where both a and b are integers and $b \neq 0$, is always a rational number.
D. A whole number is always an integer.

3. Give an example of each.

a. an integer that is not a whole number
b. a rational number that is not an integer
c. a rational number that is not a whole number

4. Explain how you know that each number is a rational number.

a. $3\frac{8}{9}$ **b.** -25
c. 1.479 **d.** -6.01

5. Is 0 a rational number? Why or why not?

Lesson 4-2

6. Find the value of each expression.

a. $\frac{13}{16}+\left(-\frac{3}{4}\right)$ **b.** $7\frac{2}{3}+6\frac{1}{4}$
c. $-\frac{7}{20}+\left(-\frac{2}{5}\right)$ **d.** $-6.98+2.75$
e. $\frac{2}{3}+\left(-\frac{5}{8}\right)+\left(-\frac{1}{6}\right)$
f. $29+(-15.7)+(-31.05)$
g. Describe a possible real-world context for the expression in item 6a.
h. Describe a possible real-world context for the expression in item 6d.

7. Bette had \$452.13 in her checking account. She wrote checks for \$53.15 and \$117.48.

a. Write an expression involving addition that you can evaluate to find the amount that remained in Bette's account.
b. Evaluate the expression.

8. Which property is illustrated by the following equation?

$\frac{3}{5}+\left(-\frac{7}{8}\right)+\frac{4}{5}=\frac{3}{5}+\frac{4}{5}+\left(-\frac{7}{8}\right)$

A. Commutative Property of Addition
B. Addition Property of Equality
C. Associative Property of Addition
D. Identity Property of Addition

9. The lowest point on Earth's surface is the shore of the Dead Sea, elevation $-1{,}344.99$ meters. The highest point, the summit of Mount Everest, is 30,380.42 meters above the Dead Sea. What is the elevation at the summit of Mount Everest?

10. Find each sum.

a. $\frac{7}{20}+(-4.8)+\left(-\frac{4}{5}\right)+4.9$
b. $5.6-1\frac{3}{8}+(-3.9)+2\frac{3}{4}$

Lesson 4-3

11. Write each subtraction problem as an addition problem.

a. $\frac{7}{8}-\frac{9}{10}$ **b.** $-6.39-10.4$

c. $5\frac{5}{9}-\left(-8\frac{3}{5}\right)$ **d.** $0.45-(-1.3)$

12. Find the value of each expression.

a. $\frac{5}{12}-\frac{2}{3}$ **b.** $-2.81-(-1.77)$

c. $12\frac{9}{16}-\left(-13\frac{1}{24}\right)$ **d.** $-46.03-21.7$

e. $-9.77-14.52-(-61.2)$

f. $\frac{5}{6}-\frac{7}{9}-\frac{1}{2}$

13. The elevation of the deepest point in the Pacific Ocean is −11,033 meters. The elevation of the deepest point in the Atlantic Ocean is −8,648 meters.

a. Write a subtraction expression you can use to find how much deeper the Pacific Ocean's deepest point is than that of the Atlantic Ocean's.

b. Evaluate your expression.

14. Greg borrowed $100 from his parents. After he did some chores, they reduced the amount of his debt by $25.

a. Let −100 represent the amount Greg owed his parents before he did chores. Write a subtraction expression you can use to find the amount Greg still owes his parents.

b. Evaluate your expression.

15. Is there a Commutative Property of Subtraction for rational numbers? Why or why not? Use examples to support your answer.

Lesson 4-4

16. Find each product or quotient.

a. $-\frac{5}{9}\left(\frac{3}{10}\right)$ **b.** $0.55(-2.6)$

c. $-25.28 \div 3.2$ **d.** $-\frac{3}{8}\div\left(-\frac{9}{16}\right)$

e. $-0.4(0.7)$ **f.** $52\left(-\frac{7}{13}\right)$

g. $1\frac{3}{4}\div\left(-4\frac{3}{8}\right)$ **h.** $2.4 \div 48$

i. $(1.8)\left(-1\frac{2}{5}\right)$ **j.** $(-9.6)\div\left(-3\frac{1}{5}\right)$

17. A glacier that was 1,076 meters thick changed in thickness at an average rate of −22.7 meters per year for 7 years.

a. Write an addition expression you can use to find the glacier's thickness after 7 years.

b. Evaluate your expression.

18. In golf, a player's score on each hole is always an integer. The more negative the score, the better it is. A golfer's combined score for the 18 holes is −5. The golfer scored −2 on each of several holes. On all the other holes the golfer scored a combined total of +1. On how many holes did the golfer score −2?

19. Naief is finding the sum $-7+4\frac{3}{4}$ on a number line.

a. What is the distance from −7 to the sum?

b. Is the sum to the left or right of −7 on the number line? How do you know?

c. What is Naief's sum?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

20. In the 3 × 3 array below, the product of the rational numbers in each row, in each column, and in each diagonal is the same number. The numbers in four of the squares are given. Find the remaining five numbers.

		−0.4
−3.6	−0.8	0.6

Rational Number Operations and Multiplying and Dividing Integers

TOP TO BOTTOM

Embedded Assessment 2
Use after Activity 4

Write your answers on a separate sheet of paper. Show your work.

The diagram at the right shows the approximate elevations of the tops and bottoms of the layers of the atmosphere (the envelope of gas above the Earth) and the zones of the ocean.

Elevations of Atmosphere Layers and Ocean Zones

10,000 km --------------------
Exosphere
600 km ------------------------
Thermosphere
90 km ------------------------
Mesosphere
50 km ------------------------
Stratosphere
20 km ------------------------
Troposphere
0 km ------------ SEA LEVEL
Epipelagic
–0.2 km ----------------------
Mesopelagic
–1 km ------------------------
Bathypelagic
–4 km ------------------------
Abyssopelagic
–6 km ------------------------
Hadalpelagic
–10.9 km ---DEEPEST POINT OF THE OCEAN

1. a. Write a subtraction expression you can use to find the difference between the elevation at the top of the exosphere and the deepest point of the ocean.
b. Write your expression as an addition expression.
c. Evaluate the expression.

2. a. How many times as thick as the ocean's epipelagic zone is the hadalpelagic zone?
b. Explain how you found the answer.

3. An airplane flew over the ocean at an elevation 7.9 kilometers below the top of the troposphere. A wheel came off and fell a total of 16.9 kilometers.
a. In which ocean zone did the wheel come to rest?
b. How far above the elevation of the deepest point in the ocean was the wheel when it stopped?

As you move upward through the lowest three layers of the atmosphere, the air grows thinner and thinner. This causes air temperatures to grow colder and colder. An average temperature at the bottom of the troposphere might be 65°F. The temperature at the top of the mesosphere might be 250°F colder than that.

4. Find the colder temperature at the top of the mesosphere.

In the thermosphere, the heat of the sun overcomes the thinness of the air and causes temperatures to rise dramatically. The hottest temperature at the top of the thermosphere can be 3700°F hotter than the temperature you found in Item 4.

5. a. Find the hotter temperature at the top of the thermosphere.
b. How many times as hot as the temperature at the top of the mesosphere is the temperature at the top of the thermosphere?

In 2012, film director James Cameron descended to the bottom of the Mariana Trench, the deepest point of the ocean, in a submarine called the *Deepsea Challenger.* The descent took 2 hours and 36 minutes.

6. a. Write the depth of the Mariana Trench and Cameron's descent time as mixed numbers.
b. Use the mixed numbers to find the average rate of descent of the *Deepsea Challenger.* Show your work. Round your answer to the nearest tenth.
c. The submarine ascended to the ocean surface in 70 minutes. Use any method you choose to find the average rate of ascent. Round your answer to the nearest tenth.

Rational Number Operations and Multiplying and Dividing Integers

TOP TO BOTTOM

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-c, 2a-b, 3a-b, 4, 5a-b, 6a-c)	• A clear and accurate understanding of operations with rational numbers and integers.	• Operations with rational numbers and integers that are usually correct.	• Operations with rational numbers and integers that are sometimes correct.	• Incorrect or incomplete computation of operations with rational numbers and integers.
Problem Solving (Items 1a-c, 2a-b, 3a-b, 4, 5a-b, 6a-c)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps that result in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-b, 2a, 3a-b, 4, 5a-b, 6a-c)	• Clear and accurately written expressions involving operations with rational numbers and integers that result in a correct answer.	• Some difficulty in writing the best expression for operations on rational numbers and integers, but with correct answers.	• Errors in writing expressions for operations on rational numbers and integers.	• Inaccurately written or missing expressions for operations on rational numbers and integers.
Reasoning and Communication (Item 2b)	• Precise use of appropriate math terms and language when explaining the process of dividing integers.	• An adequate explanation of the process of dividing integers.	• A misleading or confusing explanation of the process of dividing integers.	• Incomplete or inaccurate explanation of the process of dividing integers.

Expressions and Equations

2

Unit Overview

In this unit, you will create and solve linear equations and inequalities from tables, graphs, and verbal descriptions. You will represent equations and inequalities on number lines.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- palindrome
- media

Math Terms

- property
- numerical statement
- algebraic expression
- coefficient
- equation
- numerical expression
- variable
- algebraic statement
- constant

ESSENTIAL QUESTIONS

? Why is it important to understand how to solve linear equations and inequalities?

? How can graphs be used to interpret solutions of real-world problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 6 and 7, will give you an opportunity to demonstrate how you will use multiple representations to write and solve linear equations and inequalities.

Embedded Assessment 1:

Embedded Assessment 2:

UNIT 2

Getting Ready

Write your answers on notebook paper. Show your work.

1. A car travels 50 miles per hour.
   a. Complete the table below to show the total distance traveled for each time given.

Number of hours that have passed	Total distance traveled
1	
2	
3	

   b. Plot the data from the table.
   c. If the car has traveled n hours, write an expression for the total distance traveled.
   d. How far has the car traveled after 10 hours? Explain how you determined your answer.

2. Solve each equation below.
   a. $3x = 12$
   b. $x + 5 = -4$
   c. $2x - 5 = 7$

3. Give 3 examples of integers which are
   a. greater than -2
   b. less than or equal to 1

4. Tell 3 numbers that are less than 2 and greater than -1.

5. Evaluate each of the following
   a. 2^3 b. 3^2 c. $3 + 4 \times 2$ d. $2 \times 3 + 4$

6. Write an algebraic expression to represent each of the following.
   a. a number increased by 7
   b. 8 times a number
   c. 6 less than 3 times a number

7. Two measures of two angles of a triangle are $68°$ and $70°$. Explain how to find the measure of the third angle.

8. The Harris family is planning to buy a new 46-inch HDTV that costs $488. Mr. and Mrs. Harris will pay $200 and their three sons will split the remaining cost equally. Explain how to find the amount each of the boys will pay.

Properties of Operations

What's in a Name?

Lesson 5-1 Applying Properties of Operations

Learning Targets:

- Identify properties of operations.
- Apply properties of operations to simplify linear expressions.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Work Backward, Think-Pair-Share, Note Taking, Sharing and Responding

Words and numbers that have the property of being read the same forward and backward are called **palindromes.** The names Hannah and Otto are two examples of names that are palindromes.

1. What other names or words do you know that are palindromes?

Numbers and operations have interesting **properties** as well.

2. Determine if each statement below is true or false.
   a. $3 + 5 = 5 + 3$
   b. $3 \cdot 5 = 5 \cdot 3$
   c. $2 + (-2) = 0$
   d. $10 \div 2 = 2 \div 10$
   e. $6 - 3 = 3 - 6$
   f. $2(5 \cdot 3) = (2 \cdot 5)3$
   g. $5 + 0 = 0$
   h. $2\left(\frac{1}{2}\right) = 0$
   i. $2 + (4 + 5) = (2 + 4) + 5$
   j. $1 \cdot 3 = 3$

3. Which of the true statements above are similar to a palindrome?

Some properties of operations are listed below. As you share ideas about the information given in the table and throughout this lesson, ask your group members or your teacher for clarification of any language, terms, or concepts that you do not understand.

Property	Example
Additive Identity Property	$12 + 0 = 0 + 12 = 12$
Multiplicative Identity Property	$5 \times 1 = 1 \times 5 = 5$
Commutative Property of Addition	$7 + 3 = 3 + 7$
Commutative Property of Multiplication	$20 \times 4 = 4 \times 20$
Associative Property of Addition	$(9 + 4) + 17 = 9 + (4 + 17)$
Associative Property of Multiplication	$(10 \times 5) \times 3 = 10 \times (5 \times 3)$
Additive Inverse Property	$6 + (-6) = 0$
Multiplicative Inverse Property	$4\left(\frac{1}{4}\right) = 1$

My Notes

ACADEMIC VOCABULARY

A **palindrome** is a word, phrase, or sequence that reads the same backward or forward.

MATH TERMS

A **property** is a rule or statement that is always true.

A **numerical expression** is an expression that contains numbers and operations. For example, $12 + 0$ and $(10 \times 5) \times 3$ expression that contains numbers and operations.

A **numerical statement** is an equation that sets two numerical expressions equal. For example, $20 \times 4 = 4 \times 20$.

My Notes

4. Reason abstractly. The first seven properties listed in the table on the previous page are true for all numbers. The Multiplicative Inverse Property is true for all numbers except 0. Why does 0 not have a multiplicative inverse?

5. Reason abstractly. Do the commutative and associative properties apply for the operations of subtraction and division? Justify your response with numeric examples.

6. The number 0 is called the additive identity element because when you add 0 to a given number, the identical given number is the result. Explain why 0 is not the multiplicative identity element.

7. A hiker walked 2.75 miles directly east and then walked 2.75 miles directly west. Use the Additive Inverse Property to explain why he ends up back where he started.

You can illustrate the properties using algebraic expressions and equations.

Example A

Write an algebraic statement of the Commutative Property of Multiplication.

Step 1: Choose a variable to represent the first number.
Let $a =$ the first number

Step 2: Choose a variable to represent the second number.
Let $b =$ the second number

Step 3: Write an algebraic statement of the property.
$a \cdot b = b \cdot a$

Solution: An algebraic statement of the Commutative Property of Multiplication is $a \cdot b = b \cdot a$

Try These A

State the property illustrated by each algebraic statement.

a. $a + b = b + a$
b. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
c. $a \cdot 1 = a$
d. $a + 0 = a$
e. $a\frac{1}{a} = 1; a \neq 0$

VOCABULARY MATH TERMS

A **variable** is a letter used in place of a number.

MATH TERMS

An **algebraic expression** is an expression that contains one or more variables, such as $4a + 7$.

An **algebraic statement** is an equation or inequality that contains variables, such as $(a + b) + c = a + (b + c)$.

My Notes

Check Your Understanding

8. Explain how you can use properties of operations as strategies to evaluate each expression using mental math. Then evaluate the expression.
a. $0.6 + 12.7 + 1.4 + 2.3$
b. $4.5 + 3.9 + (-0.5) + (-4)$
c. $8\frac{3}{5} - 9\frac{1}{2} + 1\frac{2}{5}$
d. $4\frac{5}{8} + 6\frac{7}{8} - 3\frac{5}{8}$

9. Name the property of operations illustrated by each statement.
a. $x \cdot 4y = 4x \cdot y$
b. $100 \cdot 1 = 100$
c. $x(3 \cdot 2x) = (x \cdot 3) \cdot 2x$
d. $3 + (-3) = 0$
e. $-8\left(-\frac{1}{8}\right) = 1$

LESSON 5-1 PRACTICE

10. What is the value of n in $(17 \times 25) \times 4 = n \times (25 \times 4)$?

11. Name the property of operations illustrated by each statement.
a. $11.1 + 0 = 11.1$
b. $6(2.7 + 4) = 6(4 + 2.7)$
c. $\left(\frac{1}{2} \cdot 3\right) \cdot 1\frac{2}{3} = \frac{1}{2} \cdot \left(3 \cdot 1\frac{2}{3}\right)$
d. $18 + (-18) = 0$

12. Write an algebraic statement to illustrate each of the following properties. Use x, y, and z to represent the numbers.
a. Multiplicative Identity Property
b. Associative Property of Addition
c. Commutative Property of Multiplication
d. Additive Inverse Property
e. Additive Identity Property
f. Multiplicative Inverse Property
g. Commutative Property of Addition
h. Associative Property of Multiplication

My Notes

DISCUSSION GROUP TIPS

The word *factor* when used as a verb means to write a number or expression as a product of its factors.

The word *factor* when used as a noun means any of the numbers that are multiplied together to form a product.

As you work through this lesson with your partner or group, look for and identify both uses of factor.

WRITING MATH

A multiplication expression can be written in two ways: $5 \cdot n$ or $5n$.

Learning Targets:

- Apply properties to factor and expand linear expressions.
- Rewrite expressions to see how the problem and quantities are related.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Interactive Word Wall, Summarizeing, Think-Pair-Share, Quickwrite

The *Distributive Property* can be used to expand or factor an expression.

Distributive Property of Multiplication over Addition:

- To expand an expression:
  $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$
- To factor an expression:
  $ab + ac = a(b + c)$ or $ba + ca = (b + c)a$

Distributive Property of Multiplication over Subtraction:

- To expand an expression:
  $a(b - c) = ab - ac$ or $(b - c)a = ba - ca$
- To factor an expression:
  $ab - ac = a(b - c)$ or $ba - ca = (b - c)a$

Example A

Use the Distributive Property to expand $5(x + 2)$.

Step 1: Multiply 5 by the terms in the parentheses.

$$5(x + 2) = 5 \cdot x + 5 \cdot 2$$

Step 2: Simplify.

$$5 \cdot x + 5 \cdot 2 = 5x + 10$$

Solution: In expanded form, $5(x + 2) = 5x + 10$.

Try These A

Use the Distributive Property to expand each expression.

a. $4(6 - 2)$
b. $7(2 + 5)$
c. $6(a + 7)$
d. $3(4 - b)$

My Notes

Example B

Use the Distributive Property to factor $12x - 18$.

Step 1: Determine the greatest common factor of each term.
$12x - 18$
The greatest common factor of $12x$ and 18 is 6.

Step 2: Divide by the common factor.
$\frac{12x}{6} - \frac{18}{6}$
Divide both terms by 6.

Step 3: The greatest common factor, 6, is factored out of the terms and is shown outside the parentheses with the quotients of the division inside the parentheses.
Rewrite using parentheses.
$6(2x - 3)$

Solution: In factored form, $12x - 18 = 6(2x - 3)$.

Try These B

Use the Distributive Property to factor each expression.

a. $12 + 10$
b. $18 - 6$
c. $6x + 3y$
d. $2a - 10$
e. $5x + 5$
f. $12r - 24$

Equivalent expressions are two or more expressions that may look different, but represent the same quantity or have equal values when evaluated. The expression $2x + 4x$ and the expression $6x$ are equivalent expressions.

Example C

Use the Distributive Property to simplify $5a + 3a$.

Factor the expression using the distributive property.
$5a + 3a = (5 + 3)a = 8a$

Solution: The expression $5a + 3a$ can be simplified to $8a$.

Try These C

Use the Distributive Property to simplify each expression.

a. $6x + 9x$
b. $5b - 2b$
c. $12d - 8d$
d. $3h + (-7h)$

My Notes

MATH TIP

The **order of operations** is a set of rules for evaluating expressions with more than one operation. The order is as follows:

1. Do calculations inside grouping symbols first, beginning with the innermost set.
2. Evaluate expressions with exponents.
3. Multiply or divide from left to right.
4. Add or subtract from left to right.

MATH TIP

Remember that percent means hundredths, so

$85\% = \frac{85}{100} = 0.85$ and

$7\% = \frac{7}{100} = 0.07$.

1. Write an equivalent expression for $3(y - 6) + 4$.
2. **Construct viable arguments.** Izzi thinks the two expressions $2(2a - 1) + 3a$ and $7a - 2$ are equivalent. His work is shown below. Is he correct? Why or why not? List the properties of operations and the order of operations next to each step to justify your response.

   $2(2a - 1) + 3a$
   $4a - 2 + 3a$
   $4a + 3a - 2$
   $(4 + 3)\,a - 2$
   $7a - 2$

3. Naman and Ada disagree about how to find an equivalent expression for $2(4x - 3) + 6$. Who is correct? How do you know? Use properties of operations and the order of operations to justify your response.

Naman	Ada
$2(4x - 3) + 6$	$2(4x - 3) + 6$
$6 + 2(4x - 3)$	$8x - 6 + 6$
$8(4x - 3)$	$8x + 0$
$32x - 24$	$8x$

Rewriting an expression in a different form can show how quantities are related.

Example D

Use the Distributive Property to show that increasing an amount by 8% is the same as multiplying the amount by 1.08.

Step 1: Choose an amount.
50

Step 2: Add to show the original amount plus the 8% increase.
$50 + 50(0.08) = 50 + 4 = 54$

Step 3: Find the common factor of 50 and 50(0.08)
The common factor of $50 + 50(0.08)$ is 50.

Step 4: Use the Distributive Property to rewrite $50 + 50(0.08)$.
$50 + 50(0.08) = 50(1 + 0.08) = 50 + 4 = 54$

Solution: Increasing an amount by 8% is the same as multiplying the amount by $(1 + 0.08)$ or 1.08.

My Notes

Try These D

a. Louisa wants to buy an online movie subscription that is on sale for 15% off. She writes the expression $c - 0.15c$ to represent the cost of the subscription. Rewrite this expression in a different form to show what percent of the original price she will pay for the online movie subscription. Then compare your expression with Louisa's. How are the expressions related? What does each expression tell you about the problem situation?

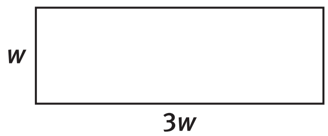
b. The Rumpart family is building a new room onto their house. The width of the new room will be 16 feet. The length of the room will be 4% greater than the width. Write an expression to find the length of the new room. What will be the area of this new room?

Check Your Understanding

4. Use the Distributive Property to write an equivalent expression for each of the following.

a. $(q - 6)7$ **b.** $3(11 + 5x)$
c. $4x - 16$ **d.** $(24 + 6x)$

5. The length of a rectangle is three times its width. One way to write an expression to find the perimeter would be $w + w + 3w + 3w$. Write the expression in two other different ways.



LESSON 5-2 PRACTICE

6. Constructing viable arguments. Explain how $a(b + c)$ can be rewritten as $(b + c)a$ and as $ba + ca$.

7. Use the Distributive Property to write an equivalent expression for each of the following.

a. $4(x - 3)$ **b.** $12x + 24y$
c. $9 - 9z$ **d.** $(7y - 1)8$

8. The expression $x - 0.10x$ gives the cost of an item that is discounted 10%. Write this expression another way.

ACTIVITY 5 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 5-1

For 1–4, name the property of operations illustrated by each statement.

1. $4 \cdot 1 = 4$
2. $3(xy + z) = (xy + z)3$
3. $7b + 0 = 7b$
4. $9x + 3y + 4x = 9x + 4x + 3y$

For 5–8 , complete each statement. Then state the property or properties illustrated by the statement.

5. $23 + \square = 23$
6. $\square \cdot (-5x) = 1$
7. $-15a + \square = 65 + (-15a)$
8. $(8 \cdot \square) \cdot (-77) = -77 \cdot (\square \cdot 8)$

For 9–12, write an algebraic statement that illustrates each property.

9. Associative Property of Addition
10. Commutative Property of Multiplication
11. Additive Identity Property
12. Multiplicative Inverse Property

Lesson 5-2

For 13–15, write an algebraic statement that illustrates each property.

13. Distributive Property of Multiplication over Addition
14. Distributive Property of Multiplication over Subtraction
15. In which properties are more than one operation used? Include an example in your response.
16. Which expression is equivalent to $2a + 13 - 0 + 65b$?
    A. $2a + 13 - 65b$
    B. $2a + 65b + 13$
    C. $(2a + 13) \cdot 65b$
    D. $67ab + 13$

For 17–19, completely factor each expression.

17. $2x + 12$
18. $24 + 8y - 16w$
19. $14 - 8m$
20. Laura is paying $1.07x$ including tax for a sweater that costs x before tax. Write this expression in another way to show the amount of tax Laura is paying.
21. The length of a rectangle is twice its width. Write two equivalent expressions for the perimeter of the rectangle. Justify your response using properties of operations and the order of operations.
22. Show how you can use properties of operations as strategies to evaluate each expression using mental math. Then evaluate the expression.
    A. $0.2 + 7.9 + 3.8 + 1.1$
    B. $-5.6 + 5 + 4.6 + 1$
    C. $3\frac{1}{6} - 4\frac{3}{4} + 1\frac{5}{6}$
    D. $2\frac{1}{12} + 3\frac{1}{6} - 1\frac{1}{12}$

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

23. Nick is buying a birthday present for a friend. The gift is on sale for 40% off. He has a coupon for an additional 20% off the sale price. Write an expression to represent the cost of the gift. Then write your expression in another form to show what percent of the original price Nick will pay for his gift.

Writing and Solving Equations

ACTIVITY 6

Melody's Music Solution

Lesson 6-1 Modeling and Writing Two-Step Equations

My Notes

Learning Targets:

- Use variables to represent quantities in real-world problems.
- Write two-step equations to represent real-world problems.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Create Representations, Look for a Pattern, Guess and Check

Melody has a new job recording for the All-Time Favorites record label. She is paid a monthly base salary of \$4,850 plus \$3 for each CD sold.

1. Complete the table below to show the monthly income for Melody.

Number of CDs Sold	Expression Used to Find Melody's Income	Melody's Income
0	4,850 + 3(0)	4,850
1	4,850 + 3(1)	4,853
2	4,850 + 3(2)	
3		
4		
5		

2. Describe any patterns you notice in the columns of the table.

3. Reason abstractly. Let *n* represent the number of CDs sold. Write an expression to represent Melody's income.

In any expression, the number the variable is being multiplied by is the **coefficient**, and the **constant** is a term that does not contain a variable.

Coefficient → $3n + 6$ ← Constant

My Notes

4. Use the expression you wrote for Melody's income to complete the table. Show your work.

Number of CDs Sold	Expression Used to Find Melody's Income	Melody's Income
10		
20		
100		

5. Assume Melody needs to make $6,000 this month to cover her expenses. Write an equation you could use to find the number of CDs that Melody needs to sell to meet her expenses.

MATH TERMS

An **equation** is a statement showing that two expressions are equal, such as $4 + 3 = 7$. An equation has an equal sign while an expression does not.

Check Your Understanding

6. Melody has hired a new accountant. He has gathered her pay stubs and is trying to determine how many CDs were sold during each month of the previous year. Her pay stub for June indicates that she made $4,889 in that month. Write an equation her accountant could use to determine how many CDs were sold in June.

7. A photography studio charges a sitting fee of $50 and $10 per enlargement ordered. Write an equation to represent the number of enlargements ordered, *n*, if the total cost was $180.

8. Does it seem reasonable that 18 enlargements were ordered in item 7? Explain.

LESSON 6-1 PRACTICE

9. The members of a Tae Kwon Do class are ordering jackets. Each jacket costs $35, and there is a one-time fee of $25 for the design. Write an equation to represent the number of jackets, *n*, that were ordered if the total cost is $620.

10. A stockbroker charges his customers $30 to open an account and $15 per month to manage the account. Write an equation to represent the number of months, *n*, an account has been open if the total cost is $360.

11. **Reason abstractly.** Lottie bought a new car for $25,000. She paid $5,000 up front and then $600 per month. Write an equation to represent the number of months, *n*, it will take Lottie to pay for her car.

12. Mrs. Carter baked 100 muffins for a bake sale. The muffins were sold in packages of 2. There were 12 muffins left. Write an equation to find how many customers bought muffins at the bake sale.

My Notes

Learning Targets:

- Solve two-step equations.
- Solve real-world problems by writing an equation of the form $px + q = r$.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Work Backward, Note Taking, Self Revision/Peer Revision

Work with your group to answer all parts of item 1. As you discuss your solutions, speak clearly and use precise mathematical language. Remember to use complete sentences and words such as *and, or, since, for example, therefore, because of* to make connections between your thoughts.

1. Melody's friend Leena earns \$15 per hour as a lab technician plus an extra \$300 per week for singing at a club on the weekend.

a. Write an equation to represent the number of hours, n, Leena must work at the lab in a week to earn \$720.

b. Which number from the set {26, 28, 30} is the number of hours Leena must work?

c. How do you know that your answer to part b is correct?

MATH TIP

Item 1 shows a problem solved algebraically using an equation with a variable. This problem can also be solved arithmetically. A possible arithmetic solution:

- Start with Leena's total earnings: \$720.
- Subtract her singing earnings (\$300) to find her lab technician earnings: $\$720 - \$300 = \$420$
- She earns \$15 per hour, so divide the lab technician earnings by \$15 to find the hours she must work: $\$420 \div \$15 = 28$ hours.

Example A

Melody needs to record a new CD. She decides she can spend as much as \$8,000 on studio time. The studio charges \$425 to reserve the space and \$75 per hour. Solve the equation $75h + 425 = 8000$ to find the maximum number of hours Melody can afford to spend in the recording studio.

Step 1: Write an equation to represent the problem.

$$75h + 425 = 8000$$

Step 2: Use inverse operations. Subtract 425 from both sides.

$$75h + 425 - 425 = 8000 - 425$$

Step 3: Simplify both sides of the equation.

$$75h = 7575$$

Step 4: Use inverse operations. Divide both sides by 75.

$$\frac{75h}{75} = \frac{7575}{75}$$

Step 5: Simplify both sides of the equation.

$$1h = 101$$

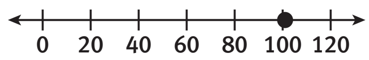
Step 6: Use the Multiplicative Identity Property to isolate the variable.

$$h = 101$$

Solution: Melody can afford 101 hours of studio time.

MATH TIP

You can graph the solution of an equation that has one variable on a number line. For the graph of the solution to Example A, the number line would have a dot at 101.



My Notes

Check the solution by substitution:

$$75(101) + 425 = 8000$$
$$7575 + 425 = 8000$$
$$8000 = 8000$$

The solution checks.

Try These A

Solve each equation below. Graph the solution to each equation on a number line.

a. $12n + 6 = 78$
b. $3x - 5 = 16$
c. $-8 = 2x + 4$
d. $2y - 3 = -9$

2. *Red Cast Records* pays a shipping company to ship CDs to retail stores. The shipping company is paid $1,650 per shipment minus $25 per hour for each hour the delivery arrives past the promised delivery time. The last shipment was late, so *Red Cast Records* was billed only $1,325.

a. Write an equation, using h to represent hours, that can be used to determine how late the delivery was made.

b. Make use of structure. Use inverse operations to solve the equation algebraically. Check your solution.

c. How late was the shipment?

So far, in this lesson and in the last lesson, you have written equations to represent problem situations expressed in words. It is also possible to work backwards; that is, write a problem situation in words that represents an equation.

3. Work with your group. Write a problem situation in words that represents each given equation. If you can, write situations that are related to music. Discuss how you will present your equations to the rest of the class. Remember to use words in your presentation that will help your classmates understand the situation.

a. $8d - 250 = 750$

b. $12d + 400 = 1000$

DISCUSSION GROUP TIPS

As you discuss ideas for your equations and presentations, make notes and listen to what your group members have to contribute. Ask and answer questions to clearly aid comprehension and to ensure understanding of all group members' ideas.

My Notes

Check Your Understanding

4. Which of the following values makes the equation $-9x + 17 = 8$ true?

A. -1 **B.** 0 **C.** 1 **D.** 3

5. a. A school reserved a banquet hall for the spring dance. In addition to a $100 deposit, each couple must pay $20. If the total cost of the banquet hall is $1,140, write and solve an equation to find the number of couples attending the spring dance.

a. Solve the problem arithmetically. Show the steps you used.

b. Compare and contrast the steps you used to solve the problem algebraically and arithmetically.

6. Mia said that $2 - 3a = 11$ and $3a - 2 = 11$ have the same solution. Is she correct? Explain.

7. Solve each equation below algebraically.

a. $5x - 2 = 13$

b. $2a + 7 = -11$

c. $\frac{1}{4}k + 3 = 6$

d. $6 - 4a = -10$

e. $15x = -15$

LESSON 6-2 PRACTICE

8. Solve each equation below algebraically.

a. $6x - 11 = 19$

b. $\frac{2}{3}y + 3 = 29$

c. $8 - a = 17$

9. Solve and then graph each solution on a number line.

a. $52 = 12 + 4w$

b. $-24 = -6p$

10. The German Club is planning a ski trip. The club will pay $500 toward the trip from money they have earned selling candy, and each member going on the trip will pay $115. If the trip costs $2,685, write and solve an equation to find the number of club members going on the trip.

11. Make use of structure. Explain the similarities and differences between guess and check and the algebraic method for solving an equation. Which method do you prefer? Why?

12. Write a problem in words that can represent this equation: $13x + 26 = 91$

ACTIVITY 6 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 6-1

1. Sam makes $400 per week plus $20 commission on each new cell phone plan she sells. Write an equation to determine how many new plans she sold to earn $680 last week.
2. Eric has a dog-walking business. He charges $13 per dog he walks and $6.00 for the water he buys for the dogs. If he made $97 on Monday, write an equation to find the number of dogs he walked on Monday.
3. In addition to dog walking, on Tuesday Eric made $45 dog sitting at one of his customer's homes. If he made $168 on Tuesday, write an equation to find the number of dogs he walked that day.
4. Rena swims every day. She burns approximately 10.6 calories per minute when swimming and about 15 total calories warming up before she swims. Write an equation to find how many minutes Rena must swim to burn 205.8 calories.
5. A cell phone company charges $10 per month for up to 500 text messages and $0.15 for each additional text message. Stanley was charged $14.50 last month for text messages. Write an equation to find the number of text messages he sent and received last month.

Lesson 6-2

6. Which of the following values makes the equation $4x - 8 = -4$ true?
   A. -3 **B.** -1
   C. 1 **D.** 3
7. Solve each equation below algebraically.
   a. $3x + 2 = 26$ **b.** $4c - 18 = 6$
   c. $6a - 2 = 10$ **d.** $-7x + 3 = 17$
8. Solve and graph the solution for each equation on a number line.
   a. $3 + 4x = 11$ **b.** $8a - 6 = 18$
   c. $\frac{1}{2x} + 3 = 9$ **d.** $17 = w - 4$
9. Lacey and Chris solved the same equation, but their solutions were different. Who is correct? Justify your response.

Lacey	Chris
$20 + 8h = 180$	$20 + 8h = 180$
$20 - 20 + 8h = 180 - 20$	$+20 \quad +20$
$8h = 160$	$8h = 200$
$\frac{8h}{8} = \frac{160}{8}$	$\frac{8h}{8} = \frac{200}{8}$
$h = 20$	$h = 25$

Jored and Sundai each want to buy a new HD movie player. They go to Electronics Superstore and find a HD movie player for $75.00. Electronics Superstore offers different payment plans. Jored is going to pay $15 now and then $7.50 per month. Sundai is going to pay $12.50 per month.

10. Write and solve an equation for each plan to show how many months it will take each person to pay the $75.00 for the HD movie player.
11. It will cost $285 to charter a bus for a class trip. The class treasurer says that there is $60 in the class treasury. The 20 students going on the trip agree to make up the difference. Write and solve an equation to find how much each student will pay.
12. A movie company sells DVDs on line. Each DVD costs $9.95. No matter how many you order, the shipping charge is $3.59. Your most recent order came to a total of $33.44.
    a. Write and solve an equation to find how many DVDs you ordered.
    b. Solve the problem arithmetically. Show the steps you used.
    c. Compare and contrast the steps you used to solve the problem algebraically and arithmetically.

MATHEMATICAL PRACTICES
Model with Mathematics

13. Jason is given the equation $3x - 12 = 36$ to solve. His first step is to divide each term by 3. Do you think Jason's method is a good one to follow? Give an example to justify your answer.

Writing and Solving Equations
FUNDRAISING FUN

Write your answers on notebook paper. Show your work.

1. Semir, Sarah, and SungSo decided to raise money for a local homeless shelter by working in a local deli. The deli agreed to donate to the shelter a portion of the profits from each meal the three sold. Semir sold 3 times as many meals as Sarah. SungSo sold 2 more meals than Sarah.
   - **a.** Write an expression for the number of meals that each sold.
   - **b.** If Sarah sold 24 meals, how many meals did Semir and SungSo each sell?
   - **c.** How many meals did the three sell in all? Explain how to use the commutative property and the associative property of addition to make finding the sum easier.

2. The three decided to participate in a walkathon to raise additional money for the shelter. Sarah walked 5 miles less than twice the number of miles that Semir walked. They each collected $18 in pledges for every mile they walked.
   - **a.** Let n represent the number of miles Semir walked. Write an expression for the number of miles Sarah walked and an expression for the amount of money Sarah collected in pledges.
   - **b.** Write and solve an equation to find how many miles Semir walked if Sarah collected $450 in pledges.
   - **c.** How many miles did Sarah walk?
   - **d.** SungSo also collected $18 in pledges for every mile he walked. In addition, his grandmother gave him a $72 donation. He collected the same amount of money as Sarah did. Write and solve an equation to find the number of miles SungSo walked.

3. Write a memo to the director of the shelter describing the total donation the three friends are sending and how it was raised. Be sure to include how much each student raised individually.

Writing and Solving Equations
FUNDRAISING FUN

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-c, 2a-d)	• A clear understanding of properties of operations. • Effective understanding of and accuracy in writing and evaluating expressions and solving equations.	• A functional understanding of properties of operations. • Writing and evaluating expressions, and solving equations that usually result in correct answers.	• Partial understanding of properties of operations. • Difficulty with writing and evaluating expressions and solving equations.	• Little or no understanding of properties of operations. • Little or no understanding of writing and evaluating expressions and solving equations.
Problem Solving (Items 1c, 2b, 2d)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a, 2a-b, 2d)	• Clear and accurate representations of problems as expressions and equations.	• Some difficulty in representing problems as expressions and equations.	• Difficulty in writing expressions and equations leading to errors.	• No understanding of representing problems as expressions and equations.
Reasoning and Communication (Items 1c, 3)	• Precise use of appropriate math terms and language to explain solutions to problems and the role of properties of operations.	• Adequate explanation of solutions to problems and the role of properties of operations.	• Misleading or confusing explanation of solutions to problems and the role of properties of operations.	• Incomplete or inaccurate explanation of solutions to problems and the role of properties of operations.

Solving and Graphing Inequalities

ACTIVITY 7

It Plays to Save

Lesson 7-1 Modeling and Writing Two-Step Inequalities

My Notes

Learning Targets:

- Represent quantities in a real-world problem.
- Construct two-step inequalities to solve problems.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Summarizing, Create Representations, Work Backward

Kerry's grandmother, Mrs. Reynoso, is helping Kerry pay for guitar lessons. She has set up a special savings account to pay for the lessons. On the first of each month, the $40 monthly fee for guitar lessons is withdrawn from the account.

1. Grandmother Reynoso started the account with $300. Record the monthly balance in the account in the table.

Month	Balance
0	**$300**
1	
2	
3	
4	

2. Write an expression for the amount of money in Kerry's guitar lesson account. Let x equal the number of months since Mrs. Reynoso deposited the initial $300.

3. Write an equation to determine when the account balance will be $100.

The bank will close the account if the balance falls below $50. To keep the account open, the balance must be greater than or equal to $50.

4. Write an inequality to represent this situation.

5. Use the pattern in the table to find the last month the balance is $50 or greater.

My Notes

Inequalities can be used to represent many situations.

6. Think of a real life situation in which you might use terms such as *more than, less than, no more than,* or *no less than.*

7. a. Write the inequality represented by the graph above.

b. Create a problem situation that could be represented by the graph above.

8. Arianna's mom deposits $80 in her lunch money account. Lunch costs $2.50 per day. Define a variable and write an inequality to represent when there will be less than $20 left in Arianna's lunch account.

9. Michelle babysits on weekends in her neighborhood. She charges $10 for transportation and $15 for each hour she babysits. At her last babysitting job she earned less than $60. Write an inequality to represent this situation.

10. Bailey can put *x* houses of his holiday miniature village on each of 7 shelves. He also has 10 houses displayed on his buffet. If he wants to display at least 45 of his houses, how many houses must he put on each of the 7 shelves? Write an inequality to represent this situation.

So far in this lesson, you have written inequalities to represent problem situations expressed in words. It is also possible work backwards; that is, write a problem situation in words that represents an inequality.

11. Work with your group. Write a problem situation in words that represents each given inequality. Remember to use real-life situations. Discuss how you will present your inequalities to the rest of the class. Remember to use words in your presentation that will help your classmates understand the situation.

a. $5x + 15 \geq 100$

b. $280 - 4m < 8$

DISCUSSION GROUP TIPS

As you discuss ideas for your equations and presentations, make notes and listen to what your group members have to contribute. Ask and answer questions to clearly aid comprehension and to ensure understanding of all group members' ideas.

My Notes

Check Your Understanding

12. Write an inequality for each situation.

a. Yiska has 56 photographs left to scan. If she can scan 6 more photographs per minute, in how many minutes will she have less than 20 left to scan?

b. Erin has a $450 gift card from a clothing store. She has spent a total of $28 so far. She wants to buy barrettes for $2.95 each. How many barrettes can she buy without going over the limit of the card?

13. What words in a real-world problem indicate that it will be represented by an inequality rather than by an equation? Explain.

LESSON 7-1 PRACTICE

14. Model with mathematics. Write an inequality for each situation.

a. A cold wave hit Chicago when the temperature hit 62°F. During the cold wave, the temperature dropped 2 degrees every hour. How many hours was it before the temperature was below 40°F?

b. Miranda is packing eggs in cartons. Each carton holds 12 eggs. She has already filled 3 cartons. How many more eggs does she need to fill at least 17 cartons?

c. Six more than three times a number is less than or equal to 96.

d. Five less than half the distance from Jerod's home to the mall is more than 6 miles.

e. The length of a rectangle is 4 more than twice the width. If the length is less than 62, find the width.

f. Miranda is buying pencils and a writing pad for her adult writing class. The writing pad she wants costs $3.50, and each pencil is $0.15. She wants to spend no more than $5 in total for these items.

15. Write a problem situation for this inequality:

$$72 - 12a < 24$$

My Notes

Learning Targets:

- Solve two-step inequalities.
- Construct two-step inequalities to solve problems.

SUGGESTED LEARNING STRATEGIES: Think Aloud, Marking the Text, Summarizing, Create Representations, Think-Pair-Share

Consider the set {5, 7, 9} and the inequality $6x - 8 < 46$.

1. Without solving the inequality, how can you determine which numbers from the set are solutions of the inequality?

2. Which numbers from the set are solutions?

3. Is 9 a solution? Why or why not?

Solving two-step inequalities is much like solving two-step equations. Use inverse operations to solve each of the following inequalities.

4. $2x - 10 < 80$

5. $5x - 8 + 7x > 40$

6. $7(x - 11) \leq 100$

7. $5x + 8.5 \geq -10.3$

There is one important difference, however, between solving equations and solving inequalities. The experiment below will help you discover this difference.

8. **a.** Work with a partner. Cut out the positive and negative number cards from the sheet your teacher will given you and stack the cards face down on your desk.

b. Draw 2 cards. Write an inequality to represent the relationship between the numbers in the table below.

c. Draw a third card. Multiply both sides of the inequality by the number indicated on this card. Record the result in the table. Is the result a true statement? If not, what can be done to make it a true statement?

Inequality Using First Two Numbers	Multiply on Both Sides By:	Inequality After Multiplication	True or False	Correction, If Necessary
$-3 < 5$	-2	$6 < -10$	False	$6 > -10$

d. Based on your results and those of your classmates, what happens in an inequality when both sides of the inequality are multiplied by a negative number? What do you believe will happen when both sides of the inequality are divided by a negative number?

To solve a two-step inequality you isolate the variable just as you did when solving an equation. Remember to switch the inequality sign if you multiply or divide by a negative number.

Example A

Solve $30 - 4x \geq 5$ and graph the solution on a number line.

Step 1: Original inequality $\quad 30 - 4x \geq 5$

Step 2: Subtract 30 from both sides. $\quad 30 - 30 - 4x \geq 5 - 30$

$$-4x \geq -25$$

Step 3: Divide by -4 and reverse the inequality. $\quad \frac{-4x}{-4} \leq \frac{-25}{-4}$

Solution: $x \leq 6.25$. The inequality symbol means less than or equal to, so 6.25 is part of the solution. This is shown by a filled-in circle on 6.25 on the graph of the solution.

Example B

Solve $2x - 50 < -70$ and graph the solution on a number line.

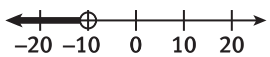
Step 1: Original inequality. $\quad 2x - 50 < -70$

Step 2: Add 50 to both sides. $\quad 2x - 50 + 50 < -70 + 50$

$$2x < -20$$

Step 3: Divide by 2. $\quad \frac{2x}{2} < \frac{-20}{2}$

Solution: $x < -10$. The inequality symbol means less than, so -10 is not part of the solution. This is shown by an open circle on -10 on the graph of the solution.



MATH TIP

Notice in Example B that you did not divide by a negative number so although the solution was negative, the inequality sign does not reverse.

My Notes

Try These A-B

Solve each inequality algebraically and graph your solution on a number line.

a. $-5x + 7 > 22$
b. $2x + 6 \geq 16$
c. $-3(x + 5) < -21$
d. $11x - 12 > 21$

9. Model with mathematics. Hamid has read 60 pages of the book he will be using for a book report. If he reads 45 pages per hour, how many hours will it take him to read at least 375 pages of the book? Define a variable, and then write and solve an inequality to represent this situation. Graph the solution on a number line.

You should interpret the graph of the solution to an inequality in terms of the problem situation.

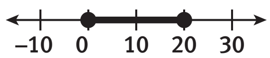
Example C

Joanne must complete a 4-question math quiz in 90 seconds or less. If she spends 30 seconds on the first question, what is the greatest amount of time on average she can spend on each of the remaining 3 questions? Write and solve an inequality to solve the problem. Then graph the solution and interpret the graph in the context of the problem.

Step 1: Write an inequality to represent the situation.
Let t represent the average amount of time in seconds for each of the remaining 3 questions.
She must complete the quiz in 90 seconds or less, so use $\leq$.
$3t + 30 \leq 90$

Step 2: Solve.
$3t + 30 \leq 90$
$3t + 30 - 30 \leq 90 - 30$
$3t \leq 60$
$3t \leq 20$

Step 3: Graph the solution.



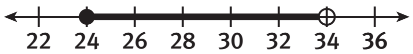
Solution: Interpret the graph.
The graph shows that Joanne could spend an average of up to 20 seconds on each of the 3 remaining problems. Even though the graph is mathematically correct, it is very unlikely that she would spend any of the lower values, such as 0, 1, 2, 3, 4, and 5 seconds, on each problem.

My Notes

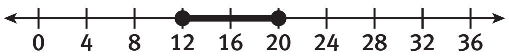
Try These C

Interpret each graph.

a. This graph shows the range in temperatures in degrees Fahrenheit during a day in February.



b. This graph shows the height of water, in inches, in a rain barrel during a rainstorm.



Check Your Understanding

10. Solve each of the following inequalities algebraically. Graph the solutions on number lines.

a. $3x - 15 < 120$ **b.** $9x - 6 + 3x > 42$

c. $7(x - 1) \leq 35$ **d.** $-3x + 4 \geq -23$

11. Falana has \$192 in her savings account. Since she is not using the account, the bank charges a monthly fee of \$5. The bank will close the account when the balance goes below \$50. Write and solve an inequality to find how many months until the bank closes Falana's account.

12. Compare and contrast the solutions of $2x - 4 > 6$ and $2x - 4 = 6$.

13. Write a short note to a friend explaining when to reverse the inequality sign when you are solving an inequality.

LESSON 7-2 PRACTICE

14. Solve each of the following inequalities algebraically. Graph the solutions on number lines.

a. $2x - 50 > 75$ **b.** $15x + 20 > 50$

c. $-4x + 10 < 26$ **d.** $3x + 70 - 7x \geq 18$

e. $18 \leq -6x - 30$ **f.** $4(x + 2) > 60$

g. $2x + 6x - 9 < 39$

15. Model with mathematics. Arianna's mom deposits \$80 in her lunch money account. Lunch costs \$2.50 per day. Write and solve an inequality to determine when there will be less than \$20 in the account.

16. Give three solutions of the inequality $5x - 2 > 7$.

17. Consider the inequality $2x + 3 \leq -3$. Find the set of all integer solutions of this inequality that are also solutions of the inequality $5x - 2 < 3$.

18. Compare and contrast solving equations and inequalities and their solutions.

19. Nilsa is working on a 60-minute math test. There are 20 questions on the test. If it takes her 20 minutes to complete 12 of the questions, what is the greatest amount of time on average she can spend on each of the remaining 8 questions?

a. Write an inequality for the problem situation and solve it.

b. Graph the solution.

c. Interpret the graph in the context of the problem.

ACTIVITY 7 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 7-1

For 1–8, write an inequality to represent each situation.

1. Twenty-two more than four times a number is less than 82.
2. There are x giraffes at the zoo. The number of elephants is 4 less than three times the number of giraffes, and there are more than 23 elephants.
3. Louie has 20 more than half as many baseball cards as Gerardo does. Together they have at least 350 cards.
4. Zasha spent \$6 on packages of gum. How many more packages of gum that cost \$1.20 each can she buy if she has a \$20 bill?
5. Dolores and four friends went to a buffet dinner. The total cost was at most \$130 including the \$20 tip they left. How much did each pay for the buffet?
6. Eight less than five times the number of marbles that Iggy has is less than or equal to 72.
7. George rented a bike for 4 hours. There was a \$10 deposit to pay in addition to the hourly rate. What was the hourly rate if the total came to less than or equal to \$65?
8. A store wants to print flyers to advertise its grand opening. A printer will charge \$50 and \$0.05 per flyer. If the store has a budget of \$100, how many flyers can the store have printed without going beyond their budget?

Lesson 7-2

For 9–15, solve each inequality and graph the solution on a number line.

9. $8x + 2 > 10$
10. $14x - 3 \leq -3$
11. $12 - 2x < 16$
12. $4 - \frac{x}{3} > 6$
13. $\frac{1}{2}x + 1 \geq 5$
14. $5(x - 3) > 15$
15. $2x + 7 - 3x \leq 10$
16. Give three solutions of the inequality $6 - 11x < 61$.
17. Which situation can be represented by the inequality $4x - 25 < 125$?
    A. Frank bought four tires for x dollars each. He had a coupon for a \$25 discount. The total came to less than or equal to \$125.
    B. Frank bought four tires for x dollars each. He paid \$25 in shipping for a total less than or equal to \$125.
    C. Frank bought 25 tires for x dollars each. He paid \$4 in shipping for a total less than or equal to \$125.
    D. Frank bought x tires for \$25 each. He paid \$4 in tax for a total less than or equal to \$125.

MATHEMATICAL PRACTICES

Reason Abstractly and Quantitatively

18. Carmine and Rachel went apple picking. Carmine has 5 more apples than Rachel. What is the minimum number of apples that Rachel has if there are at least 31 apples in all?

Write your answers on notebook paper. Show your work.

1. The ***media*** reported that Olympic gold medalist Michael Phelps regularly consumed at least 8,000 calories per day when he was training for the Olympics. In order to do this, he ate at least three extra-large meals and also consumed a maximum of 2,000 calories worth of special energy drinks each day.
   a. Write and graph an inequality to represent the number of calories from energy drinks Michael Phelps drank per day while training.
   b. Write and graph an inequality to represent the total number of calories Michael Phelps consumed each day.
2. Write a situation about Michael Phelps in which the inequality $3m + 2{,}000 \geq 8{,}000$ would represent the situation.
3. To keep from losing weight while training, athletes must not burn more calories than they consume in a day. On one day, Michael Phelps burned 1,000 calories per hour while swimming and an additional 3,000 calories while out of the pool. Write and solve an inequality to estimate the number of hours he swam that day if his daily average caloric intake was at most 9,000 calories.
4. An athlete wants to maintain a net caloric intake of no more than 2,000 calories for the day.
   a. Write and solve an inequality to determine how many hours she must train if she burns an average of 750 calories per hour and eats a total of 8,000 calories.
   b. Graph the solution to your inequality on a number line. Explain why your answer to part a is a solution to this situation.
   c. If she trains 8 hours per day, what is the greatest caloric intake she can have to keep from losing weight? Explain your reasoning.

ACADEMIC VOCABULARY

Media are various ways by which news and information are communicated to the public. Media includes television, radio, and newspapers.

Solving Inequalities
A GOLD MEDAL APPETITE

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-b, 2, 3, 4a-c)	• Effective understanding of and accuracy in writing, evaluating, and solving inequalities.	• Writing, evaluating, and solving inequalities correctly.	• Difficulty with writing, evaluating, and solving inequalities.	• Little or no understanding of writing, evaluating, and solving inequalities.
Problem Solving (Items 3, 4a, 4c)	• An appropriate and efficient strategy that results in a correct answer. • A correct and complete interpretation of the solution to an inequality.	• A strategy that may include unnecessary steps but results in a correct answer. • A correct interpretation of the solution to an inequality.	• A strategy that results in some incorrect answers. • Difficulty interpreting the solution to an inequality.	• No clear strategy when solving problems. • No understanding of interpreting an inequality or its solution.
Mathematical Modeling / Representations (Items 1a-b, 2, 3, 4a-b)	• A clear and accurate representation of a situation as an inequality. • Accurate and precise graphing of an inequality.	• Some difficulty in representing situations as inequalities. • Correct graphing of an inequality.	• Difficulty in writing inequalities leading to errors. • Some errors in graphing inequalities	• No understanding of representing situations as inequalities. • Incomplete or inaccurate graphing of inequalities.
Reasoning and Communication (Items 2, 4b-c)	• Precise use of appropriate math terms and language to explain solutions of inequalities. • Clear and accurate writing of a situation to match an inequality.	• Adequate explanation of solutions to inequalities. • Writing a situation to match an inequality.	• Misleading or confusing explanation of solutions to inequalities. • Writing a situation that partially matches an inequality.	• Incomplete or inaccurate explanation of solutions to inequalities. • An inaccurately written situation to match an inequality.

Ratio and Proportion

3

Unit Overview

In this unit, you will use pictures, graphs, tables, and verbal descriptions to study unit rates, rate of change, and proportions. You will solve problems involving scale, percentage, and proportional relationships.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- tip

Math Terms

- ratio
- rate
- unit rate
- proportion
- cross products
- conversion factor
- constant of proportionality
- constant ratio
- constant rate of change
- relative size
- scale drawing
- percent
- percent equation
- discount
- markup
- interest
- percent error

ESSENTIAL QUESTIONS

? How are ratios, unit rates, and proportions used to describe and solve real-world problems?

? How can representations, numbers, words, tables, and graphs be used to solve problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 9, 10, and 12, will give you an opportunity to demonstrate how you can use ratios and rates to solve mathematical and real-world problems involving proportional relationships.

Getting Ready

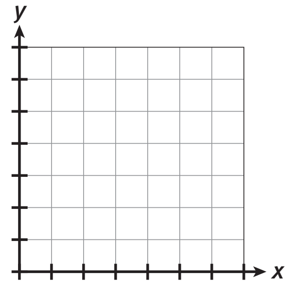
Write your answers on notebook paper. Show your work.

1. Janese can complete 7 toe touches in 10 seconds. Write a ratio of Janese's toe touches to seconds in three ways.
2. Complete the following table representing Janese's toe touches.

Janese's Toe-Touching Record							
Time (in seconds)	0	10	20	30	40	50	60
Toe Touches							

3. Use the grid below to graph Janese's toe touches. Label the horizontal and vertical axes. Provide a scale on the horizontal and vertical axes.

Janese's Toe Touches

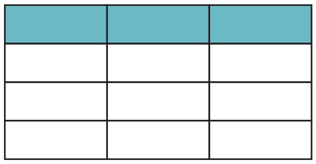


4. Write an algebraic expression for each the following.
   - **a.** The cost of each ticket, if x tickets cost \$106.25
   - **b.** The cost of g gallons of gas if each gallon costs \$3.67
   - **c.** Five less than 3 times a number
5. Solve each of the following equations.
   - **a.** $2x + 5 = 8$
   - **b.** $16 + 3x = 28$
6. Copy and complete this table to show equivalent values.

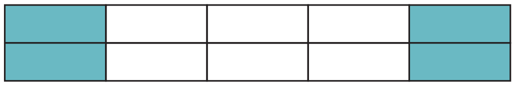
%	Decimal	Fraction
25%		
		$\frac{1}{2}$
	0.15	

7. What percent of the figures are shaded?

a.



b.



8. Explain how to determine which of the following values is the greater and tell which expression has the larger value.

$\frac{1}{3}$ of 60

25% of 60

Ratio and Proportion

Strange, But True

Lesson 8-1 Ratio and Unit Rates

ACTIVITY 8

Learning Targets:

- Express relationships using ratios.
- Find unit rates.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Paraphraing, Note Taking, Sharing and Responding, Discussion Groups

Interesting sports facts and statistics are abundant. For example, there are only *two* days of the year in which there are *no* professional sports games (MLB, NBA, NHL, or NFL): the day before and the day after the Major League Baseball (MLB) All-Star Game.

You can write a **ratio** to compare the number of days without any professional sports events each year with the total number of days in a year. There are three ways to write a ratio to express the relationship between two quantities:

a to b $\qquad a : b \qquad \frac{a}{b}$

1. Write a ratio, in all three forms, to compare the days without games to the days in a year.

2. What ratio compares days with games to days in a year? Write it three ways.

3. What ratio can you write that compares days with games to days without them? Write this ratio three ways, too.

The ratios above all compare two *like* units—*days* and *days*. A ratio that compares two *different* kinds of units is called a **rate.** One common rate in sports is miles per hour (mi/h or mph), as in car racing.

4. List some other sports statistics commonly given as rates.

You can use basketball free throws to explore rates. In a group of four, make 12 paper basketballs. Place a wastebasket about six feet from a "free-throw line." Record how many "baskets" each of you makes within the time listed in the table. Have one member of the group keep time. Then work together to answer Items 5-8 on the next page. If you do not know exact words to use during discussion, use synonyms or request assistance from group members. If you need to, use non-verbal cues such as raising your hand for help.

My Notes

CONNECT TO SPORTS

NBA = National Basketball Association

NHL = National Hockey League

NFL = National Football League

MATH TERMS

A **ratio** is a comparison of two quantities. You can write a ratio as a fraction, using the word "to," or using a colon.

A **rate** is a ratio that compares two *different* units, such as distance and time, or a ratio that compares two different things measured with the same unit, such as cups of water and cups of frozen orange juice concentrate.

My Notes

Team Member	Baskets Made	Time (in seconds)	Rate
1		30	
2		15	
3		20	
4		10	

5. What units are you comparing in this free-throw activity?

6. **Reason quantitatively.** Examine all results. What can you say about the relationship between baskets made and time allowed?

MATH TERMS

A **unit rate** is a rate expressed in terms of 1 unit.

In some cases, you find unit rates by *multiplying*. In other cases, you find them by *dividing*.

When the second term of a rate is 1, the rate is called a **unit rate** Miles *per* hour is a kind of unit rate. So is price *per* pound.

Suppose that you and your friends attend a basketball game. You buy a block of 8 tickets for $192. You want to know the price per ticket. That price can be expressed as a unit rate.

7. What is that rate? How did you figure it out?

Now look back at your made-basket rates. To see who the best shooter was, you can express each rate as a unit rate.

Think: 60 seconds = 1 minute. Let made-baskets per minute be your unit rate.

Suppose you made 7 baskets in 30 seconds. Use mental math to find how many you made in 60 seconds.

$$\frac{7}{30} = \frac{7\,(2)}{30\,(2)} = \frac{14 \text{ baskets}}{60 \text{ seconds}} = \frac{14 \text{ baskets}}{1 \text{ minute}}$$

So, your unit rate is 14 baskets per minute.

8. a. How can you find the one-minute unit rate for the baskets team member 1 made in 30 seconds? Explain your reasoning.

b. How can you find the unit rate for the baskets that team member 3 made in 20 seconds? Explain your reasoning.

My Notes

Check Your Understanding

9. There are nine position players on a baseball team:
3 outfielders, 4 infielders, 1 pitcher, 1 catcher.
Write a ratio in simplest form to express each relationship.
- **a.** infielders to number of players
- **b.** outfielders to infielders
- **c.** number of players to number of outfielders
- **d.** number of players other than infielders and outfielders to number of players
- **e.** number of pitchers to number of catchers

10. Write the rate. Then find the unit rate.
- **a.** $162 for 9 tickets
- **b.** 15 baskets in 5 minutes
- **c.** 84 yards in 14 running attempts
- **d.** 24 strikeouts in 12 innings

LESSON 8-1 PRACTICE

11. Jed made 2 free throws in 5 seconds. How many would he make in one minute?

12. Rita ran 5 miles in 48 minutes. What was her time per mile?

13. In a typical Wimbledon tennis tournament, 42,000 balls are used and 650 matches are played. About how many balls are used per match?

14. Reggie Jackson played in major league baseball for 21 years. Although this slugger was known best for his home runs, he holds the major league record for strikeouts: 2,597. What was his approximate rate of strikeouts per year?

15. In skateboarding, an *ollie* is a move in which the athlete pops the skateboard into the air, making it appear that the board and skateboarder are attached. At the 2009 X Games, one skateboarder completed 34 ollies in 30 seconds.
- **a.** If he could keep up that rate, what would be his rate of ollies per minute?
- **b.** What would be his rate per hour?

16. **Reason quantitatively.** A major-league baseball player was able to hit 70 home runs in 162 games. Create a unit rate for this information.

My Notes

Learning Targets:

- Determine whether quantities are in a proportional relationship.
- Solve problems involving proportional relationships.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Predict and Confirm, Note Taking, Create Representations

As you read the following scenario, mark the text to identify key information and parts of sentences that help you make meaning from the text.

The fastest time for running a mile while balancing a baseball bat on a finger is 7 min 5 s. This record was set by Ashrita Furman on June 20, 2009. At that rate of speed, Meg predicts that it would take 1,275 seconds, or 21 min 15 s, for Ashrita to run 3 mi. Is she right?

You can write a proportion to find out. A **proportion** is an equation. It consists of two equivalent ratios.

Example: $\frac{2}{5} = \frac{4}{10}$

MATH TERMS

A **proportion** is an equation stating that two ratios are equivalent.

To determine if Meg is correct, let n = the time it will take Ashrita to run 3 miles balancing a baseball bat.

First, convert 7 min 5 seconds to seconds: 425 seconds.

$$\frac{\text{time}}{\text{distance}} \rightarrow = \frac{425}{1} = \frac{1275}{3} \leftarrow \frac{\text{time}}{\text{distance}}$$

When two ratios are equal, their **cross-products** are equal.

For any proportion $\frac{a}{b} = \frac{c}{d}$, $ad = bc$ ← cross-products

1. Using what you know about proportions, use the proportion above involving Ashrita's speed and distance data and find the cross-products. Are the cross products equal?

READING MATH

Read the proportion $\frac{2}{5} = \frac{4}{10}$ as "the ratio 2 to 5 equals the ratio 4 to 10" or as "2 is to 5 as 4 is to 10."

2. Is Meg's prediction correct?

3. **Reason quantitatively.** Are there other ways to determine if Meg is right? Explain.

4. **Construct viable arguments.** Suppose a fast-running juggler beat Ashrita's record by half a minute. Could that person, continuing at that new world-record rate of speed, run 2 mi while juggling in 13 min 10 s? Use a proportion to find out and explain your reasoning.

Lesson 8-2
Identifying and Solving Proportions

My Notes

5. A three-toed sloth can cover a mile in 0.15 of an hour. Use proportions and sloth speed to complete the table for the distances shown.

Distance (mi)	1	2	5	12
Time (h)				

You can use proportions to solve problems about ratios and rates.

Example

Roger Bannister was the first person to break the four-minute mile. On May 6, 1954, his time was 3 minutes, 59.4 seconds. Bannister's first quarter-mile time was 57.5 seconds. Use a proportion to find his time if he had kept up this pace.

Step 1: Write a proportion.
Let $n =$ time to run the entire race.
Use 0.25 for the first quarter-mile Bannister ran.

$$\frac{\text{time (sec)}}{\text{distance (mi)}} \rightarrow \frac{57.5}{0.25} = \frac{n}{1} \leftarrow \frac{\text{time (sec)}}{\text{distance (mi)}}$$

Step 2: Solve the proportion using cross-products:
$0.25 \times n = 1 \times 57.5$
$0.25\,n = 57.5$

Step 3: Solve the equation to find n.
$n = 57.5 \div 0.25 \leftarrow$ *Think:* Divide both sides by 0.25.
$n = 230$ s
$n = 3$ min 50 s $\leftarrow$ write as minutes and seconds

Solution: Had Bannister kept up his quarter-mile pace, he would have run the mile in 230 sec, or 3 min 50 sec.

a. Model with mathematics. Why do you divide *both* sides of the equation by 0.25?

b. Reason abstractly and quantitatively. What other proportions could you have written to solve the problem?

Try These

Solve each proportion.

a. $\frac{n}{1} = \frac{2.45}{0.35}$ **b.** $\frac{n}{5} = \frac{23.4}{2}$ **c.** $\frac{3}{n} = \frac{5.4}{7.2}$

MATH TIP

Recall that in any proportion, cross-products will be equal.
For the proportion

$$\frac{a}{b} = \frac{c}{d}$$
$ad = bc$

When you write a proportion, be sure to set up the ratios in a consistent way according to the units associated with the numbers.

My Notes

Check Your Understanding

6. Use cross products to determine if the ratios are equivalent.

a. $\frac{3}{4}, \frac{6}{8}$ **b.** $\frac{8}{5}, \frac{24}{16}$ **c.** $\frac{70}{60}, \frac{6}{7}$ **d.** $\frac{1.3}{7.8}, \frac{3}{18}$

e. $\frac{4}{7}, \frac{10}{17.5}$ **f.** $\frac{9}{4}, \frac{2.1}{1.4}$ **g.** $\frac{3}{0.8}, \frac{21}{5.6}$ **h.** $\frac{0.3}{2}, \frac{0.03}{20}$

7. **Make use of structure.** Write a proportion for each situation. Then solve.

a. 336 dimples on one golf ball; 2016 dimples in n balls
b. 3 miles in 2.8 minutes; 33.3 miles in x minutes
c. 25 yards in $2\frac{1}{2}$ seconds; 100 yards in y seconds
d. 480 heartbeats in 4 minutes; z heartbeats in 1 minute

8. A zebra can run at a speed of 40 mph. Complete the table using this information.

Time (h)	1	0.25	0.5	1.75
Distance (mi)				

LESSON 8-2 PRACTICE

Solve by writing and solving a proportion.

9. There are 20 stitches per panel on a soccer ball. A soccer ball has 32 leather panels. How many stitches, in all, are on a soccer ball?

10. Jed took 2 free throws in 5 seconds. Alex took 4 free throws in 12 seconds. Did the two shoot free throws at the same rate? Explain.

11. The ratio of girls to boys on a soccer team is 2 to 3. If there are 25 players on the team, how many are girls?

12. **Model with mathematics.** A package of tickets for 4 home games costs $180. What proportion can you write to find what a 12-game package costs if all individual tickets have the same price?

13. Carlos completed 7 of 10 passes. Ty completed 21 of 30 passes. Compare their pass-completion rates.

14. Carla's team won 3 of its 5 games. Elena's team won games at the same rate and won 12 games. How many games did Elena's team play?

15. Greta completed a mile race in 5 minutes. Inez ran a mile in which each quarter-mile split was 1 min 20 seconds. Which of the two girls had the faster time? How much faster?

My Notes

Learning Targets

- Convert between measurement. Use unit rates and proportions for conversions

SUGGESTED LEARNING STRATEGIES: Visualization, Think Aloud, Discussion Groups, Sharing and Responding, Create a Plan, Identify a Subtask, Note Taking

Some problems involving measurements will require you to convert between customary and metric units of measure.

Example

A tennis court is 78 feet in length and for singles play is 27 feet in width. How many meters wide is the tennis court?

Step 1: Start by converting feet to yards: $\frac{3\text{ ft}}{1\text{ yard}} = \frac{27\text{ feet}}{x\text{ yards}}$

Step 2: Use cross-products to solve the proportion:

$27 \cdot 1 = 3x$

$27 = 3x$

$9 = x$

So, there are 9 yards in 27 feet

Step 3: Next, convert 9 yards to meters:

$\frac{9\text{ yd}}{x\text{ m}} = \frac{1\text{ yd}}{0.9144\text{ m}}$

$1x = 9(0.9144)$

$x = 8.2296$ meters

$x \approx 8.23$ meters

Solution: The tennis court is 8.23 meters wide.

Try These

a. Attend to precision. Find the length of the tennis court above in meters. Be sure to include units.

MATH TIP

In general, conversions between customary and metric systems result in approximate measurements.

The symbol $\approx$ means "is approximately equal to."

MATH TIP

Conversion factors for some common customary and metric measures:

1 yd $\approx$ 0.9144 m
1 m $\approx$ 1.094 yd
1 in. $\approx$ 2.54 cm
1 mi $\approx$ 1.61 km
1.06 qt $\approx$ 1L
1 oz $\approx$ 28.4 g
1 lb $\approx$ 0.4536 kg
2.2 lb $\approx$ 1 kg
1 cu ft (ft^3) $\approx$ 0.0283m^3

1. How do can you tell that a proportion involving conversions has been set up correctly?

2. The conversion factor for converting meters to yards is $\frac{1}{0.9144}$, and the conversion factor for converting yards to meters is $\frac{0.9144}{1}$. Use the conversion chart in the My Notes column to find the conversion factors for converting grams to ounces and converting liters to quarts.

My Notes

Five hundred years ago, the toy that we now call a yo-yo was bigger and used as a weapon in the Philippines. Each weighed about 4 pounds and was attached to a 20-ft cord.

3. About how much did one of those killer yo-yos weigh, in kilograms? Find out by writing and solving a proportion using a conversion factor as one of the ratios. Use a calculator to speed computation.

Check Your Understanding

4. Use appropriate tools strategically. Convert rounding your answers to the nearest tenth when necessary.

a. 8 in. ≈ ____ cm **b.** ____ mi ≈ 20 km **c.** 16 cm ≈ ____ in.
d. ____ L ≈ 50 qt **e.** ____ km ≈ 100 mi **f.** 60 g ≈ ____ oz
g. 44 lb ≈ ____ kg **h.** 500 g ≈ ____ lb **i.** 1.5 oz ≈ ____ g

5. Write a short note to your teacher explaining how you would estimate the number of kilometers in 19 miles.

LESSON 8-3 PRACTICE

6. The 50-km walk is the longest track event at the Olympics. To the nearest mile, about how long is the race in miles?

7. The Tour de France bicycle race is not only challenging; at 2,300 miles, it is long! In kilometers, about how long is the race?

8. The fastest ball game in the world may well be Jai-Alai. In it, players use a scoop attached to their hand to throw a small hard ball as fast as 188 mph at a granite wall. To the nearest tenth of a kilometer, about how fast is that speed in km/h?

9. Reason abstractly and quantitatively. A baseball used in major league games weighs at least 5 oz and not more than 5.25 oz. About what is that range measured in grams? Explain your reasoning.

10. About 50 years ago, the Yankees' Mickey Mantle was one of baseball's great sluggers. He is credited with hitting the longest homerun ever. It traveled a distance of 643 feet. How many kilometers did the ball travel, rounded to the nearest hundredth?

11. Reason abstractly and quantitatively. How many km/h equals 880 ft/min? Explain how you solved this problem.

12. Make sense of problems. Ed can run a mile in 6 min 30 sec. Ned can run a kilometer in 4 min. Who runs at a faster rate? Explain.

ACTIVITY 8 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 8-1

There are five position players on a starting basketball team: 2 guards, 2 forwards, 1 center.

Write a ratio in simplest form to express each relationship.

1. centers to forwards
2. forwards to guards
3. guards to players on the team
4. guards to players on the court
5. players who are not centers to players on the court

For Items 6–9, determine the rate and the unit rate.

6. \$279 for 9 tickets
7. \$18 for 6 volleyballs
8. 4 fouls in 20 minutes
9. 36 strikeouts in 54 innings

A total of 180 students and 35 chaperones are going on a field trip to the Smithsonian Institution in Washington, D.C.

Write each ratio in simplest form.

10. the ratio of students to chaperones
11. the ratio of chaperones to students
12. the ratio of students to people on the trip.

Determine the unit rate. Use mental math when you can.

13. 6 golf balls for \$15
14. 2 dozen tennis balls for \$36
15. 4 lb meat for \$18
16. 24 tickets for \$480

Determine if each pair of ratios are equivalent.

17. $\frac{8}{5}, \frac{24}{20}$ 18. $\frac{0.5}{10}, \frac{5}{100}$ 19. $\frac{1.3}{5.2}, \frac{12}{48}$

Density is the ratio of mass to volume. A 3-liter jug of honey has a mass of 4.5 kg.

20. Write the density of honey as a ratio in three different ways.
21. Write the density of honey as a unit rate.

Lesson 8-2

Solve.

22. In 2002, Takaru Kobyashi ate 50 hot dogs in 12 minutes! At that rate, and assuming that he wouldn't explode, how many dogs could Takaru eat in an hour?
23. If $\frac{3}{4}$-cup of packed brown sugar is needed for one batch of chocolate chip cookies, how much packed brown sugar is needed for five batches?
    A. $\frac{15}{100}$ cup
    B. $3\frac{3}{4}$ cups
    C. 3 cups
    D. $5\frac{3}{4}$ cups
24. **Make use of structure.** If a person walks $\frac{1}{2}$ mile in $\frac{1}{4}$ hour, how far does that person walk in $1\frac{3}{4}$ hours at that rate?
    A. $\frac{1}{8}$ of a mile
    B. $\frac{7}{8}$ of a mile
    C. 5 miles
    D. $3\frac{1}{2}$ miles

Solve by writing and solving a proportion.

25. One recipe for pancakes says to use $1\frac{1}{2}$ cup of mix to make 7 pancakes. How much mix is needed to make 35 pancakes?

26. At the local pizza parlor, game tickets can be traded for small toys. The rate is 10 tickets for 4 small toys. If Meg won 55 tickets playing skeeball, for how many small toys can she trade her tickets?

27. The ratio of boys to girls on a swimming team is 4 to 3. The team has 35 members. How many are girls?

28. Jay made 8 of 10 free throws. Kim made 25 of 45. Who made free throws at the better rate? How do you know?

Troy is going to Spain and needs to convert his dollars to Euros. He knows that when he goes, $5.00 is equivalent to about 3.45 Euros.

29. Find the unit rate of Euros per dollar

30. How many Euros will he get for $125?

31. About how many more or fewer Euros would Troy get for $125 if the exchange rate had changed to 0.75 Euros per dollar?

Lesson 8-3

Convert. Round your answers to the nearest hundredth, as needed.

32. 6 in. ≈ ____ cm

33. 500 g ≈ ____ oz

34. 24 lb ≈ ____ kg

35. ____ mi ≈ 40 km

36. ____ oz = 170.4 g

Solve. Use the conversion factors provided on page 85. As needed, round answers to the nearest hundredth.

37. How many ounces are in 80 grams?

38. What might weigh 20 kg: a small car, a tablet, a heavy suitcase, or a watermelon?

39. A recipe calls for 8 oz of raisins. The raisins come in 100-gram packages. How many packages do you need to buy?

40. A golf ball weighs about 45.9 grams. About how many ounces would a dozen golf balls weigh?

41. A regulation volleyball can weigh anywhere from 260 grams to 280 grams. In ounces, what is the least a volleyball can weigh?

42. The most a bowling ball can weigh is 7,258 grams. What is the most it can weigh when measured in pounds?

43. Lisa can run a mile in 7 minutes. At that rate of speed, how long would it take her to run 2 kilometers?

44. Jen can run a mile in 8 minutes. Which is the most reasonable time for her to run a 10-km race: 1.6 min, 5 min, 50 min, or 500 min?

An official rugby ball can weigh anywhere from 383 grams to 439 grams.

45. What is the least one of these balls can weigh, measured in ounces?

MATHEMATICAL PRACTICES
Make Sense of Problems

46. The record for the most Major League Baseball career innings pitched is held by Cy Young, with 7,356 innings. If the average length of an inning is19 minutes, how many minutes did Young play in Major League games? How many hours is this?

Proportional Reasoning

Scrutinizing Coins

Lesson 9-1 Equations Representing Proportional Relationships

My Notes

Learning Targets:

- Given representations of proportional relationships, represent constant rates of change with equations of the form $y = kx$.
- Determine the meaning of points on a graph of a proportional relationship.
- Solve problems involving proportional relationships.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Summarizing, Use Manipulatives, Look for a Pattern, Predict and Confirm, Discussion Groups

Ratios and proportions are used to solve all kinds of problems in the real world. For example, ratios and proportions are used in cooking to double recipes, by travelers to find distances on maps, and by architects to make scale models.

Work with your group to explore the proportional relationship between the number of pennies in a stack and their heights in millimeters. You will need a centimeter ruler and 25 pennies. As you work with your group, you may hear math terms or other words that are unfamiliar. Record words that are frequently used in your math notebook. Ask for clarification of their meaning and make notes to help you remember how they are used.

1. Without using your pennies or ruler, predict the height of a stack of 150 pennies, and explain why you made this prediction. Be sure to include units in your prediction.

2. **Attend to precision.** Explore this finding by measuring and recording the height of a stack of each number of pennies in the table below.

Number of Pennies	10	15	20	25
Height of Stack (mm)				

3. Write a ratio in fraction form that relates the number of pennies to the height of a stack.

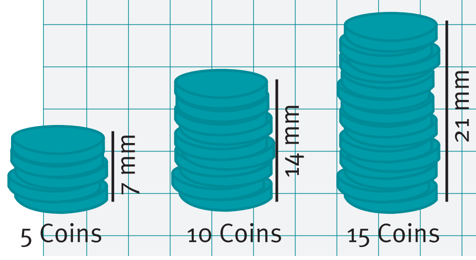
a. 10 pennies

b. 15 pennies

c. 20 pennies

d. 25 pennies

4. Write a ratio that relates the number of pennies in each stack at the right to the height of the stack.



My Notes

5. What do you notice about the ratios you wrote in Item 3?

6. Use the ratio you found in Item 4 and proportional reasoning to complete the table below.

Number of Pennies	10	15	20	25
Height of Stack (mm)				

7. Use your table in Item 6 to answer the following.

a. Write two ratios in fraction form relating the number of pennies to the height of the stacks.

b. Write these ratios as an equation.

c. Is your equation a proportion? Explain why or why not.

8. When quantities are proportional, they have a **constant rate of change**.

a. What is the rate of change of the stack of coins in Item 4?

b. Explain what the rate of change in Item 4 means.

MATH TERMS

If the rate of change remains the same throughout a problem situation, it is a **constant rate of change.**

9. **Make use of structure.** How could you find the height of a stack of 60 pennies without having 60 pennies to measure? Determine a reasonable estimate of the height and explain your method.

10. Now suppose you wanted to find the height of a stack of 372 pennies. Determine a reasonable estimate and explain your method.

11. Compare and contrast your methods for answering Items 9 and 10.

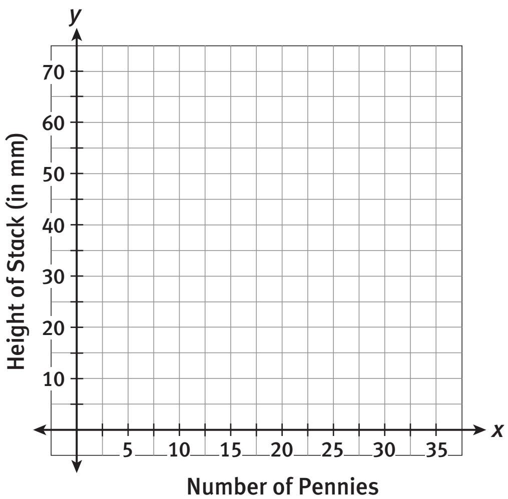
My Notes

12. Why might the value you determined for height in Items 9 and 10 be different from the actual measured height of a stack of 60 pennies or 372 pennies?

13. Write and solve a proportion to determine the number of pennies, *x*, in a stack that is 100 mm high. Use numbers, words, or both to explain your method.

The proportional relationship between the number of pennies in a stack and the height of the stack that you recorded in the table in Item 7 can also be represented in a graph. The graph will help you predict the height of a stack of pennies.

14. Graph the data from Item 7 onto the graph.



15. What does a point (x, y) on the graph mean for this situation?

16. **Construct viable arguments.** Does it make sense to include the point (0, 0) on your graph? Explain. If yes, plot (0, 0) on your graph.

My Notes

MATH TIP

Remember to think about whether or not you should connect the points on your graph.

17. If the points on your graph were connected what would the graph look like?

18. How does the graph in Item 14 show a constant rate of change?

19. Does it make sense to include the point $\left(\frac{1}{2}, \frac{5}{14}\right)$ on your graph? Explain.

20. Use your graph to predict the height of a stack with only one penny in it. Explain your method.

21. What does it mean for the ratio of number of pennies to height of the stack of pennies to be in the ratio 1:1.4?

22. Find the height of a stack of 30 pennies.

a. Use the graph. Explain your reasoning.

b. Using the height of 1 penny that you found in question 21. Explain your reasoning.

WRITING MATH

Another way to write a proportional relationship is as an *equation* of the form $y = kx$, where the constant of rate of change is k.

23. What equation could you write to find the height y in millimeters of any number of pennies x?

24. Use your equation in Item 24 to find the height of a stack of 35 pennies. Confirm this solution using your graph.

My Notes

Check Your Understanding

25. Model with mathematics. Look back at your original prediction for the height of a stack of 150 pennies.

a. Use a proportion to revise your original prediction. Explain your reasoning.

b. Use the equation you wrote in Item 24 to revise your original prediction. Justify your reasoning.

c. Explain how you could use your graph to revise your original prediction.

LESSON 9-1 PRACTICE

26. Solve the proportion $\frac{4}{5} = \frac{28}{x}$ using two different methods. Explain each method.

27. Construct viable arguments. Solve $\frac{x}{42} = \frac{3}{7}$ using two different strategies. Explain each strategy.

28. Is the ratio 4.2:1.5 proportional to the ratio12.6:4.5? Explain.

29. Is the ratio 35 to 10 proportional to the ratio 7 to 5? Explain.

30. At Lake Middle School, the average ratio of boys to girls in a classroom is 3:2. Use a proportion to predict the number of girls in a classroom that has 15 boys.

31. Complete the ratio table below to show ratios equivalent to 4:18.

48	160		8	
		20		90

32. Use the graph at the right.

a. Predict the number of chocolate chips in nine pancakes. Explain.

b. Predict the number of pancakes that would have 48 chocolate chips. Explain.

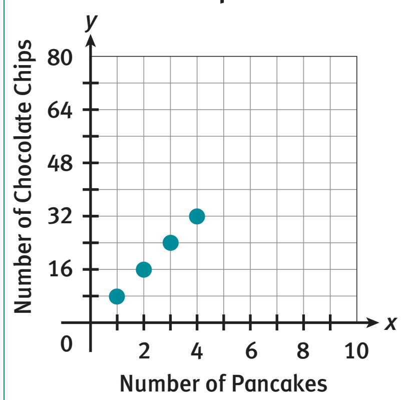
c. What does the point (1, 8) mean in this situation?

d. Which of the equations below represents this situation?

A. $y = 16x$ **B.** $y = 8x$

C. $y = x$ **D.** $y = 48x$

Chocolate Chips in Pancakes

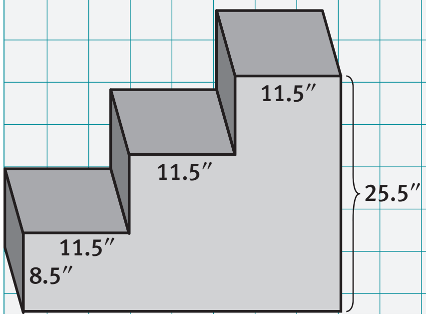


33. Three steps of a staircase are shown here.

a. What is the ratio of the width of a step to its height?

b. Explain why the staircase represents a constant rate of change.

c. What does the rate of change mean in the context of a staircase?



My Notes

Learning Targets:

- Determine the constant of proportionality from a table, graph, equation, or verbal description of a proportional relationship.

SUGGESTED LEARNING STRATEGIES: Shared reading, Marking the Text, Interactive Word Wall, Note Taking, Self Revision/Peer Revision

When two quantities are proportional they have a constant rate of change. A **constant ratio** can be found between the output values and their corresponding input values.

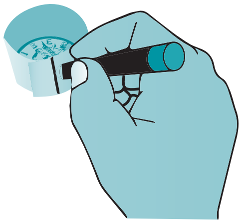
MATH TERMS

A **constant ratio** occurs when the ratio between two variables is constant.

Work with a partner and use the relationship between the circumference and diameter of a circle to explore finding a **constant of proportionality**. You will need a centimeter ruler, tape, a penny, a nickel, a dime, and a quarter.

MATH TIP

To measure the circumference, wrap a piece of tape around the edge of your coin. Make a mark on the tape to show where the tape begins to overlap.



Unwrap the tape and place it along the edge of your centimeter ruler.

1. Use your string and ruler to measure the circumference of each coin to the nearest millimeter. Record the measurement in the table below.

	Penny	Nickel	Dime	Quarter
Circumference (mm)				
Diameter (mm)				

2. Use your ruler to measure the diameter of each coin to the nearest millimeter. Record the measurement in the table above.

3. For each coin write the ratio of the length of the circumference to the length of the diameter as a fraction and as a decimal to the nearest hundredth.

MATH TIP

Find the average of a set of data items by adding the items and then dividing by the number of data items.

4. Because the ratios are very close to being the same, there is a proportional relationship. Calculate the average of their decimal ratios.

5. Suppose you had a coin with a diameter of 30 mm. What would you expect its circumference to be? Explain.

6. **Model with mathematics.** Write an equation in the form $y = kx$ using the constant of proportionality you found in Item 4 above to determine the circumference, y, of a coin with a diameter x. Explain.

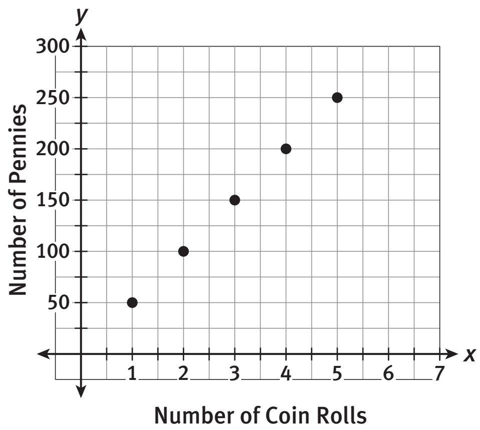
My Notes

The factor k that you multiplied by in Item 6 also represents the **constant rate of change** in the situation.

7. What is the constant rate of change in the equation you wrote?

Graphs can also be used to find a constant of proportionality in proportional relationships.

The graph below shows the number of pennies in a number of standard coin rolls.



8. Plot a point at (0, 0) and connect the points with a line. What does the point (0, 0) represent?

9. Create a table showing this information in your My Notes column.

10. Why do the points in the graph lie on a straight line?

11. What is the ratio of number of pennies to the number of coin rolls?

12. Define the variables and write an equation in the form $y = kx$ for this situation.

13. What is the constant of proportionality in this situation?

14. Describe what the constant of proportionality means in this situation.

My Notes

Check Your Understanding

Describe how to find the constant of proportionality in each representation below.

15. A ratio table

16. A graph of a proportional relationship

17. The equation of a proportional relationship

LESSON 9-2 PRACTICE

18. There are 40 nickels in every standard coin roll.

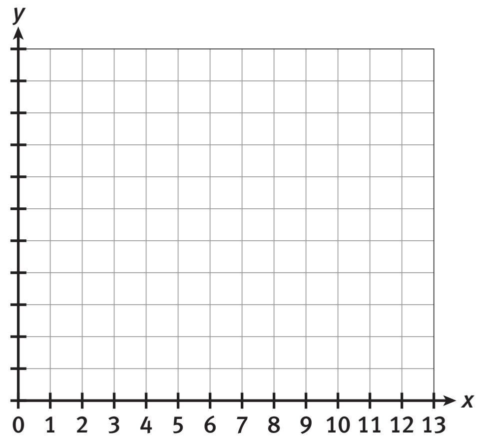
a. What is the constant of proportionality?

b. Model with mathematics. Define the variables and write an equation that can be used to show this relationship.

c. Create a table of this information.

Number of Coin Rolls						
Number of Nickels						

d. Represent this information in the graph below.



e. How many nickels are needed to fill 8 coin rolls? Explain how you determined your answer.

ACTIVITY 9 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 9-1

1. Complete the ratio table to show ratios equivalent to 16:10.

2			8	
	36	9		72

2. Solve the proportion $\frac{3}{8} = \frac{21}{x}$ using two different methods. Explain each method.

3. Solve $\frac{x}{48} = \frac{5}{6}$ using two different strategies. Explain each strategy.

4. Is the ratio 25 to 16 proportional to the ratio 5 to 4? Explain.

5. Are the ratios 2.5:3.5 and 5:7 proportional? Explain.

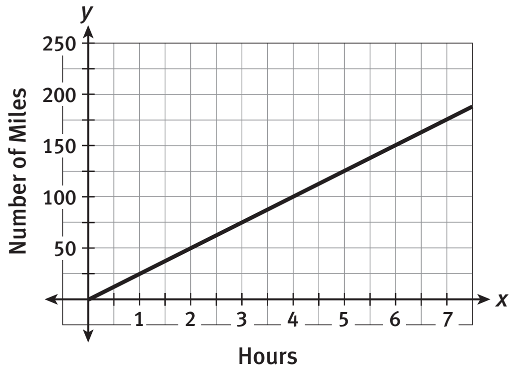
6. Is the ratio 4.2:1.5 proportional to the ratio 12:5? Explain.

7. At the library, the average ratio of hardbound books to paperback books on a shelf is 5:3.

a. Use a proportion to predict the number of hardbound books on a shelf that has 75 paperback books.

b. Use a proportion to predict the number of paperback books on a shelf that has 75 hardbound books.

For Items 8–12, use the following graph to make predictions.



8. Use the graph to predict the number of miles driven in 8 hours. Choose the correct answer below.

A. 150 miles
B. 175 miles
C. 200 miles
D. 250 miles

9. Use the graph to predict the number of hours it would take to drive 162.5 miles. Choose the correct answer below.

A. 15.5 hours
B. 6 hours
C. 6.5 hours
D. 7 hours

10. What does the point (0, 0) mean in this situation?

11. What does the point (1, 25) mean in this situation?

12. Write an equation in $y = kx$ form to represent this situation.

Lesson 9-2

For Items 13–20, use the following information.

A fruit punch uses 1.5 cups of orange juice for every cup of apple juice.

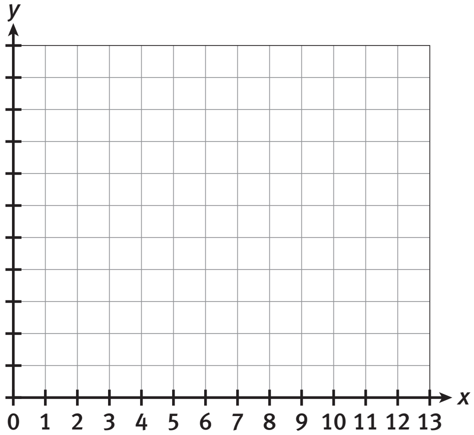
13. What is the constant of proportionality used to find the number of cups of orange juice needed for any amount of apple juice?

14. Define the variables and write an equation that can be used to show this relationship.

15. Create a table of this information.

Apple Juice (cups), x					
Orange Juice (cups), y					

16. Represent this information in the graph below.



17. How many cups of orange juice are needed for 12 cups of apple juice?

18. How many cups of apple juice are needed for 8 cups of orange juice?

19. What does the point (0, 0) mean for this situation?

20. What does the point (1, 1.5) mean for this situation?

21. Therese is on a trip overseas. She uses the table below to determine the conversion rate of her U.S. dollars to British pounds. What is the constant of proportionality?

British Pound, x	5	10	15	20	25
U.S. Dollar, y	8	16	24	32	40

22. Use the table in Item 21. Write an equation to convert British pounds to U.S. dollars.

23. Use your equation in Item 22 to determine the number of U.S. dollars Therese would spend if she bought an item that cost 110 British pounds.

MATHEMATICAL PRACTICES
Model with Mathematics

24. A tire maker produces 20,000 tires to be shipped. They inspect 200 of the tires and find that 16 are defective. How many tires of the 20,000 tires would you expect to be defective?

Ratios, Proportions, and Proportional Reasoning
WEIGHING IN ON DIAMONDS

Write your answers on notebook paper. Show your work.

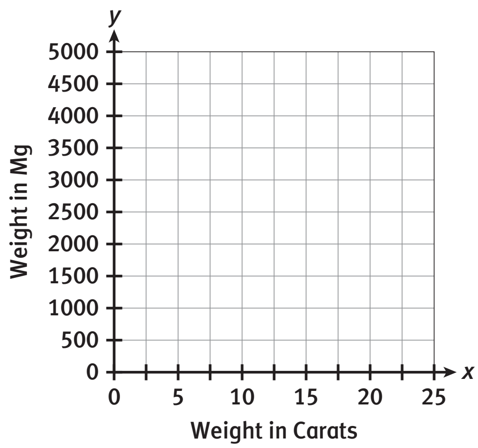
You may have had diamonds in your mouth before. Many dentists' drills are embedded with diamonds. In fact, 18% of your body is made up of carbon, and diamonds are also made of compressed carbon. That must mean you are priceless!

For Items 1–8, use the following information.

Diamonds are weighed in units called carats. Carat weight is based on the diamond's weight in milligrams. The table at the right shows the relationship between carats and milligrams.

Weight in Carats	Weight in Milligrams
$\frac{1}{2}$	100
2	400
4	800
6	1200
10	2000
25	

1. Write an equation to convert a diamond's weight in carats to its weight in milligrams. Be sure to define your variables.
2. What is the constant of proportionality represented in the table at the right?
3. Complete the last row of the table by using the constant of proportionality.
4. Use your equation to find the weight in milligrams of the Tiffany Yellow Diamond, which weighs 287.42 carats.
5. Create a graph of the information in the table.
6. Explain the meaning of the point (0, 0) on your graph.
7. Use your graph to determine the weight in milligrams of a diamond weighing 8 carats.
8. Give the ordered pair for the point on the graph that shows how many milligrams a 1-carat diamond weighs.



Solve.

9. The Cullinan is the largest rough gem-quality diamond ever found. It was 3,106.75 carats. It weighed about 0.62 kg uncut. Recall that 1 kg is equal to 2.2 pounds. What was the uncut Cullinan weight in pounds?
10. How many pounds would a 0.5 kg diamond weigh?
11. The ratio of a diamond's hardness to its specific gravity is 10:3.515, and the ratio of the hardness to specific gravity for a ruby is 9:4.05. Are these ratios in proportion? Explain your answer.

For Items 12–13, use the following information.

The largest diamond is thought to be Lucy, a star consisting of diamonds. Its weight is *10 billion trillion trillion carats*. Lucy is about 50 light-years away from Earth. One light-year is about 5.87 trillion miles, or the distance light travels through space in one year.

12. Use a proportion to determine how many trillion miles away from Earth Lucy is.

13. Write an equation in $y = kx$ form to represent this situation. Use the equation to check your answer from Item 11.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13)	• Clear and accurate understanding of ratios, unit rates, and solving proportions.	• An understanding of ratios, unit rates, and solving proportions that usually results in correct answers.	• An understanding of ratios, unit rates, and solving proportions that sometimes results in correct answers.	• Incorrect or incomplete understanding of ratios, unit rates, and solving proportions.
Problem Solving (Items 4, 7, 9, 10, 12)	• Accurate interpretation of the solution of a proportion to solve a problem.	• Interpretation of the solution of a proportion to solve a problem.	• Difficulty interpreting the solution of a proportion to solve a problem.	• Incorrect or incomplete interpretation of the solution of a proportion to solve a problem.
Mathematical Modeling / Representations (Items 1, 5, 6, 7, 8, 13)	• Accurate representation of a problem situation with a proportional equation or a graph.	• A mostly correct representation of a problem situation with a proportional equation or a graph.	• Difficulty representing a problem situation with a proportional equation or a graph.	• An incorrect or incomplete representation of a problem situation with a proportional equation or a graph.
Reasoning and Communication (Items 6, 11)	• Precise use of appropriate math terms and language to explain proportional relationships.	• An adequate explanation of solutions using proportional relationships.	• A misleading or confusing explanation of solutions using proportional relationships.	• An incomplete or inaccurate description of solutions using proportional relationships.

Ratio and Proportion

ACTIVITY 10

Patriotic Proportions
Lesson 10-1 Using Scale Drawings

My Notes

Learning Targets:

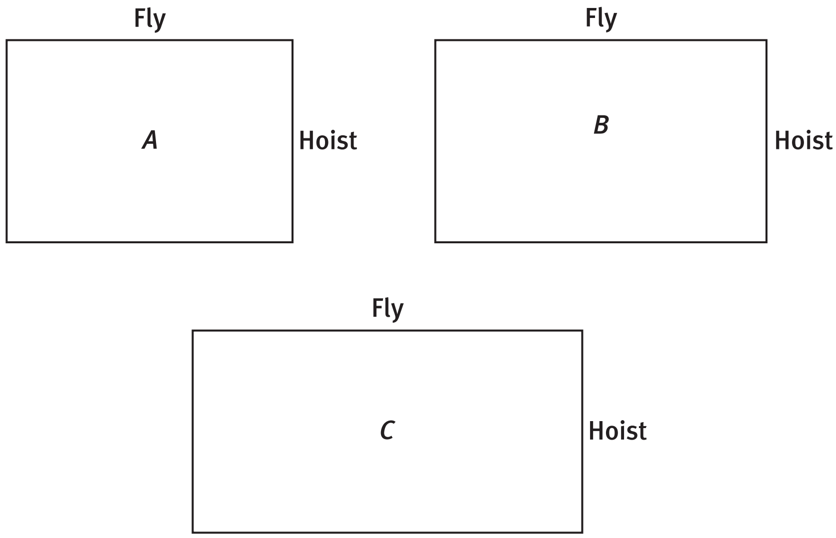
- Represent proportional relationships by equations.
- Determine the constant of proportionality from a table, graph, equation, or verbal description of a proportional relationship.
- Solve problems using scale drawings.

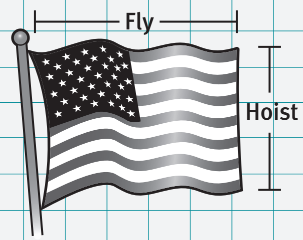
SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Paraphrasing, Critique Reasoning, Create Representations

Martha Rose Kennedy was watching an old black-and-white movie about World War II, and during the parade scene she noticed that the flags seemed to have a "funny shape." Martha did a little research and learned that according to the *U.S. Code, Title 4, Chapter 1,* the ratio of the **hoist** (height) to the **fly** (width) of the flag should be 1:1.9.

However, in the 1950s, President Dwight D. Eisenhower eased the restrictions on the dimensions of the U.S. flag to accommodate current standard sizes, such as 3 feet × 5 feet, 4 feet × 6 feet, and 5 feet × 8 feet.

1. Without using a ruler, predict which of the following rectangles will have a ratio: $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$. Explain how you made your decision.





2. a. If $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$ and the hoist is 1 ft, calculate the fly.

My Notes

b. This relationship can also be shown using the equation $y = 1.9x$, where y = the length of the fly and x = the length of the hoist. Show how the fly can be found using this equation.

c. What is the constant of proportionality in the equation in part b?

d. If the ratio remains constant and the hoist of the flag is 2 ft, calculate the fly using the constant of proportionality.

e. Check your work from part c by using a proportion to calculate the fly.

3. a. Complete the following table, which displays the correct hoist and fly of the U.S. Flag according to the *U.S. Flag Code*. Assume that the units of measure are the same for the hoist and fly and that $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$.

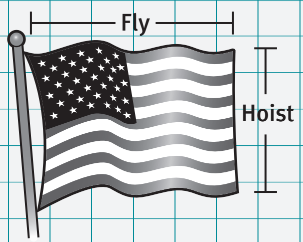
Hoist	Fly
1	
2	
3	
4	
5	
	17
	19
	25
	38

b. Use appropriate tools strategically. How can the constant of proportionality be determined from a table? Explain using the table above as an example.

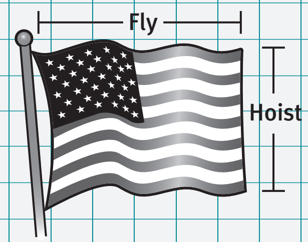
WRITING MATH

Another way to write a proportional relationship is as an equation of the form $y = mx$, where the constant of proportionality is m.

Example: The cost of a flag is $5. If x = the number of flags purchased, the equation $y = \$5.00x$ could be used to determine the cost of x flags.



My Notes



4. Suppose that the proportion used to make a flag is 3:5.

a. Use the method of determining the constant of proportionality from Item 3 to determine the constant of proportionality for this flag. Let $x =$ the length of the hoist.

b. Write the equation for determining the length of the hoist or fly.

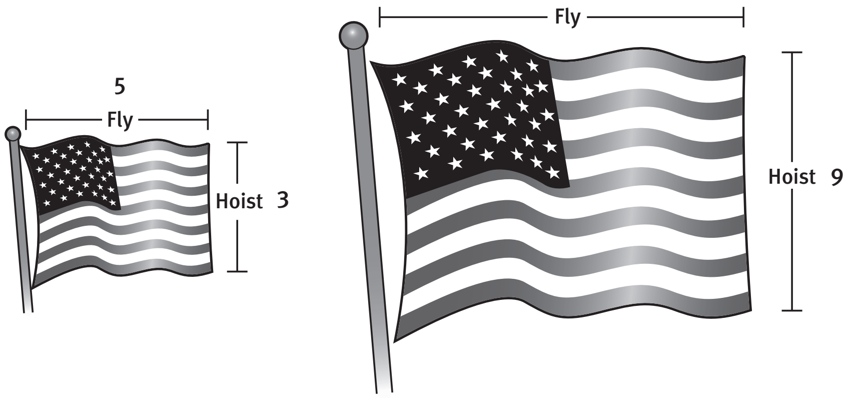
c. If the length of the hoist is 7.5 ft, determine the length of the fly.

d. Check your work from part c by using a proportion to calculate the fly.

e. Find the fly if the hoist is 1 ft. Show your work.

5. Find the difference in the lengths of a 3:5 flag and the flag measurements in the table in Item 3 whose hoist is 3 ft. Which flag has the longer fly?

Sometimes a scale drawing is used to represent the data in a problem or to show the **relative size** of two items. Use the drawings below to solve.



MATH TERMS

The **relative size** of two items shows how the size of one item is larger or smaller than the other item.

For example: The relative size of a baby is small next to a full grown adult.

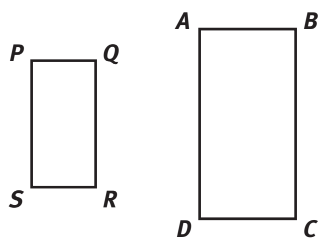
6. a. Write a proportion to represent the drawing.

b. Use the proportion to calculate the length of the unknown fly.

My Notes

MATH TERMS

A **scale drawing** of a figure is a copy of the drawing with all lengths in the same ratio to the corresponding lengths in the original. If Quad *ABCD* is a scale drawing of Quad *PQRS* then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{AD}{PS}$.



Check Your Understanding

Martha Rose decides to investigate further by creating a **scale drawing** of the U.S. flag including the thirteen stripes and the blue field for the stars. She chooses the following characteristics for her flag.

- $\frac{\text{height}}{\text{width}} = \frac{3}{5}$
- There are 7 red stripes and 6 white stripes, all of which have the same height, the hoist.
- The height of the blue field equals the height of seven stripes.
- $\frac{\text{height of the blue field}}{\text{width of the blue field}} = \frac{2}{3}$

7. The height of one stripe is what fraction of the height of the entire flag?

8. The height of the blue field is what fraction of the height of the flag?

9. **Reason quantitatively.** Since all of the information concerning the dimensions of the flag and its parts are given in terms of the height, Martha decides to begin her scale drawing by choosing 13 cm for the height. Explain why 13 cm is a good choice for the height.

LESSON 10-1 PRACTICE

10. Write an equation for the following proportional relationships.

a. $\frac{1}{2.5}$ **b.** $\frac{6}{18}$ **c.** $\frac{5}{22.5}$ **d.** $\frac{4}{17}$

11. Determine the constant of proportionality for the following.

a. a ratio of 3 to 4
b. the point (2, 5) on a graph
c. three red to two blue
d. $7 = 5m$

12. The long side of a rectangle is 4 times the side of a square of length 3. What is the length of the side of the rectangle?

13. **Make sense of problems.** Two stripes on the American flag represent what fraction of the height of the flag?

My Notes

Learning Targets:

- Given the scale of a map and a distance on a map, find the actual distance.
- Convert scale factors with units to scale factors without units

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Summarizing, Think Aloud, Create Representations

The Green family set out by car from Boston to visit the Statue of Liberty in New York City, the Liberty Bell in Philadelphia, and the nation's capital in Washington, D.C. They had a map of the Northeast region of the United States. In the corner was a scale that showed this:

1 inch

|---------|---------|

0 50 100,

which gives the proportion $\frac{\text{inches}}{\text{miles}} = \frac{1}{100}$.

1. a. Write the proportion to find the number of miles from Boston to Philadelphia if the distance measures 3.08 inches on the map.

b. Solve the proportion for the number of miles.

2. The next stop was Washington, D.C., which was 1.38 inches farther from Boston. What was the total mileage from Boston to Washington, D.C.?

3. Reason quantitatively. The distance from Boston to New York is 216 miles.

a. If this represents 4 inches on the map, what is the scale used?

b. Calculate the number of inches on the map if the scale is $\frac{\text{inches}}{\text{miles}} = \frac{1}{50}$?

c. What is the difference in miles travelled over 4 inches using the scales in parts a and b?

My Notes

The Green family thought that they would travel to Mount Rushmore for their next trip. Since that is in Keystone, South Dakota, they need to use a different map. The scale of that map is $\frac{\text{inches}}{\text{miles}} = \frac{1.5}{250}$.

4. The distance on the map is approximately 11.5 inches. How many miles is this?

5. How many inches represent $\frac{3}{4}$ of the trip?

After visiting Washington, D.C., Joey Green wanted to make a scale model of the Washington Monument. The actual height is 555 feet.

6. a. Is the scale $\frac{\text{inches}}{\text{feet}} = \frac{1}{25}$ a reasonable scale for the model?

b. What is the scale if the height of the model is 10 inches?

The scales that the Green family have been using have different units. If they want to eliminate the units, both need to be the same. For example, $\frac{1\text{ inch}}{1\text{ foot}} = \frac{1\text{ inch}}{12\text{ inches}} = \frac{1}{12}$.

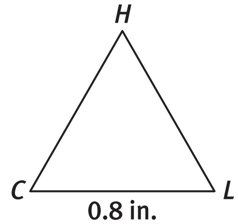
7. As a class project, you are asked to make a scale drawing of your home.

a. What unit should be used for a scale drawing of your bedroom?

b. What unit should be used for a scale drawing of your yard?

8. If feet were used as the unit for the model of the Washington Monument, what would the scale be if the model is only 10 inches high?

My Notes



9. Model with mathematics. On a neighborhood map shown at the right, 1 inch = 5 miles. If the distance from your house (H) to the convenience store (C) is 3 miles, the distance from your house to the library (L) is 6 miles, and the distance from the convenience store to the library is 4 miles, label the path from H to C and from H to L in inches.

Check Your Understanding

10. If the scale on a map is $\frac{\text{inches}}{\text{miles}} = \frac{1}{10}$, find the following actual distances.

a. 2 inches **b.** $\frac{1}{2}$ inch **c.** 3.5 inches

11. Two towns on a map are $2\frac{1}{4}$ inches apart. The actual distance between the towns is 45 miles. Which of the following could be the scale on the map?

a. $\frac{1}{2}$ inch : 10 miles **b.** 1 inch: 5 miles **c.** 1 inch : 20 miles

LESSON 10-2 PRACTICE

12. If the scale on a map is $\frac{\text{inches}}{\text{miles}} = \frac{1}{50}$, find the following actual distances.

a. 2.5 inches **b.** 4.75 inches **c.** 7.6 inches

13. If the scale on a map is $\frac{\text{inches}}{\text{feet}} = \frac{2}{10}$, how would the following lengths be represented?

a. 15 feet **b.** 30 feet **c.** 55 feet

14. On a map, the scale is $\frac{\text{inches}}{\text{miles}} = \frac{1}{75}$. Going from home to the first destination is 1.5 inches, and then from there to the next destination is 2.25 inches. How many miles were traveled?

15. Reason abstractly. You are making a scale model of the White House using blocks that represent 15 feet. If the length of the White House is 170 feet, can you use these blocks without having a part of the block extend outside the length?

16. a. A scale drawing of your classroom is 3 inches by 5 inches. If one inch represents 6 feet, what is the actual size of your classroom?

b. Using another scale, the shorter side of the classroom is 1.5 in. in a scale drawing. What is the length of the longer side of the classroom in the same scale drawing?

My Notes

Learning Targets:

- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing.
- Reproduce a scale drawing at a different scale.

SUGGESTED LEARNING STRATEGIES: Close Reading, Visualization, Create a Plan, Create Representations, Look for a Pattern

The Constitution of the United States is a document with dimensions $23\frac{5}{8} \times 28\frac{3}{4}$ inches.

The Declaration of Independence is $24\frac{1}{2} \times 29\frac{3}{4}$ inches.

Recall that the scale of a figure "redraws" the figure with a new size.

1. a. If the Declaration of Independence is reproduced at half scale, what would be its dimensions?

b. Attend to precision. What is the area of this document? (Round to the nearest hundredth.)

c. What would be the dimensions of the Constitution reproduced at $\frac{1}{4}$ scale?

MATH TIP

To find the scale you will need to use division.

2. A student version of the Constitution fits on a piece of paper that is $8\frac{1}{2} \times 11$ inches. Which of the following is the largest scale that can be used so that it fits on the paper?

A. $\frac{1}{8}$ **B.** $\frac{1}{4}$ **C.** $\frac{3}{8}$ **D.** $\frac{1}{2}$

My Notes

3. a. How would you determine the largest scale to use for the Declaration of Independence to allow it to fit on an $8\frac{1}{2} \times 11$ inch piece of paper?

b. What is the largest scale that can be used?

The New England Patriots are a professional football team. Some information about football fields is given below.

- The dimensions are 160 ft by 360 ft.
- The end zone is 160 ft by 30 ft.
- The upright (goal post) is 10 ft off the ground.
- The width of the upright is 18 ft, 6 in.

4. a. Make sense of problems. What are the dimensions of a field that is drawn $\frac{1}{8}$ scale?

b. The area of a full-sized field is how many times larger than the area of a field at $\frac{1}{8}$ scale?

c. Sketch and label a diagram of a football field reproduced at $1\frac{1}{4}$ scale.

5. For a backyard game of football you make an upright that is 5 ft off the ground and 9 ft 3 in. high. What scale did you use?

6. Express regularity in repeated reasoning. A football field scaled to $\frac{1}{4}$ its size is then scaled to $\frac{1}{4}$ of its size again. What are the dimensions of this field?

My Notes

Check Your Understanding

7. The dimensions of a professional basketball court are 50 ft by 94 ft. A game maker is producing a video basketball game and the court must be reproduced to fit on a computer screen that is 13 inches by $10\frac{1}{2}$ inches. Which of the following scales is the greatest they can use to fit on the screen?

A. $\frac{1}{2}$ **B.** $\frac{1}{8}$ **C.** $\frac{1}{100}$ **D.** $\frac{1}{150}$

8. A book cover is 8 inches by $10\frac{1}{4}$ inches. What are the dimensions of the cover when it was reproduced for a catalog picture using a $\frac{1}{12}$ scale?

9. A car model box has "$\frac{1}{8}$ scale" printed on the outside of the box. If the actual car is 178 inches long, what is the length of the model?

LESSON 10-3 PRACTICE

10. A document is 20 in. by 34 in. What are the dimensions of documents using the following scales?

a. $\frac{1}{4}$ **b.** $\frac{1}{2}$ **c.** $\frac{3}{4}$ **d.** $1\frac{1}{2}$

11. a. A teacher wants a large poster of the Declaration of Independence that is three times its actual size. What are the dimensions of the poster?

b. What is the area of the poster?

12. Reason quantitatively. A copy of a document that was originally 24 in. by 36 in. is now 8 in. by 12 in. What scale was used for the reduction?

Football Field Measurements

- The dimensions are 160 ft by 360 ft.
- The end zone is 160 ft by 30 ft.
- The upright (goal post) is 10 ft off the ground.
- The width of the upright is 18 ft 6 in.

13. a. A giant paper football field is made for the floor of the gym. The length of the gym is 90 ft. Using this same scale, determine the width of the paper football field.

b. What is the area of the paper field?

c. Compare the area of the paper to the area of an actual field.

d. Using this scale, what would the height and width of the upright be?

14. A document that is 30 in. × 40 in. is redrawn at $1\frac{1}{2}$ scale and redrawn again at $\frac{1}{2}$ scale. What are the final dimensions?

ACTIVITY 10 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 10-1

1. The ratio of the hoist to the fly of an American Flag is $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$.

a. Using this ratio, determine the fly of a flag that has a hoist of 3 feet.

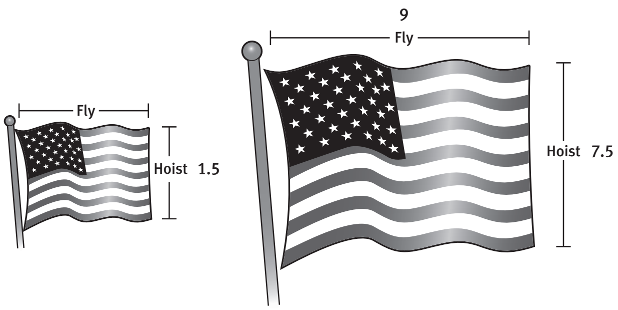
b. If the ratio is changed to 3:5, determine the fly that has a hoist of 4.5 feet.

2. In the equation $y = 1.9x$, 1.9 represents the constant of proportionality.

a. Find the constant of proportionality for the ratio $\frac{2}{5}$.

b. Find the constant of proportionality for the ratio $\frac{4}{7}$.

3. Use the following drawings to determine the missing dimension.



Lesson 10-2

4. The scale on a map has the proportion $\frac{\text{inches}}{\text{miles}} = \frac{1}{50}$.

a. How many miles is 3.25 inches on the map?

b. How many miles is 5.4 inches on the map?

5. a. If 3 inches on a map covers 225 miles, what is the scale of inches to miles?

b. If 4.5 inches on a map covers 270 miles, what is the scale of inches to miles?

6. The Statue of Liberty is 305 feet tall. What scale would be used to make a model 20 inches high?

7. A sketch of the Roosevelt Room in the White House is drawn to $\frac{1}{15}$ scale. The sketch shows a room that is 3 feet by 4.5 feet. What are the actual dimensions of the room?

Lesson 10-3

8. To make a large poster of the Bill of Rights to hang on the classroom wall, a poster that is 22 inches by 46 inches is copied. If the classroom version is to be $3\frac{1}{2}$ times scale, what are the dimensions of the new document?

9. Complete the following table.

Scale	Length	Width
1	14	8
1.5		
2		
2.5		

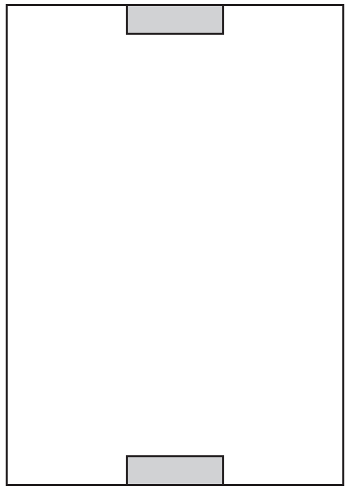
MATHEMATICAL PRACTICES
Reason Quantitatively

10. a. Using the data in the table for Item 9, what is the area of the item with scale 1.5?

b. What is the area of the item with scale 2.5?

Write your answers on notebook paper. Show your work.

A competitive youth soccer team is preparing for a soccer tournament.



1. The coach uses a scale drawing of the soccer field, shown in the diagram, to review plays with the team. The diagram uses a scale of $\frac{1}{4}$ in.:30 ft
   a. Explain how to use the scale and the scale drawing to find the actual dimensions of the soccer field.
   b. What are the actual length and the width of the soccer field?
   c. What is the actual area of the soccer field?
2. The shaded box indicates the goal box. How long is the actual goal box?
3. The center circle is not included in the diagram. On the field, the center circle has a diameter of 60 feet. How long would the diameter of the center circle be if it were included on the scale drawing? Explain your thinking.
4. The coach wants to make a larger version of the scale drawing to distribute to team members.
   a. Use the scale $\frac{1}{2}$ in.:15 ft to reproduce the scale drawing. Explain your thinking.
   b. The actual width of the goal box is 18 feet. Include the goal box, to scale, in the new scale drawing.
5. The soccer team must travel to the tournament. On a map, the tournament is 6.5 centimeters away. The map scale is 2 cm = 25 mi.
   a. What is the actual distance represented by 1 cm on the map
   b. How far will the team have to travel to the tournament?

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-c, 2, 3, 4a-b, 5a-b)	• Effective understanding and accuracy in using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings. • Correctly determining a scale factor.	• Using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings with few errors. • Determining a scale factor.	• Using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings with multiple errors. • Errors in determining a scale factor.	• Incorrect or incomplete understanding of using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings. • No understanding of determining a scale factor.
Problem Solving (Items 1b-c, 2, 3, 5b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Item 1a-c, 2, 3, 4a-b, 5b)	• Clear and accurate understanding of writing a proportion to solve a problem involving scale. • Creating a clear and accurate scale drawing.	• Writing a proportion to solve a problem involving scale. • Creating a scale drawing that is largely correct.	• Difficulty writing a proportion to solve a problem involving scale. • Difficulty creating a scale drawing.	• Little or no understanding of writing a proportion to solve a problem involving scale. • An inaccurate or incomplete scale drawing.
Reasoning and Communication (Item 1a, 3, 4a)	• Precise use of math terms and language to explain scale and scale drawings.	• An adequate explanation of scale and scale drawings.	• A misleading or confusing explanation of scale and scale drawings.	• An incomplete or inaccurate explanation of scale and scale drawings.

Ratio and Proportion

Well, There is More Than One Way

Lesson 11-1 Basic Percent Problems

My Notes

Learning Targets:

- Find a percent of a number.
- Find the percent that one number is of another.
- Given the percent and the whole, find the part.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Note Taking, Think-Pair-Share, Simplify the Problem, Look for a Pattern

There are several methods you can use to solve **percent** problems.

One way to solve such problems is to use a proportion like the one below.

$$\text{percent} = \frac{\text{number}}{\text{hundred}} \text{ and } \frac{\text{number}}{100} = \frac{\text{part}}{\text{whole}}$$

MATH TERMS

A **percent** is a ratio that compares a number to 100 and uses the % symbol.

Example A

The percent of moviegoers under age 30 is 31%. There were a total of 1,290,000,000 moviegoers in 2011. What number of moviegoers in 2011 were under age 30?

Step 1: Use the given information to set up a proportion.

$$\frac{31}{100} = \frac{m}{1{,}290{,}000{,}000}$$

The fraction, 31 over 100, represents the percent of moviegoers under 30.
The variable m represents the number of moviegoers under 30. This is the part of the total number of moviegoers, or the whole.
The 1,290,000,000 under the m represents the total number of moviegoers in 2011.

Step 2: Cross-multiply and write the cross products as an equation.

$$31 \cdot 1{,}290{,}000{,}000 = 100 \cdot m$$

Step 3: Solve the equation for m, the number of moviegoers under age 30.

$$31 \cdot 1{,}290{,}000{,}000 = 100 \cdot m$$

$$m = \frac{31 \cdot 1{,}290{,}000{,}000}{100}$$

$$m = 399{,}900{,}000$$

Solution: The number of moviegoers under 30 was 399,900,000.

My Notes

If we go back to the original proportion $\frac{\text{number}}{100} = \frac{\text{part}}{\text{whole}}$ and consider that $\frac{\text{number}}{100}$ can be written as decimal, we now have the equation:
percent (as a decimal) $= \frac{\text{part}}{\text{whole}}$

In Example A, we wanted to know the **part**, so we can solve the above equation for **part** by multiplying both sides by the **whole**.

$$\textbf{whole} \cdot \text{percent (as a decimal)} = \cancel{\textbf{whole}} \cdot \frac{\text{part}}{\cancel{\text{whole}}}$$

The two wholes on the right hand side cancel each other out, and the result is the percent equation:

$$\text{whole} \cdot \text{percent (as a decimal)} = \text{part}$$

Substitute the information from the movie situation in the percent equation.

$$1{,}290{,}000{,}000 \cdot 0.31 = \text{part}$$
$$399{,}900{,}000 = \text{part}$$

The percent equation can also be used when the percent and the part are known, and you want to find the whole.

Example B

We know that 68% of a number is 204 but want to know the total that 204 is part of.

Step 1: Convert the percent to a decimal.

$$68\% = 0.68$$

Step 2: Set up the percent equation using the information you know.

$$\text{percent} \cdot \text{whole} = \text{part}$$
$$0.68x = 204$$

Step 3: Solve the equation for x by dividing both sides by 0.68.

$$\frac{0.68x}{0.68} = \frac{204}{0.68} \qquad x = \frac{204}{0.68} = 300$$

Solution: 68% of 300 is 204. Check using the percent equation.

$$0.68 \cdot 300 = 204$$

My Notes

The percent equation has been used to solve for the part and for the whole. Lastly, use the equation to solve for the percent.

Example C

What percent is 208 of 320?

Step 1: Set up the percent equation using the given information.

$$\text{percent} \cdot \text{whole} = \text{part}$$
$$p \cdot 320 = 208$$

Step 2: Solve the equation for p by dividing both sides by 320.

$$\frac{p \cdot 320}{320} = \frac{208}{320}$$
$$p = \frac{208}{320}$$
$$p = 0.65 = 65\%$$

Solution: 208 is 65% of 320.

Try These A-B-C

a. What percent is 49 of 50?
b. 30 is 6% of what number?
c. 32% of 250 is what number?

Check Your Understanding

1. Women make up 5% of all film directors. There are 122,500 film directors working. How many women are directing films?
2. There were 548 films released in 2011. Of these, approximately 8.75% were rated PG (parental guidance suggested). How many PG movies were released in 2011?
3. 35% of the total length of the screen at the IMAX is 51.8 feet. What is the total length of the screen?
4. 7.5% of the box office receipts on a Friday night are $270. What are the total box office receipts?
5. **Make sense of problems.** The movie *Avatar* is the number one movie in U.S. history, earning $760,507,625. The second place movie is *Titanic*, earning $600,788,188. What percent is $600,788,188 of $760,507,625?
6. **Attend to precision.** There are 143 people in an audience. Out of this number, 63 are female. What percent of the people in the audience are male?

My Notes

LESSON 11-1 PRACTICE

Round your answers to nearest hundredth, if needed.

7. In a group of 75 fourth graders, 20% do not like hot chocolate. How many students like hot chocolate?

8. The center on the basketball team scored 19 of the team's 98 points. What percent of the points did he score?

9. Ed spent 8.5% of his savings on lunch, which cost $5.25. How much did he have in savings before lunch?

10. The school band sold T-shirts to fund a trip to play in a parade. They collected $570, and the band made 34% of that amount. How much money do they have for their trip?

11. **Construct viable arguments.** Is 10% of 10% of 100 the same as 20% of 100?

12. A ticket agency charges a 9.5% fee on all tickets sold. If a ticket costs $40, what is the fee?

13. **Reason quantitatively.** The student population of a school consists of 300 girls and 500 boys. If 53% of the girls play a sport, and 39% of the boys play a sport, who plays more sports?

14. In a bag of marbles, 12% were red, 14% were blue, and the rest were white. If the bag has 250 marbles, how many were red or blue?

15. Fifty-six out of 128 students went on a trip during vacation. What percent of students went on trips?

16. The school chorus has 52 students, which represents 26% of the seventh graders. How many students are in the seventh grade?

My Notes

Learning Targets:

- Solve problems about sales tax, tips, and commissions.

SUGGESTED LEARNING STRATEGIES: Close Reading, Think Aloud, Summarizing, Create a Plan, Identify a Subtask

A sales tax is the amount added to the cost of an item. Sales taxes are most often a percent of the purchase price. Sales tax can be determined using the percent equation. A ***tip*** is calculated the same way.

ACADEMIC VOCABULARY

A ***tip*** is an optional payment to someone who provides good service. This is in addition to the cost of the bill.

Example A

In Massachusetts, the sales tax is 6.25%. If someone buys a sweater for \$24.95, what is the total cost, including sales tax?

Step 1: First, determine the sales tax on the item using the formula

$$\text{percent} \cdot \text{whole} = \text{part}$$

Step 2: Substitute the decimal form of the percent into the equation and determine the amount of the sales tax.

$$0.0625 \cdot \$24.95 = \$1.56$$

Step 3: Add the sales tax to the original amount of the sweater to determine the total cost.

$$\$24.95 + \$1.56 = \$26.51$$

Solution: The total cost is \$26.51.

Example B

A real estate agent earns a commission when he or she sells a home. The commission is 5%. If the agent sells a \$400,000, a \$250,000, and a \$300,000 house in one month, what is the commission for the month?

Step 1: Add to find the total value of the homes.

$$\$400{,}000 + \$250{,}000 + \$300{,}000 = \$950{,}000$$

Step 2: Determine the commission using the formula

$$\text{percent} \cdot \text{whole} = \text{part}$$

$$0.05 \cdot \$950{,}000 = \$47{,}500$$

Solution: The agent gets paid \$47,500 in commissions.

CONNECT TO BUSINESS

A ***commission*** is a fee paid for services and is usually a percent of the total cost.

Try These A-B

a. A salesperson at a car dealership has a salary of \$1,200 per week plus a 2% commission on sales. If a salesperson had sales of \$135,000 in one week, what was he paid that week?

b. Two businesswomen are having lunch at a restaurant. Their bill came to \$47.85, and they want to leave an 18% tip. What is the amount of the tip?

My Notes

Check Your Understanding

1. A skier needs to buy new ski poles during a ski trip to Utah. The price of the poles is $24 and the sales tax is 4.7%. What is the total cost of the poles, rounded to the nearest cent?
2. Josephina likes the service she receives at her favorite café and wants to leave a 20% tip. Her bill is $22.58. What tip should she leave?
3. Film studios make a commission from every ticket sold. The movie tickets at Carbob Theatres cost $8.50. The studios earn a 40% commission. What amount of the movie ticket do they earn?
4. Nick is selling software and earns 12% of his sales as a commission. If he sells a total of $668 in software this week, how much is his commission?

LESSON 11-2 PRACTICE

5. Bobby delivers newspapers in his neighborhood. In addition to a weekly salary, he earns tips from the people he delivers to. If he delivers $200 worth of papers each week and earns 17% in tips, how much does he make from tips each week?
6. The sales tax on a piece of furniture that cost $450 was $28.13. What was the percent sales tax?
7. An art dealer makes a 17.5% commission on every painting sold. If a painting sold for $1,500, what was the commission?
8. **Construct viable arguments.** Sam lives in Massachusetts and wants to buy a television that costs $2,000. The sales tax in Maine is only 5%, which is less than the 6.25% in Massachusetts. Dave tells Sam to just buy the television in Massachusetts. If gas costs $30 to get to Maine, who is correct? Explain.
9. Art supplies in Washington, D.C., cost $28. If the sales tax is 6%, what was the total cost of supplies?
10. A family went out to dinner and the bill was $113. If they want to leave a 19% tip, how much should they leave?
11. A salesperson earning a 22.5% commission takes home $1,200 as a commission. What was the dollar value of the items she sold?
12. If a new CD player in North Carolina costs $225 and the sales tax is 4.75%, how much will a customer save during a tax-free holiday, when tax does not need to be paid?
13. **Reason quantitatively.** Joe leaves $3 as a tip for a meal that cost $22. If a good tip is between 15% and 20%, is this a good tip?

ACTIVITY 11 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 11-1

1. Find the percent of the number.
   a. 35% of 750
   b. 24% of 480
   c. 105% of 400
   d. 54% of 220
2. Solve for the number.
   a. 75 is 20 percent of what number?
   b. 82 is 70 percent of what number?
   c. 160 is 120% of what number?
   d. 18 is 9% of what number?
3. Find the percent.
   a. 32 is what percent of 160?
   b. 15 is what percent of 165?
   c. 120 is what percent of 90?
   d. 36 is what percent of 144?
4. Thirteen out of twenty-one people surveyed prefer fresh fruit over canned fruit. What percent of people prefer fresh fruit?
   A. 38.1%
   B. 162%
   C. 61.9%
   D. 161.9%
5. A survey at Lake Middle School shows that only 8% of the student population walks to school each day. If 1,200 students attend the school, about how many walk to school? Explain your reasoning.
6. The population of a school is 400 students. Next year it is expected to be about 120% of what it is now. Choose next year's student population from the answers below.
   A. 320 students
   B. 80 students
   C. 400 students
   D. 480 students
7. On Monday night, 21% of the television shows are comedy shows. There are 13 comedy shows on Monday night. How many total shows are on Monday night? Explain your reasoning.
8. The soccer team made a total of 36 goals this season and 22 goals last season. What percent of this year's total did the team make last year?
9. Out of 139 seventh graders, 86 voted math as their favorite subject. What percent is this?

Lesson 11-2

10. A realtor sells a house for $460,000. If she earns 3% of the house's price as a commission, how much is her commission?

11. Nicole made 24% of her daily salary in tips, receiving $28. What is her daily salary?

12. Kevin bought some souvenirs in Missouri for $13.50. The sales tax is 4.225%. What is the total cost of the souvenirs?

13. Terry has a goal of making $300 in commissions for the week. If her commission is based on 18% of sales, how much does she need to sell to make her goal?

14. Karl bought a pair of jeans for $25 at a store in California, where the sales tax is 6.5%. He was charged $1.45 in sales tax. Is this correct? Explain why or why not.

15. Five friends went out to dinner, and the bill came to $85. They wanted to leave a 17% tip because they thought the waiter did a good job. If they split the bill evenly, how much does each person need to pay?

MATHEMATICAL PRACTICES
Construct Viable Arguments

16. Max's bill at a restaurant is $8.67. He uses the following method to figure out a tip of about 15%.

Round the amount of the bill to the nearest ten cents. Move the decimal point one place to the left. Then find half of the result. Add the last two answers, and round to the nearest five cents.

How much will the tip be for this bill? Why does Max's method work?

Ratio and Proportion

ACTIVITY 12

Like Animals? Have I Got a Job for You!

Lesson 12-1 Percent Increase and Decrease

My Notes

Learning Targets:

- Solve problems about percent increase, percent decrease, markups, and discounts.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Summarizing, Group Presentation, Think-Pair-Share, Create a Plan

Veterinarians care for the health of animals. They diagnose, treat, or research animal diseases in homes, zoos, and laboratories. Although most veterinarians work in clinics, others travel to farms, work outdoors, or work in labs.

There were 61,400 veterinarians practicing in the United States in 2010.

The U.S. government predicts that the need for veterinarians will rise in the future. In fact, they predict that 22,000 more veterinarians will be needed by 2013.

1. a. To find the **percent increase** predicted for veterinarians from 2010 to 2013, first find the difference in the number of veterinarians from 2010 to 2013.

b. What was the original number of veterinarians?

c. Use the equation to determine the percent increase.

2. What would the **percent decrease** be if there were to be 10,000 fewer veterinarians in 2013?

3. A pet album that usually sells for \$12.50 was put out on sale for \$8.50. What is the percent decrease?

4. The price of a set of feeding bowls for dogs had been \$9.25, but was increased to \$11.10. What was the percent increase?

MATH TIP

Percent increase or decrease can be found using an equation.

$$\left(\frac{\text{difference}}{\text{original amount}}\right) \cdot 100 = \text{\% of change}$$

My Notes

Check Your Understanding

5. Dr. Detwiler saw 500 dogs. In 2010, 21.4% were considered to be obese, and in 2011, 20.6% were considered to be obese. To find the **percent decrease** from 2010 to 2011, students Marley and Leland used two different methods.

Marley		Leland	
$500 \cdot 0.214 = 107$ $500 \cdot 0.206 = 103$ $107 - 103 = 4$ $\left(\frac{4}{107}\right) \cdot 100$ $= 3.74\%$		$21.4 - 20.6 = 0.8$ $\left(\frac{0.8}{21.4}\right) \cdot 100 = 3.74\%$	

a. Use the empty space in the table to explain what each student did.

b. Explain which method makes better sense to you and why.

LESSON 12-1 PRACTICE

6. Dr. North Piegan's practice sees 125 cats. In 2010, 24.8% of cats were classified as obese, compared to 21.6 percent in 2011. Find the percent decrease in obese cats in one year.

7. Dr. Piegan also treated 440 dogs. The percentage of obese dogs increased from 52.5 percent in 2010 to 55.0 percent in 2011. What was the percent increase?

8. Susie's dog weighed 32 pounds in 2010. At the end of 2011, the dog weighed 43.5 pounds. What was the percent increase in weight from 2010 to 2011?

9. **Reason quantitatively.** Pet Food, Inc. created a new package size that was 25% larger than the old package. If the old package contained 20 pounds, how much does the new package contain?

10. Sunil bought a new crate for his dog. The old crate was 8 square feet, and the new crate was 12 square feet. What is the percent increase in space?

11. Pet Store, Inc., found that they were selling more guinea pigs and fewer birds. In 2010, they had 42 birds in stock, and in 2011, they decided to only keep 34 birds in stock. What was the percent decrease in inventory from 2010 to 2011?

12. Jay has a pet grooming business. In 2010, he groomed 200 dogs. In 2011, he groomed 450 dogs. What was the percent increase in the number of dogs groomed?

13. In 2006, there were 72 million dogs in U.S. homes. That number declined by 2 million in 2010. What was the percent decline in the number of dogs in U.S. homes?

My Notes

Learning Targets:

- Solve problems about percent increase, percent decrease, markups, and discounts.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Interactive Word Wall, Summarize, Think/Pair/Share, Share and Respond, Note Taking, Create a Plan, Identify a Subtask

Veterinarians markup medicine and supplies so that they can earn a salary and pay their bills. A **markup** is an amount added to the original cost of an item to find the selling price. Items can also be **marked down** by decreasing the original price. Markdowns are also considered to be **discounts**.

MATH TERMS

A **discount** is a reduction in price.

~~$10~~
$8

Here the discount is $2.

Sometimes discounts are a percent, such as a 10% discount.

Example A

A pet owner bought a leash at the veterinarian's office. The leash cost the vet $2.25 to buy from the supplier. The vet marked the price up 45% before selling it to the pet owner. What is the price the pet owner paid?

Step 1: To determine the amount of the markup, use the percent equation.

$$\$2.25 \cdot 0.45 = \$1.01$$

Step 2: Add this amount to the original amount to find the final cost.

$$\$2.25 + \$1.01 = \$3.26$$

Solution: The final cost is $3.26.

Example B

Suppose the vet wanted to make $0.75 profit on each leash. What is the percent markup applied?

Use the percent increase equation.

$$\left(\frac{\text{difference}}{\text{original amount}}\right) \cdot 100 = \text{percent markup}$$

$$\left(\frac{0.75}{2.25}\right) \cdot 100 = 33.33\%$$

Solution: The markup is 33.33%.

Try These A-B

a. A dog food company sells its canned dog food to stores for $0.35 per can. One of the stores sells the can of dog food for $0.79. What percent markup did the store use?

b. A bird food company sells its box of birdseed to the pet shop for $1.89. The pet store uses a 55% markup. What is the pet store price for the box of birdseed?

GROUP DISCUSSION TIPS

Read the problem scenarios and look for key information. Organize the information and describe the math concepts you will use to solve the problems.

My Notes

Check Your Understanding

1. A cat's catnip toy is on sale for 10% off the regular price of $3.30. Find the sale price.
2. Dr. Star Blanket gives a 20% discount if his customers pay cash for their office visits. Determine the cost of a $65 office visit if the customer pays cash.
3. A horse that is ill needs medication. The medication costs the veterinarian $17.50 to buy. He marked up the medication 54.2% before selling it to the customer. Find the final selling price of the horse's medication.

LESSON 12-2 PRACTICE

4. Kim purchases pet insurance for her cat. In 2011, she paid $40 per month. She switched to discounted insurance she bought online for $34 dollars a month. What was the percent discount?
5. A new dog leash with an attached flashlight was on sale for 20% off. If the original price was $16.50, what is the discounted price?
6. **Make sense of problems.** A sale on dog food that costs $10 a bag says buy 1, get one $\frac{1}{2}$ off. What is the true percent discount on the dog food?
7. A veterinarian purchases some medication for $19.50 and wants to make a $2 profit. What is the percent markup on this medication?
8. The grooming department at a local pet store is running a sale of 35% off grooming. If the regular cost is $47.75, what is the sale price?
9. A doggie day care charges $21.50 per day. If you leave your dog 5 days a week, the discount is 20%. What is the cost of leaving your dog for 5 days in a week?
10. A bag of birdseed that normally costs $22.25 is marked up to $25.30. What is the percent markup?
11. **Reason quantitatively.** A gerbil cage, priced at $18, is discounted 20% for one week, and then this price is marked up 20%. Is the price back to $18 Explain.

My Notes

Learning Targets:
- Solve problems about interest.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Summarizing, Interactive Word Wall, Create a Plan

Starting a veterinary clinic often includes taking out loans that the veterinarians need to pay back. Paying back a loan includes **interest**.

Example

Dr. Blevins is a first-year veterinarian and plans to rent space for an office for her new hospital. She will take out a loan of \$94,000 over 7 years at 6.25% interest to remodel the office. How much will she pay in interest on this loan?

The problem is asking for the simple interest, so use the formula for finding interest:

$$Interest = P \cdot r \cdot t$$

where p is the principal, or the amount of money borrowed, r represents the rate of interest charged, and t is the time in years.

Substitute in the values and solve for I.

$$I = P \cdot r \cdot t$$
$$I = \$94{,}000 \cdot 0.0625 \cdot 7$$
$$I = \$41{,}125$$

Solution: The interest paid on the loan will be \$41,125.

MATH TIP

The variables in the equation represent:

I = *interest*

P = *principal*

r = *rate*

t = *time*

Try These

Work with your group to find the amount of simple interest to the nearest cent. As needed, refer to the Glossary to review the meaning of key terms. Use your understandings of the terms in group discussions to confirm your knowledge and correct use of math language.

a. \$12,000 at 2.5% for 3 years

b. \$15,000 at 4% for 6 months

MATH TIP

If time is given in another unit, such as months or days, it needs to be converted to years when using the interest formula.

My Notes

Check Your Understanding

1. Dr. Blevins will have to take out another loan for other costs, such as equipment, furniture, products, and advertising. The loan will be for $25,900 over 5 years at 6.25%. Find what the interest will be on the loan for these other costs.
2. **Attend to precision.** The local shelter decided to build an addition for cats. They borrowed $6300 from a local bank at a rate of 8%. The high school held a fundraiser that allowed the shelter to pay off the loan after only 9 months. How much interest did the shelter pay on this loan?

LESSON 12-3 PRACTICE

3. **a.** Pet owners may have to take out a loan when their animal has a major operation. Buddy needs to have surgery on his leg. The cost of the surgery is $2500. His owner will get a loan for this amount over 2 years at a rate of 7.5%. How much will the interest be on this loan?

   b. What is the total cost of the surgery including the loan and the interest?
4. The average cost of an education to become a veterinarian is $147,656, which includes veterinary school. To cover these costs, most students take out loans. What would the interest on a loan be if the rate is 3.25% and the term is 15 years?
5. **Reason abstractly.** How is interest similar to sales tax and markups? How is it different?
6. Dr. Jones took out a loan for a van that could be equipped with veterinarian supplies for visits to animal shelters. The van cost $22,500 and the loan was for 4 years. If the interest rate was 4.5%, how much will be paid?
7. An X-ray machine for animals costs $125,000. If the loan is for 10 years, and the interest paid is $65,625, what is the interest rate on the loan?
8. **Make sense of problems.** Dr. Jones would like a computer system for the office to help with management of the clinic. The cost is $15,000, and the interest rate is 4.6%. The amount of interest to be paid is $3,450. How many months is the loan for?

My Notes

Learning Targets:

- Solve problems about percent error.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Discussion Groups, Interactive Word Wall, Create a Plan, Identify a Subtask

Veterinarians sometimes have to give animals transfusions to help them recover from illness or accidents. Cats have their own unique blood types. Cats are either type A, type B, or, rarely, type AB.

There are about 600,000 cats living in Las Vegas, Nevada. Local veterinarians assume that 90% of these cats have type A blood, a total of 540,000 cats.

It is found that actually 95% of cats have type A blood, or 570,000 cats.

CONNECT TO MEDICINE

A ***transfusion*** is the transfer of whole blood or blood products from one individual to another.

Example

What is the **percent error** between the veterinarians' earlier estimate and the actual number?

Step 1: Find the absolute value of the difference between the estimate and the actual number.

$$540{,}000 - 570{,}000 = -30{,}000$$
$$|-30{,}000| = 30{,}000$$

Step 2: Divide this difference by the actual number.

$$\frac{30{,}000}{570{,}000} = 0.0526$$

Step 3: Multiply this value by 100 to make it a percent.

$$0.0526 \cdot 100 = 5.26\%$$

Solution: The percent error is 5.26%.

MATH TERMS

Percent error is the amount of error between the assumed value and the actual value.

Try These

Work with your group.

a. A vet's assistant weighs a cat at 5.2 kg. The cat's actual weight is 4.8 kg. What is the percent error between the two weights?

b. The vet estimated that the facility had kept 23 dogs in their overnight boarding. The actual number of dogs kept overnight was 18. What is the percent error between the vet's estimate and the actual number?

GROUP DISCUSSION TIPS

Summarize the information needed to create a reasonable solution. Be sure to describe the math concepts your group will use to solve.

My Notes

Check Your Understanding

1. An adult Labrador retriever weighing 86 pounds came into the veterinarian's office for treatment. The vet estimated that 45 pounds of the dog's weight was fluid. When he did the math, he found that 51.6 pounds of the dog's weight was fluid. What was the percent error between the vet's estimate and the actual amount?
2. On a busy day, a vet thought the practice saw 28 cats. The actual number was 24 cats. What is the percent error between the vet's estimate and the actual amount?

LESSON 12-4 PRACTICE

3. Sally estimated that the cost of a new fish tank would be $45. The actual cost was $37.79. What was the percent error between Sally's estimate and the actual amount?
4. Marco estimated that he took his dog on a 2.5-mile walk. When he drove the same route in his car, he found that it was 2.65 miles. What is the percent error between Marco's estimate and the actual mileage?
5. A new horse owner estimated that the horse would eat 30 pounds of food a day. The horse was training for a race, so it actually ate 42 pounds per day. What is the percent error between the owner's estimate and the actual amount?
6. Kyle brought his guinea pig to the vet to see what was wrong with it. He thought the visit would cost $25, but the actual cost was $33. What is the percent error between what Kyle thought it would cost, and the actual cost?
7. **Reason quantitatively.** Pat helps run an animal shelter. The actual number of animals brought in during a particular month was 150. The percent error was 12%. If Pat estimated on the high side (more than 150), how many animals did Pat think were brought to the shelter?
8. Shea's mother wanted to get him a doctor's bag for a graduation present. She thought it would cost $125, but the actual cost was $95.99. What is the percent error between her estimate and the actual cost?
9. The average number of prescriptions written by Dr. Jones on a daily basis is 12. The assistant thought that number was 17. What is the percent error between the actual amount and the estimate by the assistant?

ACTIVITY 12 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 12-1

1. The last time Tia bought pet food, she purchased 25 pounds. This time she needed 38 pounds. What is the percent increase in the amount of food she needed?
2. The number of people and dogs taking an obedience class increased from 1400 students to 1600 students. Choose the percent increase in enrollment from the choices below.
   A. 14.3%
   B. 87.5%
   C. 114.3%
   D. 187.5%
3. If the number of veterinarians in California increased by 12.5%. and the original number was 1,215, what is the new number of veterinarians?
4. The value of the building owned by two veterinarians went from $125,000 to $117,500. What is the percent decrease in value?

Lesson 12-2

5. What is the discount on a stethoscope that is marked down from $300 to $250?
6. A pet shop pays $40 for a dog bed, and then sells it to a customer for $75. What is percent markup?
7. A supply company marks up their doctor scrubs by 40%. Find the doctor's cost for a set of scrubs that the supply company buys for $34.
8. **a.** Noah wants to buy a purebreed golden retriever puppy that he sees for $499. By the time he goes to buy it, the price has increased to $599. Find the percent markup in the price of the puppy.
   b. If the price increases another 5%, what is the final price of the puppy?
9. **a.** A fancy new litter box is sold at a pet store for $47.50. It is on sale for a discount of 15%. What is the final sale price?
   b. Would it be better or worse if the store had simply lowered the original price by $7.10? Explain your answer.
10. The clownfish Kay wanted to buy for $12 last week has been marked up 8%. What is the new price of the fish?

Lesson 12-3

11. Angie borrowed $1,485 to purchase a new laptop for the veterinarian's assistant. The interest on the loan was 7.75%. How much interest will Angie pay if the loan is for 2 years?
12. Mark takes out a loan for $30,000 to add on a new exam room for his practice. If the bank charges 5.4% interest, how much interest will he pay in 5 years?
13. Dr. Jones took out a loan for a new medical instrument cleaning system. The interest paid on the loan was $650. The loan was for 24 months, and the interest rate was 2.75%. How much money did Dr. Jones borrow?
14. Dr. Lee went to a bank to get a loan for some new furniture for the waiting area at her clinic. She wanted to borrow $5,000 to be paid back over 3 years. Last week the interest rate was 4.25%, but this week the interest rate went up to 4.50%. How much additional interest will Dr. Lee pay because she waited the extra week?
15. A vet borrows $12,000 for 4 years. The total interest paid was $875. What was the interest rate on the loan?
16. **a.** A veterinary student takes a loan to pay for the last year of school. The loan is for $25,000 at 3.25% to be paid back over 10 years. How much interest will be paid?
    b. If the student chose to pay back the loan in 7 years instead, how much interest would be saved?

Lesson 12-4

17. The local zoo contracts with one of the local veterinary clinics for pet checkups and sick visits. The zoo manager thought the vet made more than 375 visits to the zoo in one year, but the actual number was 412. What was the percent error between the actual number of visits and the number of visits the manager thought the vet made?

18. A veterinary student estimated the weight of a snapping turtle to be 147 pounds. The actual weight was 175 pounds. What is the percent error between the actual weight of the turtle and the estimated weight?

19. It was estimated that the United States would need 6200 more veterinarians in 2012. The percent error was 10%. If the estimate was low, how many were actually needed?

20. The aquarium estimated that they would need 520 cubic feet of water for a new exhibit. They got more fish than expected and actually needed 620 cubic feet of water. What was the percent error between the actual amount needed and the estimate?

21. It was estimated that a giraffe would be 15.5 feet tall when fully grown. Its actual height was 16.25 feet. What was the percent error between the actual and estimated values of height?

22. The percent of women in veterinary school is 78%. If you had guessed 50%, what would the percent error be?

23. Judy thought she spent $140 on pet accessories during the year. When she looked at all her receipts, she found she actually spent $112. What was the percent error between the actual amount she spent and her estimate?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

24. Which two of the following situations would give the same amount of interest? Explain.
A. $6,000 borrowed for 4 years at 4.6%
B. $15,000 borrowed for 3 years at 5%
C. $12,000 borrowed for 8 years at 2.3%
D. $7,500 borrowed for 6 years at 5%

Write your answers on notebook paper. Show your work.

Facebook and eBay are two popular Web services with many users. Facebook is a social media site where members connect with others. eBay is a marketing site where people buy and sell merchandise.

There were 845 million Facebook users at the end of 2011, which was up from 608 million users at the end of 2010.

1. What percent is 608 million of 845 million?
2. What is the percent increase in users between 2010 and 2011?
3. 51.5% of all users access Facebook on mobile devices. How many users used mobile devices in 2011?
4. Facebook is now used by one in every 13 people on Earth. What percent is this?

eBay allows people to sell any item for what others will pay for it.

5. The price of a digital camera sold in stores is $129. It is sold on eBay for $65. What is the percent decrease from the store's price to the online price?
6. What percent discount would this be in a store?

Some people have others sell items for them on eBay and then give them a part of the profit as a commission.

7. One forklift sells every 4 hours on eBay. If a forklift sells for $2,999 and the seller receives a 12% commission, how much does the seller receive for the sale?
8. One family spent $2,000 to start an online business selling goods. They now make $1,240,000 each year. If they invested half of their yearly earnings in a savings account earning 4% simple interest, how much would the account have in it at the end of 5 years?
9. Sales tax is added to the price of items sold on eBay. If someone in Arizona buys an item online for $22.50 and the state sales tax rate is 6.6%, what is the final selling price?
10. Businesses use many terms in their daily operation. Separate the terms below into groups. Each group should contain words that are related in some way. Label each group to show how the words are related.

percent increase	markup	percent decrease
commission	sales tax	discount
tip	interest	markdown
fee	principal	rate

Percents and Proportions
SOCIALIZING AND SELLING

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)	• Effective understanding and accuracy in calculating percents, percent increase, percent decrease, and finding a part given a percent. • Effective understanding and accuracy in calculating sales tax, commissions, and interest.	• Few if any errors in calculating percents, percent increase, percent decrease, and finding a part given a percent. • Few if any errors in calculating sales tax, commissions, and interest.	• Multiple errors in calculating percents, percent increase, percent decrease, and finding a part given a percent. • Multiple errors in calculating sales tax, commissions, and interest.	• Incorrect or incomplete understanding of calculating percents, percent increase, percent decrease, and finding a part given a percent. • Incorrect or incomplete understanding of calculating sales tax, commissions, and interest.
Problem Solving (Items 2, 5, 7, 8, 9)	• An appropriate and efficient strategy that results in a correct answer. • Accurate interpretation of a percent to solve a problem.	• A strategy that may include unnecessary steps but results in a correct answer. • Interpretation of a percent to solve a problem.	• A strategy that results in some incorrect answers. • Difficulty interpreting a percent to solve a problem.	• No clear strategy when solving problems. • Incorrect interpretation of a percent to solve a problem.
Mathematical Modeling / Representations (Item 2, 4, 5, 7, 8, 9)	• Clear and accurate interpretation of a percent problem to write and solve an equation.	• Interpreting a percent problem to write and solve an equation	• Difficulty interpreting a percent problem to write and solve an equation	• Incorrect or incomplete interpretation of a percent problem to write and solve an equation
Reasoning and Communication (Item 2, 5, 6, 7, 8, 9, 10)	• Effective understanding and command of terms relating to percents.	• An adequate knowledge of terms relating to percents.	• Difficulty understanding and distinguishing terms relating to percents.	• An incomplete or inaccurate understanding of terms relating to percents.

Geometry

4

Unit Overview

In this unit you will extend your knowledge of two- and three-dimensional figures as you solve real-world problems involving angle measures, area, and volume. You will also study composite figures.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- unique
- orientation
- decompose

Math Terms

- angle
- complementary angles
- complement
- vertical angles
- included angle
- similar figures
- corresponding parts
- plane
- circumference
- radius
- semicircle
- prism
- pyramid
- lateral face
- lateral area
- slant height
- complex solid
- vertex
- supplementary angles
- supplement
- conjecture
- included side
- congruent
- circle
- center
- diameter
- composite figure
- inscribed figure
- net
- cross section
- right prism
- surface area
- volume

ESSENTIAL QUESTIONS

? Why is it important to understand properties of angles and figures to solve problems?

? Why is it important to be able to relate two-dimensional drawings with three-dimensional figures?

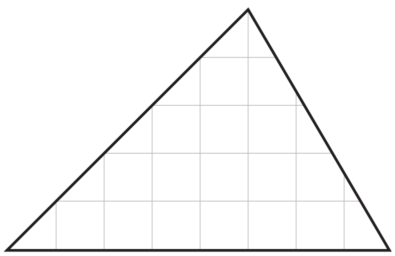
EMBEDDED ASSESSMENTS

These assessments, following Activities 14, 17, and 19, will give you an opportunity to demonstrate how you can use your understanding of two- and three-dimensional figures to solve mathematical and real-world problems involving area and volume.

Getting Ready

Write your answers on notebook paper. Show your work.

1. Write three ratios that are equivalent to $\frac{4}{6}$.
2. Solve each of the following equations.
   a. $3x + 4 = 21$
   b. $2x - 13 = 3x + 18$
   c. $\frac{6}{51} = \frac{3}{x}$
3. Sketch each of the following figures.
   a. square
   b. triangle
   c. parallelogram
   d. trapezoid
   e. right triangle
   f. $50°$ angle
4. Write an expression that can be used to determine the area of each figure.
   a. circle
   b. trapezoid
   c. parallelogram
   d. triangle
5. Determine the area of each plane figure described or pictured below.
   a. Circle with radius 5 inches. Round your answer to the nearest tenth.
   b. Right triangle with leg lengths 4 inches and 7 inches.
   c. Rectangle with length 6 inches and width 10 inches.
   d. Trapezoid with base lengths 3 inches and 7 inches and height 12 inches.
   e.



6. Compare and contrast the terms *complementary* and *supplementary* when referring to angles.
7. Think about triangles.
   a. List three ways to classify triangles by side length.
   b. List four ways to classify triangles by angle measure.
8. Polygons are named by the number of sides they have. Give the names of four different polygons and tell the number of sides each has.

Angle Pairs

Some of the Angles

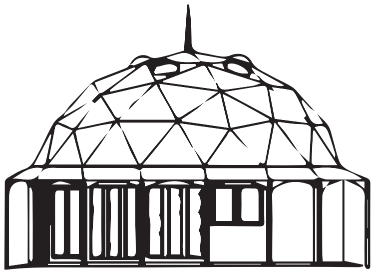
Lesson 13-1 Complementary, Supplementary, and Adjacent Angles

Learning Targets:

- Use facts about complementary, supplementary, and adjacent angles to write equations.
- Solve simple equations for an unknown angle in a figure.

SUGGESTED LEARNING STRATEGIES: Close Reading, Think Aloud, Create Representations, Marking the Text, Critique Reasoning, Sharing and Responding, Look for a Pattern

Architects think about angles, their measure, and special angle relationships when designing a building.



Two rays with a common endpoint form an **angle**. The common endpoint is called the **vertex**.

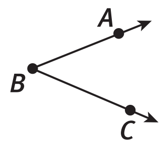
1. Angles are measured in degrees and can be classified by their relationship to the angle measures of 0°, 90°, and 180°. What angle measures characterize an acute angle, a right angle, an obtuse angle, and a straight angle?

Some angle relationships have special names. Two angles are **complementary** if the sum of their measures is 90°. Two angles are **supplementary** if the sum of their measures is 180°.

2. Compare and contrast the definitions of complementary and supplementary angles.

My Notes

READING MATH



To read this angle, say "angle *ABC*," "angle *CBA*," or "angle *B*."

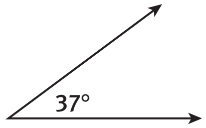
READING MATH

A small square at the vertex of an angle denotes a right angle.

My Notes

3. Name pairs of angles that form complementary or supplementary angles. Justify your choices.

a.



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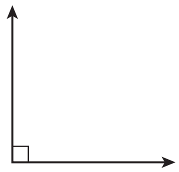
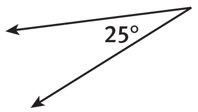


c.



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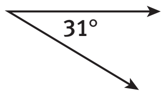
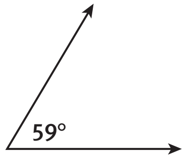
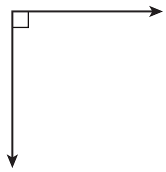


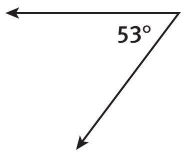
e.



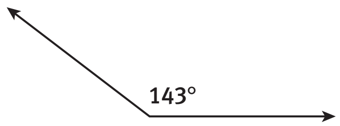
f.



g.



h.



4. Find the **complement** and/or **supplement** of each angle or explain why it is not possible.

a. 32°

b. 113°

c. 68.9°

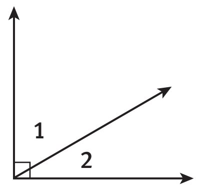
MATH TERMS

In a pair of complementary angles, each angle is the **complement** of the other.

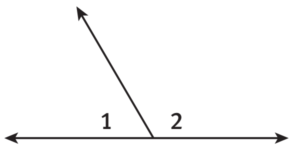
In a pair of supplementary angles, each angle is the **supplement** of the other.

My Notes

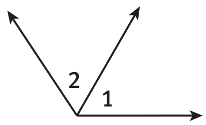
5. Why are angles 1 and 2 in this diagram complementary?

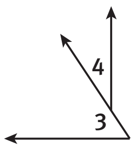


6. Why are angles 1 and 2 in this diagram supplementary?



7. Which of the following is a pair of **adjacent angles**? Justify your answer.





8. Angle A measures 32°.

a. Angle A and $\angle B$ are complementary. Find $m\angle B$.

b. Write an equation that illustrates the relationship between the measures of $\angle A$ and $\angle B$.

c. Solve your equation from Part b to verify your answer in Part a.

MATH TERMS

Adjacent angles have a common side and vertex but no common interior points.

READING MATH

Read $m\angle B$ as "the measure of $\angle B$." This form indicates the size of the angle.

My Notes

9. Model with mathematics. Two angles are complementary. One measures $(2x)^\circ$ and the other measures 48°.

a. Draw a pair of adjacent, complementary angles and label them using the given information.

b. Write an equation to show the relationship between the two angles and solve for the value of x.

c. Find the measure of both angles. Show your work.

10. Angle C measures 32°.

a. Angle C and $\angle D$ are supplementary. Find $m\angle D$.

b. Write an equation you could use to find the measure of $\angle D$.

c. Solve your equation from Part b to verify your answer in Part a.

11. Make use of structure. Two angles are supplementary. One angle measures $(3x)^\circ$ and the other measures 123°.

a. Draw a pair of adjacent supplementary angles and label them using the given information.

b. Write an equation to show the relationship between the two angles. Solve the equation for x.

c. Find the measure of both angles. Show your work.

My Notes

Check Your Understanding

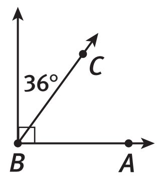
12. Explain how to find the complement and the supplement of an angle that measures 42°.

13. Reason quantitatively. What angle is its own complement? What angle is its own supplement? Explain.

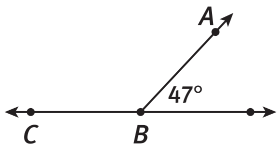
LESSON 13-1 PRACTICE

Find $m\angle ABC$ in each diagram.

14.



15.



Find the complement and/or supplement of each angle. If it is not possible, explain.

	Angle	Complement	Supplement
16.	14°		
17.	98°		
18.	53.4°		

19. $\angle P$ and $\angle Q$ are complementary. $m\angle P = 52°$ and $m\angle Q = (3x + 2)°$. Find the value of x. Show your work.

20. $\angle TUV$ and $\angle MNO$ are supplementary. $m\angle TUV = 75°$ and $m\angle MNO = (5x)°$. Find the value of x and the measure of $\angle MNO$. Show your work.

21. $\angle ABC$ and $\angle TMI$ are complementary. $m\angle ABC = 32°$ and $m\angle TMI = (29x)°$. Find the measure of $\angle TMI$.

22. Make use of structure. $\angle ZTS$ and $\angle NRQ$ are supplementary. $m\angle ZTS = (5x - 3)°$ and $m\angle NRQ = (2x + 1)°$. Find the measure of each angle.

23. Model with mathematics. The supplement of an angle has a measure that is three times the angle. Write and solve an equation to find the measure of the angle and the measure of its supplement.

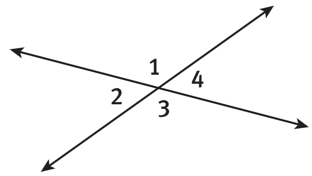
My Notes

Learning Targets:

- Write and solve equations using geometry concepts.
- Solve problems involving the sum of the angles in a triangle.
- Solve equations involving angle relationships.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think-Pair-Share, Create Representations, Identify a Subtask

Vertical angles are pairs of nonadjacent angles formed when two lines intersect. They share a vertex but have no common rays. ∠1 and ∠3 are vertical angles, as are ∠2 and ∠4.



MATH TERMS

A **conjecture** is a statement that seems to be true but has not been proven to be either true or false.

1. Visually inspect the diagram above. Then make a **conjecture** about the measures of a pair of vertical angles.

2. Find $m\angle 1$ and $m\angle 3$ above using your protractor.

3. Find the measure of ∠2 and ∠4 using geometry concepts.

4. **Construct viable arguments.** Do your answers to Items 2 and 3 support your conjecture? Support your reasoning.

5. **Reason quantitatively.** Two angles are vertical angles. One angle measures $(3x)°$ and the other measures 63°.
   a. Draw the pair of vertical angles and label them using the given information.

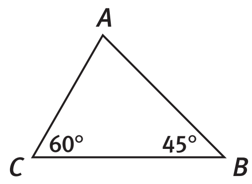
   b. Write an equation to show the relationship between the two angles. Then solve the equation for x.

Lesson 13-2
Vertical Angles and Angle Relationships in a Triangle

My Notes

A *triangle* is a closed figure made of three line segments that meet only at their endpoints. The sum of the angle measures of a triangle is 180°.

6. Triangle *ABC* is shown.

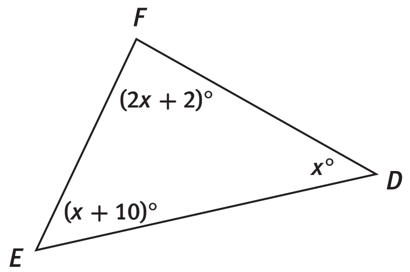


a. Find the measure of ∠*A*. Explain your answer.

b. Write an equation that illustrates the relationship between the measures of ∠*A*, ∠*B*, and ∠*C*.

c. Solve your equation from Part b to verify your answer in Part a.

7. Reason quantitatively. Triangle *DEF* is shown.



a. Write an equation that illustrates the relationship between the measures of ∠*D*, ∠*E*, and ∠*F*.

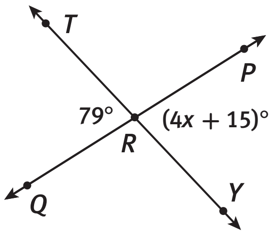
b. Solve the equation to find the value of *x*.

c. Find the measure of all three angles of △*DEF*.

My Notes

Check Your Understanding

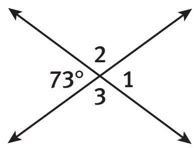
8. Explain how to find the value of x in the diagram shown.



9. **Construct viable arguments.** If you know the measure of one non-right angle of a right triangle, can you always find the measure of the third angle? Explain.

LESSON 13-2 PRACTICE

Use the diagram for Items 10–12.

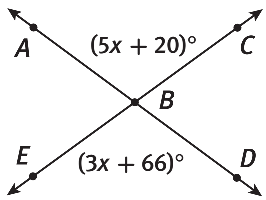


10. Find the measure of $\angle 1$.

11. Find the measure of $\angle 2$.

12. Find the measure of $\angle 3$.

Use the diagram for Items 13–15.

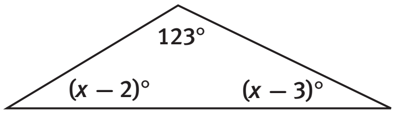


13. Find x.

14. Find the measure of $\angle ABC$.

15. Find the measure of $\angle ABE$.

16. **Reason quantitatively.** Find the measure of each of the angles in the triangle shown.



17. In triangle GKJ, $m\angle G = 72°$ and $m\angle J = 28°$. Write and solve an equation to find the measure of $\angle K$.

18. In a right triangle, one of the angles measures 29°. What are the measures of the other angles in the triangle? Explain.

19. **Reason quantitatively.** In an isosceles triangle, the two base angles are congruent. One of the base angles measures 36°. What are the measures of the other two angles in the triangle? Support your answer with words and equations.

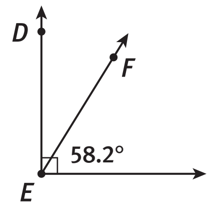
ACTIVITY 13 PRACTICE

Write your answers on notebook paper. Show your work.

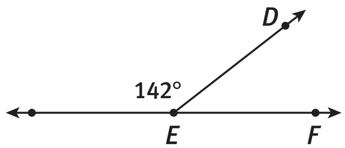
Lesson 13-1

Find the measure of angle DEF in each diagram.

1.



2.



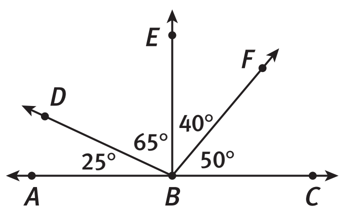
3. $\angle JKL$ and $\angle RST$ are complementary. $m\angle JKL = 36°$ and $m\angle RST = (x + 15)°$. Find the value of x and the measure of $\angle RST$.

4. $\angle SUN$ and $\angle CAT$ are supplementary. $m\angle SUN = (2x)°$ and $m\angle CAT = 142°$. Find the value of x and the measure of $\angle SUN$.

5. $\angle P$ and $\angle Q$ are supplementary. $m\angle P = (5x + 3)°$ and $m\angle Q = (x + 3)°$. What is the measure of $\angle Q$?

A. 17° **B.** 26°
C. 29° **D.** 32°

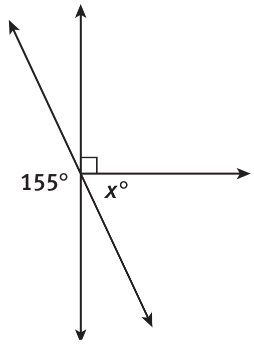
6. In the diagram shown, which angle pairs form complementary angles?



A. $\angle ABD$ and $\angle CBF$ **B.** $\angle DBE$ and $\angle FBE$
C. $\angle ABD$ and $\angle DBC$ **D.** $\angle EBF$ and $\angle FBC$

Lesson 13-2

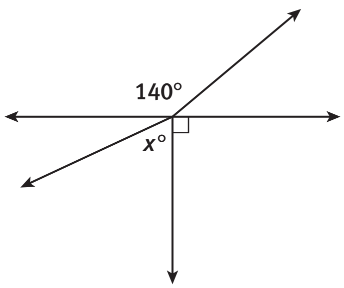
7. Find the measures of the vertical angles in the diagram. Then find x in the diagram.



8. $\angle B$ and $\angle D$ are vertical angles. $m\angle B = (2x + 1)°$ and $m\angle D = (x + 36)°$.

Find the measure of each angle.

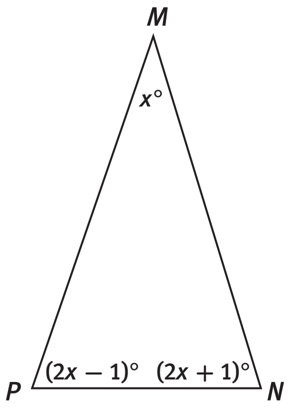
9. Find x in the diagram shown.



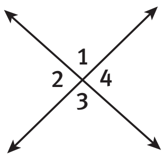
10. In right triangle TWZ, $\angle W$ is a right angle and $m\angle Z = 41°$. Find $m\angle T$.

11. In triangle QRS, $\angle Q$ and $\angle S$ have the same measure. If $m\angle R = 58°$, find the measures of $\angle Q$ and $\angle S$.

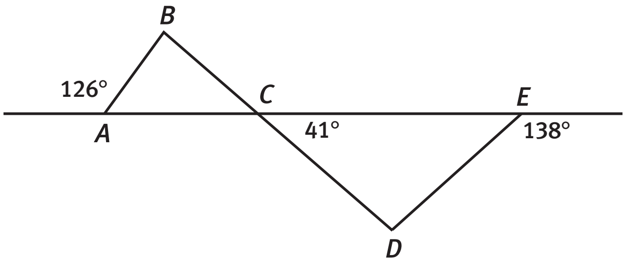
12. For triangle MNP shown, find the value of x and the measure of each of the three angles.



13. In the diagram, $m\angle 2 = 86°$. Erica says that $m\angle 3 = 86°$ because the diagram shows vertical angles and all vertical angles are congruent. Is her statement reasonable? Explain.



14. Use the diagram shown to find the measures of the each of the angles of $\triangle ABC$ and $\triangle CDE$.



15. The angles of an equilateral triangle are congruent. What are the measures of the angles?

16. In isosceles triangle ABC, $\angle B$ is a right angle. What are the measures of angles A, B, and C? Justify your answer.

MATHEMATICAL PRACTICES
Reason Abstractly

17. Consider the angle pair classifications from this activity: adjacent, complementary, supplementary, and vertical angles. Can two angles fit all four categories? Explain.

Triangle Measurements

Rigid Bridges

Lesson 14-1 Draw Triangles from Side Lengths

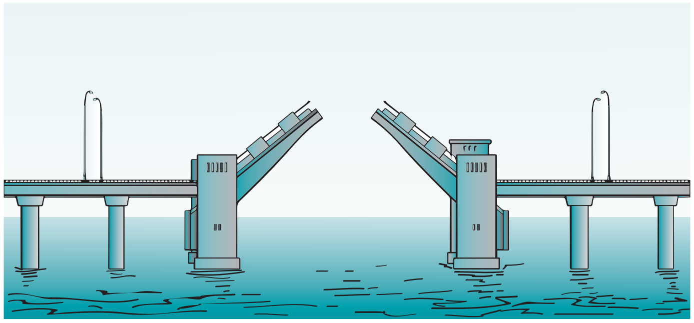
My Notes

Learning Targets:

- Decide if three side lengths determine a triangle.
- Draw a triangle given measures of sides.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Use Manipulatives, Predict and Confirm, Shared Reading, Visualization

When the sides of a drawbridge are raised or lowered, the sides move at the same rate.



CONNECT TO ENGINEERING

A drawbridge is a bridge that can be drawn up, let down, or drawn aside, to permit or prevent ships and other watercraft from passing below it.

1. Look at the drawbridge.

a. What will happen if the sum of the lengths of the moving sides is greater than the opening? Draw an illustration to explain your answer.

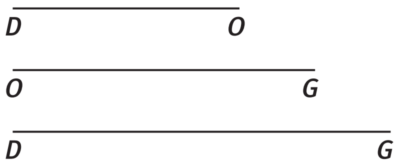
b. What will happen if the sum of the lengths of the moving sides is equal to the length of the opening? Draw an illustration to explain your answer.

c. What will happen if the sum of the lengths of the moving portions of the bridge is less than the length of the opening? Draw an illustration to explain your answer.

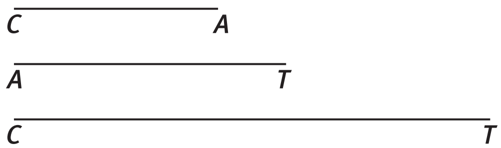
My Notes

2. Work with a partner or with your group. Use paper or string to cut out segments that are the same length as the segments shown. If possible, connect the segments at the endpoints to create a triangle and draw the triangle. If it is not possible, explain.

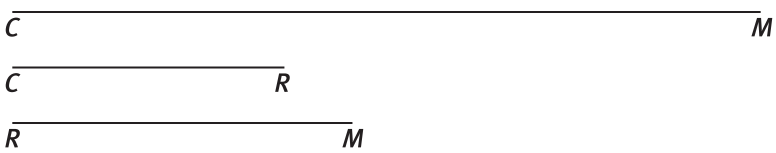
a. $\triangle DOG$



b. $\triangle CAT$



c. $\triangle RCM$



3. **Use appropriate tools strategically.** Measure the lengths in centimeters of the segments in Item 2. Then write an equation or inequality to compare the sum of two side lengths to the longest side length.

4. Based on your results for Items 2 and 3, make a conjecture about what side lengths can form a triangle. As you share your ideas with your group about making the conjecture, be sure to explain your thoughts using precise language and details to help the group understand your ideas and reasoning.

My Notes

5. Use a ruler to draw each triangle described below. You may want to cut out segments and use the segments to help form the triangle.

a. Draw a triangle with side lengths that each measure 3 centimeters. Can you form more than one triangle with the given side lengths? Explain.

b. Draw a triangle with side lengths that are 3 centimeters, 3 centimeters, and 5 centimeters long. Can you form more than one triangle with the given side lengths? Explain.

c. Draw a triangle with side lengths that are 3 centimeters, 4 centimeters, and 5 centimeters long. Can you form more than one triangle with the given side lengths? Explain.

6. Construct viable arguments. When a triangle is formed from three given side lengths, is the triangle a ***unique*** triangle or can more than one triangle be formed using those same side lengths? Explain.

ACADEMIC VOCABULARY

The term *unique* means "only" or "single." In geometry, a unique triangle is a triangle that can be drawn in only one way.

My Notes

Check Your Understanding

7. Is it possible to draw a triangle with sides that are 4 inches, 5 inches, and 8 inches long? Justify your answer.

8. Draw a triangle with sides that are 2 inches, 2 inches, and 3 inches long. Can you form more than one triangle with the given side lengths? Explain.

Lesson 14-1 PRACTICE

Determine whether it is possible to draw a triangle with the given side lengths. Justify your answers.

9. 7 feet, 5 feet, and 2 feet

10. 3 meters, 4 meters, and 6 meters

11. 5 inches, 7 inches, and 15 inches

12. 6 yards, 12 yards, and 6 yards

13. 8 millimeters, 11 millimeters, and 4 millimeters

14. 30 feet, 9 feet, and 16 feet

15. **Express regularity in repeated reasoning.** To check that three side lengths can form a triangle, you only have to check the sum of the two shortest lengths. Explain why.

16. **Look for and make use of structure.** Two sides of a triangle measure 3 inches and 6 inches. What is one possible length for the third side of the triangle? Explain.

17. Draw a triangle with side lengths that are 6 centimeters, 8 centimeters, and 10 centimeters long.

18. Draw a triangle with side lengths that are 2 centimeters, 6 centimeters, and 7 centimeters long.

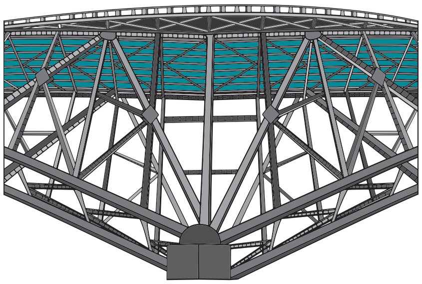
19. **Express regularity in repeated reasoning.** A triangle is formed using three given side lengths. Do these side lengths always form a unique triangle? Explain.

My Notes

Learning Targets:

- Draw a triangle given measures of angles and/or sides.
- Recognize when given conditions determine a unique triangle, more than one triangle, or no triangle.

SUGGESTED LEARNING STRATEGIES: Create Representations, Graphic Organizer, Think-Pair-Share, Visualization

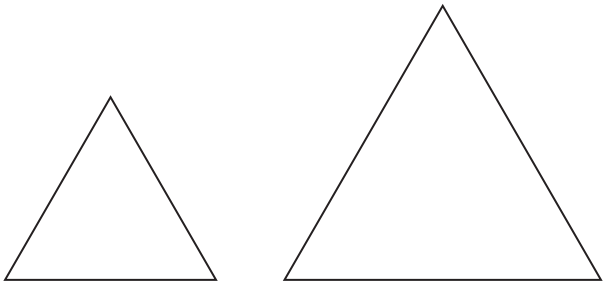


Triangles are rigid shapes. Structures, such as bridges and towers, are reinforced with triangles that give the structures added strength. Notice how triangles within triangles are used to support the bridge in the picture.

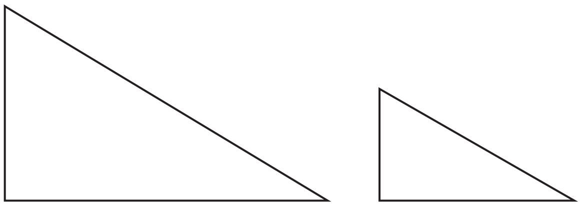
In the last lesson, you learned that three given side lengths determine a unique triangle. Other conditions can also determine a unique triangle.

1. Use a protractor to measure each angle in the triangles.

a.



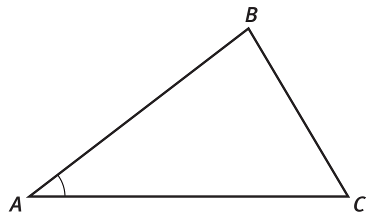
b.



My Notes

2. **Reason abstractly.** When a triangle is formed from three given angle measures, is the triangle a unique triangle, or can more than one triangle be formed using those same angle measures? Explain.

READING MATH



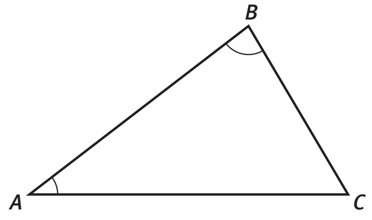
Angle *A* is the **included angle** between sides *AB* and *AC*.

Two sides of a triangle form an angle called an **included angle**.

3. a. Use a ruler and protractor. Draw a triangle with two sides that measure 4 centimeters each and an included angle of 30°.

b. Is there only one triangle that fits the description given in Part a? Explain.

READING MATH



Side *AB* is the **included side** between angles *A* and *B*.

An **included side** is the side between two angles.

4. a. Use a ruler and protractor. Draw a triangle with two angles that each measure 30° and an included side that measures 5 centimeters.

b. Is there only one triangle that fits the description given in Part a? Explain.

My Notes

5. **Construct viable arguments.** Decide if the given conditions create a unique triangle.
   a. When a triangle is formed from two side lengths and an included angle measure, is the triangle a unique triangle, or can more than one triangle be formed? Explain.

   b. When a triangle is formed using two given angle measures and an included side length, is the triangle a unique triangle, or can more than one triangle be formed? Explain.

Two known angle measures and the length of a non-included side also form a unique triangle. However, two given side lengths and the measure of a non-included angle may or may not form a unique triangle.

6. Two angles of a triangle measure 40° and 110°. The side opposite the 40° angle is 6 inches long. Can more than one triangle be drawn with these conditions? Explain.

7. Two sides of a triangle are 4 inches and 7 inches long. The included angle has a measure of 35°. Can more than one triangle be drawn with these conditions? Explain.

8. **Make use of structure.** Two angles of a triangle each measure 70° and 55°. The side adjacent to the 70° angle is 3 inches long. Can more than one triangle be drawn with these conditions? Justify your answer.

My Notes

Check Your Understanding

9. Is it possible to draw a unique triangle with angle measures of 35°, 65°, and 100°? Explain.

10. Is it possible to draw a unique triangle with two sides that are each 5 centimeters long and an included angle that measures 40°? Explain.

LESSON 14-2 PRACTICE

Determine whether the given conditions determine a unique triangle or more than one triangle. Justify your answers.

11. Two angles of a triangle measure 40° and 75°. The side between the angles is 3 feet long.

12. Two angles of a triangle each measure 55°. The side opposite one of the 55° angles is 2 meters long.

13. The angles of a triangle measure 40°, 60°, and 80°.

14. The sides of a triangle are 5 inches, 12 inches, and 13 inches long.

15. Two sides of a triangle are 10 centimeters and 13 centimeters long. One of the nonincluded angles measures 95°.

16. Two angles of a triangle measure 61° and 48°. One of the sides formed by the 48° angle is 15 millimeters long.

17. The two sides that form the right angle of a right triangle are 9 centimeters and 12 centimeters long.

18. **Look for and make use of structure.** If the measures of the angles of a triangle are known, is the length of one side of the triangle sufficient to determine if the triangle formed is a unique triangle? Explain.

ACTIVITY 14 PRACTICE

Write your answers on a separate piece of paper. Show your work.

Lesson 14-1

For 1–6, determine whether it is possible to draw a triangle with the given side lengths. Justify your answers.

1. 8 feet, 5 feet, and 9 feet
2. 3 centimeters, 2 centimeters, and 7 centimeters
3. 14 inches, 6 inches, and 10 inches
4. 3 yards, 2 yards, and 5 yards
5. 1.5 meters, 1.1 meters, and 2 meters
6. 42 feet, 18 feet, and 23 feet
7. Draw a triangle with side lengths that are 3 inches, 5 inches, and 6 inches long. Is this the only triangle that you can draw using these side lengths? Explain.
8. Multiple Choice: Which of the following cannot be the side lengths of a triangle?
   A. 4 inches, 4 inches, and 4 inches
   B. 3 inches, 3 inches, and 5 inches
   C. 15 centimeters, 16 centimeters, and 17 centimeters
   D. 2 centimeters, 10 centimeters, and 20 centimeters
9. Multiple Choice: Which of the following could be the length of the third side of a triangle with side lengths 2 feet and 10 feet?
   A. 12 feet
   B. 20 feet
   C. 11 feet
   D. 8 feet
10. **Express regularity in repeated reasoning.** Explain how to determine whether a triangle can be formed from three given segment lengths.

Lesson 14-2

Determine whether the given conditions determine a unique triangle or more than one triangle. Justify your answers.

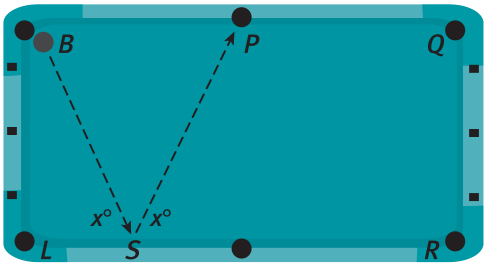
11. Two angles of a triangle measure 36° and 102°. One of the sides formed by the 36° angle is 9 inches long.
12. The angles of a triangle measure 25°, 73°, and 82°.
13. Two angles of a triangle measure 86° and 67°. The side between the angles is 2.5 meters long.
14. Two sides of a triangle are 3.6 meters and 5.2 meters long. One of the non-included angles measures 48°.
15. The two sides that form the right angle of a right triangle are 6 inches and 4 inches long.
16. The sides of a triangle are 10 centimeters, 12 centimeters, and 14 centimeters long.
17. Two angles of a triangle each measure 62°. The side opposite one of the 62° angles is 34 inches long.

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

18. **a.** If the lengths of two sides of a triangle are known, is the measure of one of the angles of the triangle enough to determine a unique triangle? Explain.
    b. If the measures of two angles of a triangle are known, is the length of one side of the triangle sufficient to determine if the triangle formed is a unique triangle? Explain.

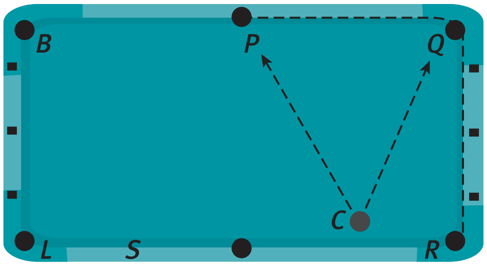
Angles and Triangles
POOL ANGLES

Pool is a game that requires talent and a knowledge of angles to play well. Bank and kick shots involve hitting a ball (B) into a rail of a rectangular pool table, and then into a pocket, somewhere on the other side of the table. As shown below, the angle at which the ball hits the side is equal to the angle at which it leaves the side.



1. Name a pair of adjacent, supplementary angles in the diagram.
2. Angle QPS is supplementary to $\angle PSR$. Also, $m\angle QPS = (2x + 12)°$ and $m\angle PSR$ is $x°$. Answer each question below and show your work.
   - **a.** Find the value of x.
   - **b.** Find $m\angle PSL$.
3. The measure of $\angle BLS$ is 90°. Explain why $\angle LSB$ and $\angle LBS$ must be complementary.
4. The measures of segment BS and segment PS are both 4.5 feet and $m\angle PBS = 56°$.
   - **a.** Find the measure of $\angle BSP$. Item 2 says that "Angle QPS is supplementary to $\angle PSR$."
   - **b.** Is $\triangle BPS$ a unique triangle, or can more than one triangle be formed using the given segment lengths and angle measures? Explain.

Another type of pool shot involves aiming the ball at point C directly at a pocket, as shown below.



5. The measure of $\angle RQC = (x + 12)^\circ$ and $m\angle PQC = (2x)^\circ$. Set up and solve an equation to find the value of x.

6. In $\triangle QPC$, $m\angle C = (x + 11)^\circ$, $m\angle Q = (2x + 6)^\circ$, and $m\angle P = 70^\circ$.
   a. Set up and solve an equation to find the value of x.
   b. Use the value you found for x to find $m\angle C$ and $m\angle Q$. Show your work.
   c. Is $\triangle QPC$ a unique triangle, or can more than one triangle be formed using the three angle measures? Justify your answer.

7. Is it possible for $\triangle QPC$ to have side lengths of 4 feet, 1.5 feet, and 2 feet? Justify your answer.

8. The measures of two side lengths of a triangle are 6 centimeters and 8 centimeters, and the measure of one angle is 35°.
   a. Use a ruler and a protractor to draw a triangle or triangles that meet these conditions
   b. **Attend to precision.** Is there only one triangle or more than one triangle that meets these conditions? Explain.

Angles and Triangles
POOL ANGLES

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2a-b, 3, 4a-b, 5, 6a-c, 7, 8b)	• Clear and accurate understanding of adjacent angle relationships and angle relationships in a triangle.	• An understanding of adjacent angle relationships and angle relationships in a triangle.	• Partial understanding of adjacent angle relationships and angle relationships in a triangle.	• Incorrect or incomplete understanding of adjacent angle relationships and angle relationships in a triangle.
Problem Solving (Items 2a-b, 4a, 5, 6a-b)	• An accurate interpretation of a problem in order to find missing angle measurements.	• A somewhat accurate interpretation of a problem to find missing angle measurements.	• Difficulty interpreting a problem to find missing angle measurements	• Incorrect or incomplete interpretation of a problem.
Mathematical Modeling / Representations (Items 4b, 6c, 8a-b)	• An accurate drawing of a triangle given information on the side lengths and angles.	• A drawing of a triangle given information on the side lengths and angles.	• Difficulty in drawing a triangle given information on the side lengths and angles.	• An incorrect drawing of a triangle given information on the side lengths and angles.
Reasoning and Communication (Items 1, 3, 4b, 6c, 7, 8b)	• Precise use of appropriate terms to describe angle relationships and triangles.	• Use of appropriate terms to describe angle relationships and triangles.	• A partially correct use of terms to describe angle relationships and triangles.	• An incomplete or inaccurate use of terms to describe angle relationships and triangles.

Similar Figures

The Same but Different

Lesson 15-1 Identify Similar Figures and Find Missing Lengths

My Notes

Learning Targets:

- Identify whether or not polygons are similar.
- Find a common ratio for corresponding side lengths of similar polygons.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Summarize/Paraphrase/Retell, Visualization, Create Representations, Identify a Subtask

The Pentagon, the headquarters of the United States Department of Defense, is located in Arlington County, Virginia. This building is named for its shape.

1. Study these two aerial photos of the Pentagon.

Photo 1

Photo 2

a. How are the photos alike?

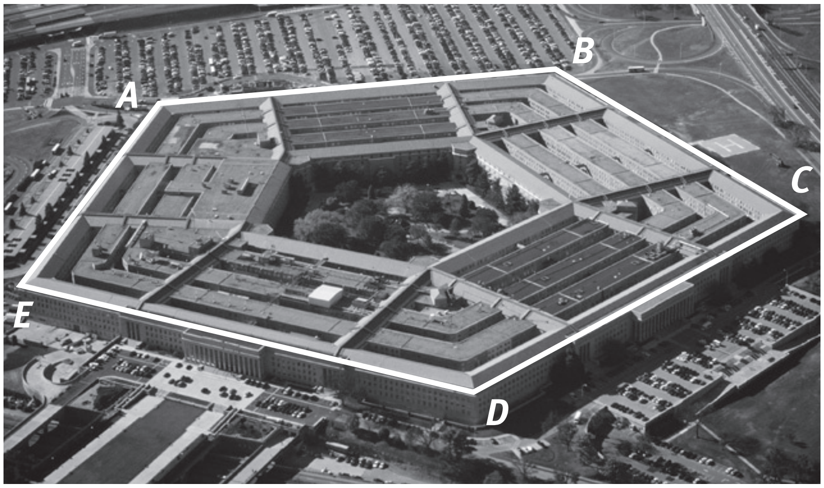
b. How are the photos different?

My Notes

MATH TIP

The shape of the Pentagon building is actually a regular pentagon, but the perspective in this picture makes the lengths appear to be different.

2. Use a protractor and ruler to measure the line segments and angles in both photos below. Measure segments to the nearest millimeter and angles to the nearest degree.



a. $AB =$ _______ **b.** $m\angle A =$ _______
c. $BC =$ _______ **d.** $m\angle B =$ _______
e. $CD =$ _______ **f.** $m\angle C =$ _______
g. $DE =$ _______ **h.** $m\angle D =$ _______
i. $EA =$ _______ **j.** $m\angle E =$ _______

k. $FG =$ _______ **l.** $m\angle F =$ _______
m. $GH =$ _______ **n.** $m\angle G =$ _______
o. $HI =$ _______ **p.** $m\angle H =$ _______
q. $IJ =$ _______ **r.** $m\angle I =$ _______
s. $JF =$ _______ **t.** $m\angle J =$ _______

My Notes

3. Use the measurements from Item 2 to find the following ratios to the nearest tenth.

a. $\frac{AB}{FG} =$ ______ **b.** $\frac{BC}{GH} =$ ______ **c.** $\frac{CD}{HI} =$ ______

d. $\frac{DE}{IJ} =$ ______ **e.** $\frac{EA}{JF} =$ ______

4. What can you conclude about the ratio of the lengths of the segments and the measures of the angles in the photos?

Similar figures are figures in which the lengths of the corresponding sides are in proportion and the corresponding angles are **congruent**. **Corresponding parts** of similar figures are the sides and angles that are in the same relative positions in the figures.

5. Construct viable arguments. Are the two photographs of the Pentagon *similar*? Justify your reasoning.

MATH TERMS

Figures that are **congruent** have exactly the same size and the same shape

In a *similarity statement*, such as $\triangle ABC \sim \triangle DEF$, the order of the vertices shows which angles correspond. So, $\triangle ABC \sim \triangle DEF$ means that $\angle A$ corresponds to $\angle D$, $\angle B$ corresponds to $\angle E$, and $\angle C$ corresponds to $\angle F$. The corresponding sides follow from the corresponding angles. They are AB and DE, BC and EF, and CA and FD.

READING MATH

The symbol $\sim$ means "is similar to." Read the similarity statement $\triangle ABC \sim \triangle DEF$ as "Triangle *ABC* is similar to triangle *DEF*."

6. The lengths of the sides of quadrilateral *ABCD* are 4, 6, 6, and 8 inches. The lengths of the sides of a similar quadrilateral *JKLM* are 6, 9, 9, and 12 inches.

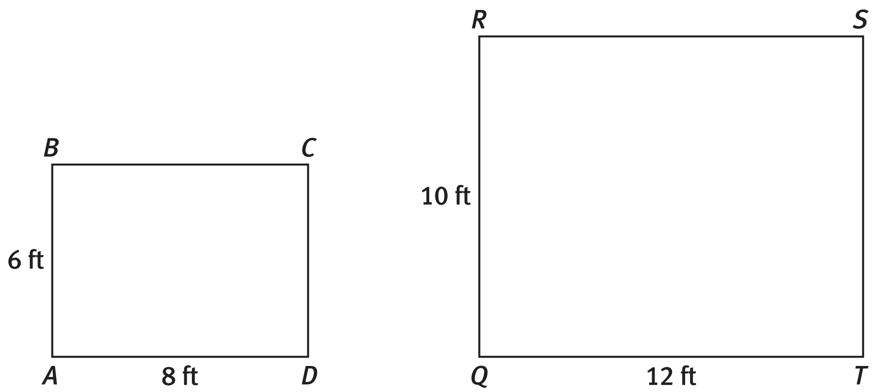
a. Write the ratios for the corresponding sides of the quadrilaterals.

b. What do you notice about the ratios of the sides of the similar quadrilaterals?

My Notes

If two figures have the same shape with corresponding angles congruent and proportional corresponding sides, then the two figures are similar. When the corresponding sides of similar figures are in proportion, they are equivalent to a common ratio.

7. Rectangle *ABCD* is 6 feet by 8 feet. Rectangle *QRST* is 10 feet by 12 feet.



a. Name the corresponding angles.

b. Are the corresponding angles congruent? Explain.

c. Name the corresponding sides.

d. Write the ratios of the corresponding widths and lengths of the rectangles.

e. Are the corresponding sides in proportion? Explain.

f. Is rectangle *ABCD* similar to rectangle *QRST*? Explain.

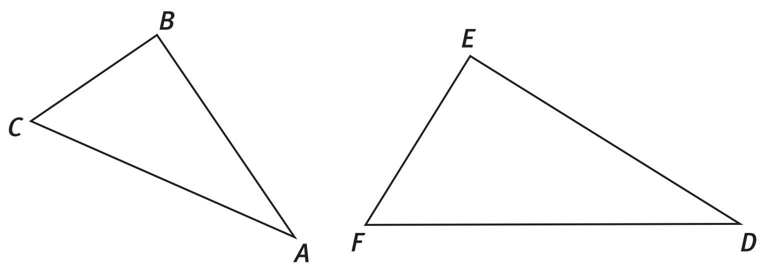
My Notes

Check Your Understanding

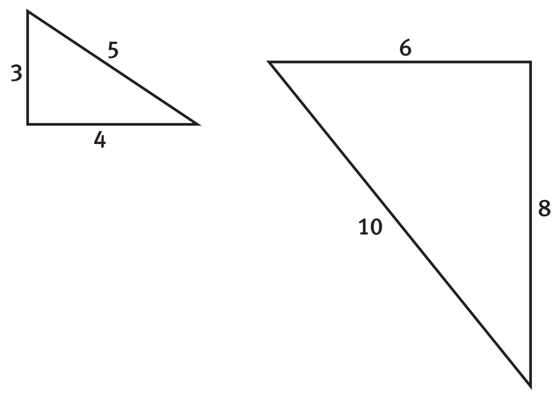
8. Are two congruent figures similar? Explain.
9. When are two polygons with the same number of sides not similar?

LESSON 15-1 PRACTICE

Use a protractor and a ruler to measure the angles and sides of triangles *ABC* and *DEF*.



10. Are the corresponding angles congruent? Explain.
11. Are the corresponding sides in proportion? Explain.
12. Are the triangles similar? If so, explain why and write a similarity statement. If not, explain why not.
13. **Model with mathematics.** Sketch two similar rectangles. Explain why they are similar.
14. Are the ratios of the corresponding sides of the right triangles shown equal? Explain.



15. **Make sense of problems.** Rectangle *A* is 3 meters wide and 5 meters long. Rectangle *B* is 2.5 meters wide and 4.5 meters long. Rectangle *C* is 10 meters wide and 18 meters long. Are any of the rectangles similar? Explain.
16. **Reason abstractly.** Are all squares similar? Explain.

MATH TIP

When two figures have a different *orientation*, it can be tricky to identify corresponding sides or angles. Try turning and/or flipping one of the figures until its shape looks like the other figure.
In Item 14, turn the larger triangle to the right until the 8-unit side is the base. Then flip the triangle over an imaginary vertical line. This can help you identify the corresponding sides.

ACADEMIC VOCABULARY

The ***orientation*** of a figure is the way in which the figure is positioned.

My Notes

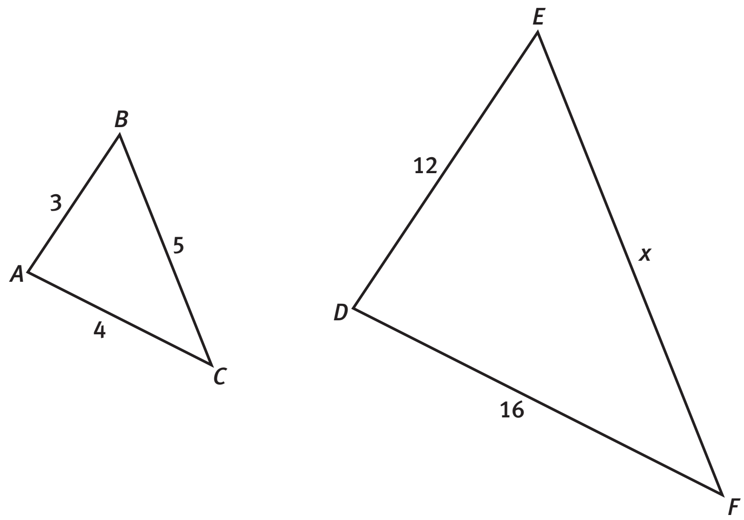
Learning Targets:

- Apply properties of similar figures to determine missing lengths.
- Solve problems using similar figures.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, KWL Chart, Think Aloud, Visualization, Discussion Groups, Create Representations

The corresponding sides of similar figures are in proportion and form a common ratio. This relationship can be used to find missing lengths.

1. $\triangle ABC \sim \triangle DEF$



a. Write the ratios of the corresponding sides.

b. What is the common ratio of the corresponding sides with known lengths?

c. Use the common ratio to write a proportion to find the value of x, the missing side length. Solve the proportion.

2. Model with mathematics. A rectangular postcard of a painting is similar to the original painting. The postcard is 4 inches wide and 6 inches long. The original painting is 34 inches wide.

a. Draw similar rectangles to model the problem.

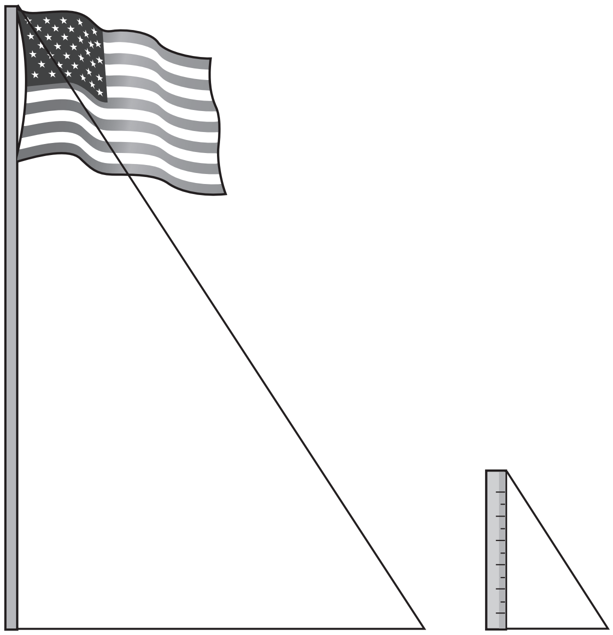
b. Write and solve a proportion for the situation. Let x represent the length of the original painting.

c. How long is the original painting?

d. Is your answer reasonable? Explain.

Some objects may be too tall to measure with rulers. Similar triangles can be used to indirectly measure the heights of these objects in real life.

3. A flagpole casts a shadow 8 feet long. At the same time, a yardstick 3 feet tall casts a shadow 2 feet long. The drawing shows how similar triangles can be used to model the situation.



a. Label the picture with the appropriate lengths. Let x represent the height of the flagpole.

b. Use the corresponding sides of similar triangles to write and solve a proportion for the situation.

c. How tall is the flagpole?

d. Is your answer reasonable? Explain.

e. Lars claims that he can solve the flagpole problem using measures within each figure and writes $\frac{x}{8} = \frac{3}{2}$. Is Lars correct? Explain.

My Notes

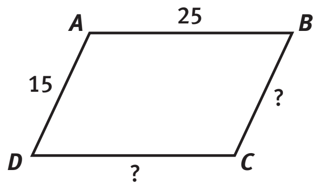
Check Your Understanding

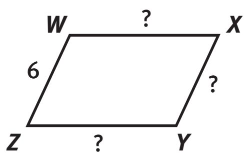
4. If you know the length and width of one rectangle and the length of a second rectangle, can you always use corresponding sides to find the width of the second rectangle? Support your answer.

5. Do you need to know the lengths of all the sides of one triangle to find a missing length of a similar triangle? Explain.

LESSON 15-2 PRACTICE

parallelogram *ABCD* ∼ parallelogram *WXYZ*

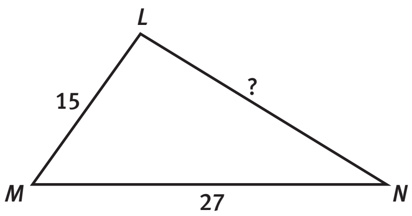


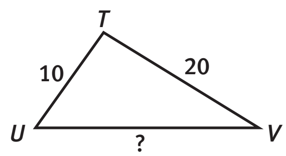


6. What is the common ratio of side *AD* to side *WZ*?

7. Find the length of segment *WX*.

$\triangle LMN \sim \triangle TUV$. Use what you know about common ratios to answer Items 8–9.





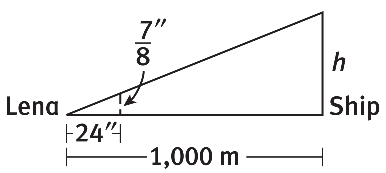
8. How long is segment *LN*?

9. How long is segment *UV*?

10. A 4-meter-tall flagpole casts a 6-meter shadow at the same time that a nearby building casts a 24-meter shadow. What is the height of the building? Solve this problem two different ways. First, set up and solve a proportion in which each ratio compares corresponding side lengths in the two figures. Then set up and solve a proportion in which each ratio compares side lengths within each figure.

11. Make sense of problems. Lena is standing on the beach when she sees a tall sailing ship pass by 1,000 meters offshore. She holds a ruler vertically 24 inches in front of her eyes, and the ship appears to be $\frac{7}{8}$ inch high. The figure in the My Notes column represents the situation as two similar right triangles. Find the approximate height (h) of the sailing ship above the water. Explain your answer.

(Figure not to scale)

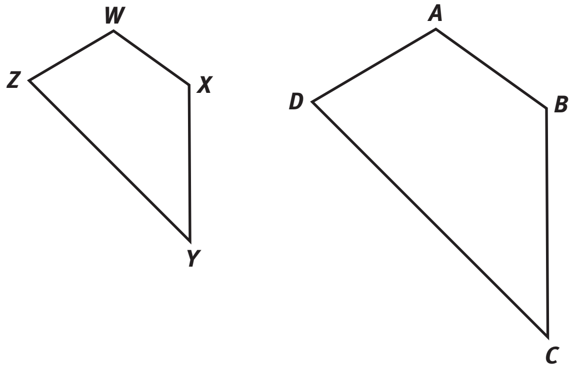


ACTIVITY 15 PRACTICE

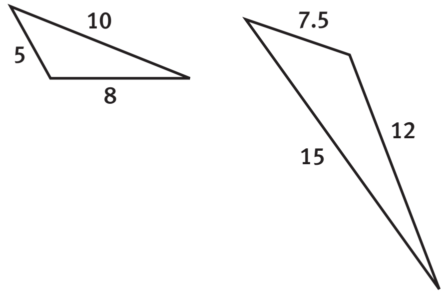
Write your answers on a separate piece of paper. Show your work.

Lesson 15-1

1. Use a ruler and protractor to decide if the figures are similar. Justify your decision.



2. a. Identify the corresponding sides on the figures below.
   b. Are the ratios of the corresponding sides of the triangles equal? Explain.



3. $\triangle ABC \sim \triangle EFG$, $m\angle A = 25°$, and $m\angle F = 100°$. What is $m\angle C$?
   A. 125°
   B. 100°
   C. 55°
   D. 25°

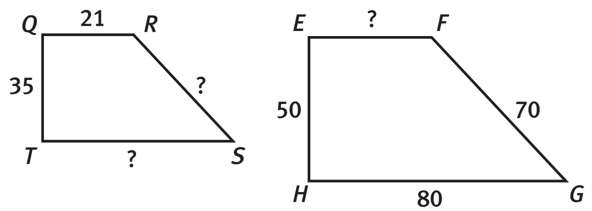
4. Rectangle J is 6 feet wide and 9 feet long. Rectangle K is 9 feet wide and 12 feet long. Rectangle L is 15 feet wide and 22.5 feet long. Are any of the rectangles similar? Explain.

5. a. Are all equilateral triangles similar? Explain.
   b. Are all right triangles similar? Explain.

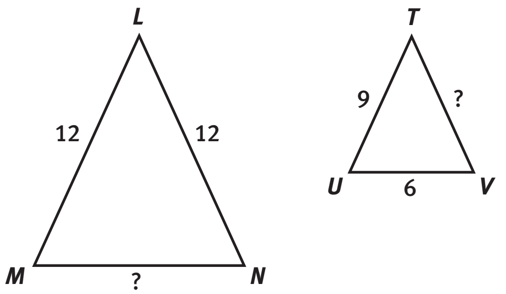
6. How is the mathematical meaning of the word "similar" the same as or different from the word "similar" in everyday conversation?

Lesson 15-2

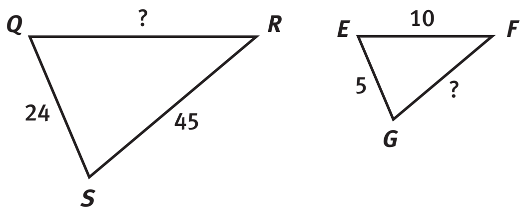
7. Trapezoid *QRST* ~ trapezoid *EFGH*. Find the measures of the missing sides.



8. △*LMN* ~ △*TUV*. Find the measures of the missing sides.

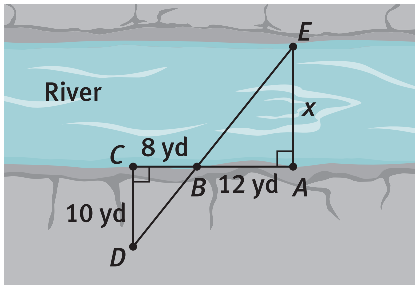


9. △*QRS* ~ △*EFG*. Find the measures of the missing sides.



10. A rectangular room is 42 feet wide and 69 feet long. On a blueprint, the room is 14 inches wide. How long is the room on the blueprint?

11. John wants to find the width of a river. He marks distances as shown in the diagram. Which of the following ratios can be used to find the width of the river?



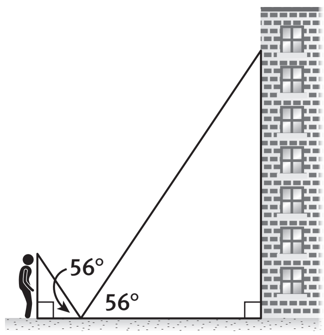
A. $\frac{10}{8} = \frac{12}{x}$ **B.** $\frac{10}{12} = \frac{8}{x}$

C. $\frac{8}{10} = \frac{12}{x}$ **D.** $\frac{x}{12} = \frac{8}{10}$

12. Miguel is 5 feet 10 inches tall. On a sunny day he casts a shadow 4 feet 2 inches long. At the same time, a nearby electric tower casts a shadow 8 feet 9 inches long. How tall is the tower?

MATHEMATICAL PRACTICES
Make Sense of Problems

13. Sam wants to find the height of a window in a nearby building but it is a cloudy day with no shadows. Sam puts a mirror on the ground between himself and the building. He tilts it toward him so that when he is standing up, he sees the reflection of the window. The base of the mirror is 1.22 meters from his feet and 7.32 meters from the base of the building. Sam's eye is 1.82 meters above the ground. How high up on the building is the window?



Circles: Circumference and Area

Gardens Galore

Lesson 16-1 Circumference of a Circle

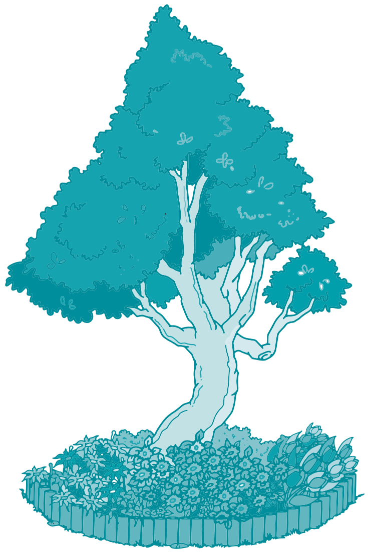
My Notes

Learning Targets:

- Investigate the ratio of the circumference of a circle to its diameter.
- Apply the formula to find the circumference of a circle.

SUGGESTED LEARNING STRATEGIES: Think Aloud, Create Representations, Discussion Groups, Summarize/Paraphrase/Retell

Rose wants to create several circular gardens in her yard. She needs to find the distance around and the area of each garden.

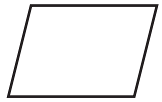


A **circle** is the set of points in the same **plane** that are an equal distance from a given point, called the **center**. The distance around a circle is called the **circumference**.

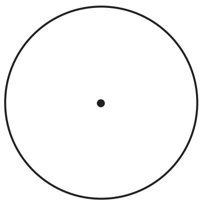
A line segment through the center of a circle with both endpoints on the circle is called the **diameter**. A line segment with one endpoint on the center and the other on the circle is called the **radius**.

MATH TERMS

A **plane** is a flat surface that extends in all directions. A parallelogram is usually used to model a plane.



1. A circle is shown.



   a. Use the information given above to label the center and the circumference of the circle
   b. Draw and label a diameter and a radius in the circle.

2. What is the relationship between the length of the diameter and the length of the radius of a circle?

My Notes

There is also a relationship between the circumference and the diameter of a circle. Work with your group to complete the activity below to find the relationship.

3. Measure the circumference and diameter of the circular objects provided by your teacher. Use the table to record the data. Then calculate the ratios.

Description of object	Object 1	Object 2	Object 3	Object 4	Object 5
Circumference					
Diameter					
Ratio of circumference to diameter (as a fraction)					
Ratio of circumference to diameter (as a decimal)					

The ratio of the circumference to the diameter of a circle is called **pi**, denoted by the Greek letter π. A commonly used approximation for *pi* is $\pi \approx 3.14$.

WRITING MATH

The digits following the decimal point in the decimal representation for *pi* never end or repeat, so all values found using *pi* are approximations. The symbol $\approx$ means "approximately equal to."

4. Which measurement tools used by your class gave the most accurate approximation of *pi*? Why do you think this is true?

5. **Express regularity in repeated reasoning.** Using the data on your table, write the equation that relates the circumference of a circle, C, to *pi* (π) and the diameter, d, of a circle.

CONNECT TO LANGUAGE

The Greek letter π is the first letter in the Greek words for perimeter and periphery.

6. Rewrite the equation from Item 5 to show the relationship of the circumference of a circle, C, to its radius (r) and π.

The equations you wrote for Items 5 and 6 above are the formulas for finding the circumference of a circle given its diameter or radius.

My Notes

7. Sometimes $\frac{22}{7}$ is used as an approximation of π. Why is this fraction a good approximation?

8. **Attend to precision.** Should the circumference of a circle be labeled in units or in square units? Explain.

Now you have an equation you can use to find the circumference of a circle. Use what you know to help Rose find the distance around one of her gardens.

9. One of the circular gardens Rose wants to make has a diameter of 6 feet.
   a. Use a circumference formula to find the amount of decorative fencing that Rose needs to enclose this garden. Use 3.14 or $\frac{22}{7}$ for π. Show your work. Tell which value you used for *pi*.

   b. Decorative fencing is sold in packages of 12-foot sections. How many packages must Rose buy to enclose this garden? Explain your reasoning and show your work.

My Notes

Check Your Understanding

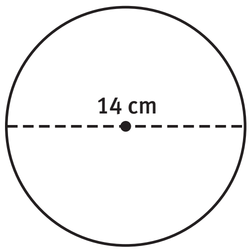
10. Explain how you could decide which approximation of π—3.14 or $\frac{22}{7}$—to use to compute the circumference of a circle.

11. Explain how the circumference of a circle and the definition of *pi* are related.

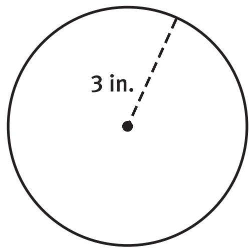
LESSON 16-1 PRACTICE

12. For Items a–d, find the circumference of each circle expressed as a decimal.

a.



b.



c. a circle with a radius of 8 ft

d. a circle with a diameter of 25 m

13. Find the circumference of a circular dog pen that has a radius of 35 meters. Use $\frac{22}{7}$ for π.

14. A window shaped like a circle has a diameter of 15 centimeters. What is the circumference of the window? Use 3.14 for π.

15. A circular tablecloth has a radius of 18 inches. What is the circumference of the tablecloth? Use 3.14 for π.

16. **Make use of structure.** A circle has a circumference of 125.6 centimeters. What is the diameter of the circle to the nearest centimeter? What is the radius to the nearest centimeter?

17. **Make sense of problems.** The diameter of a bicycle wheel is 26 inches. About how many revolutions does the wheel make during a ride of 500 feet? Use 3.14 for π. Explain your answer.

My Notes

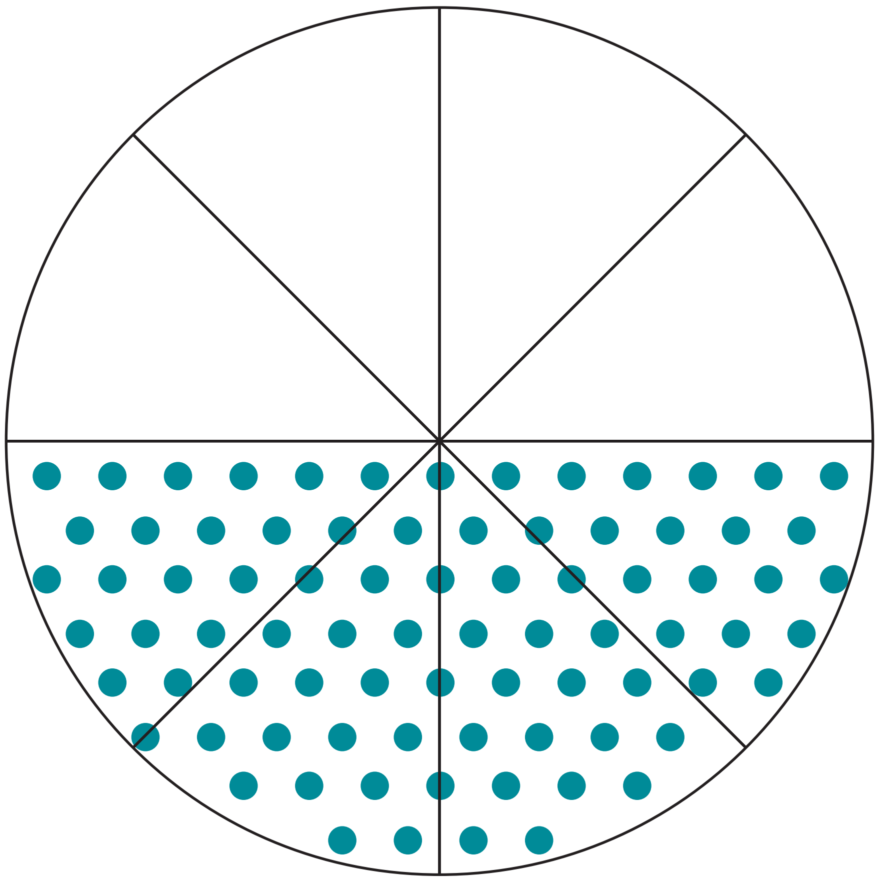
Learning Targets:

- Approximate the area of a circle.
- Apply the formula to find the area of a circle.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think Aloud, Think-Pair-Share, Use Manipulatives

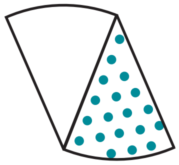
Rose also needs to know the area of each of the circular gardens.

1. On a separate sheet of paper, trace the circle shown. Shade half of the circle. Then cut your circle into eight congruent pie-shaped pieces.



My Notes

2. Arrange the eight pieces using the alternating pattern shown.



3. Sketch the shape you made with the circle pieces.

a. What geometric shape does the shape resemble?

b. What do you know about the area of the circle and the area of the figure you make?

4. On your sketch, draw and label the height of the figure. What part of the circle does the height represent?

5. What other measure of the circle do you need to know to determine the area of the shape you sketched? Label it on your sketch and explain your reasoning.

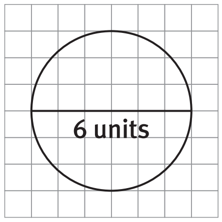
6. **Model with mathematics.** Use words, symbols, or both to describe how you can now calculate the area of the circle. Start with the formula $A = b \times h$ and substitute into the formula. Refer to your labeled sketch as needed.

7. Use your answer from Item 6 to write the formula for the area of a circle A in terms of its radius r and π. Explain how you found the formula.

8. Should the area of a circle be labeled in units or in square units? Explain.

My Notes

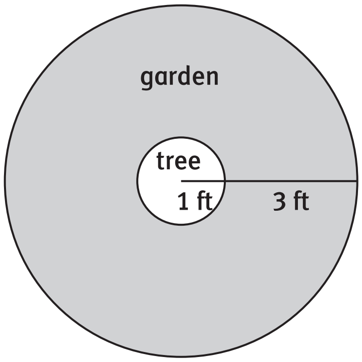
9. A circle with a radius of 3 units is graphed.



a. Estimate the area by counting the number of enclosed squares. Show the calculations that lead to your estimation.

b. Use the formula you wrote in Item 7 to find the area of the circle. Use 3.14 for π.

10. Make sense of problems. One of Rose's gardens will have a tree in the center as shown. The radius of the large outer circle will be 4 feet while the radius of the inner circle for the tree will be 1 foot. Find the area of the garden without the tree. Show your work.



Check Your Understanding

11. Is the circumference of a circle enough information to determine the area of the circle? Explain.

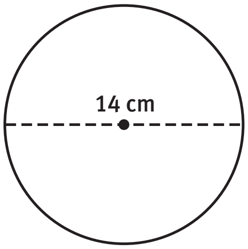
12. What is the formula for the area of a circle in terms of its diameter, d?

My Notes

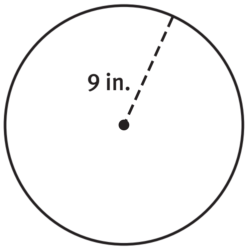
LESSON 16-2 PRACTICE

13. Find the approximate area of each circle. Use the value for π that makes the math simplest for you.

a.



b.



c. a circle with a radius of 8 km

d. a circle with a diameter of 25 ft

14. What is the approximate area of a circular pond that has a radius of 35 meters? Tell what value you used for π.

15. A penny has a diameter of about 26.5 millimeters. What is the area of a penny to the nearest hundredth? Use 3.14 for π.

16. One trampoline has a diameter of 12 feet. A larger trampoline has a diameter of 14 feet. How much greater is the area of the larger trampoline? Use 3.14 for π.

17. **Make sense of problems.** A painting shaped like a circle has a diameter of 20 inches. A circular frame extends 2 inches around the edge of the painting. How much wall space does the framed painting need? Use 3.14 for π.

18. The circumference of a circle is 31.4 centimeters. What is the area of the circle?

19. **Reason abstractly.** In ancient Egypt, a scribe equated the area of a circle with a diameter of 9 units to the area of a square with a side length of 8 units. What value of π does this method produce? Explain.

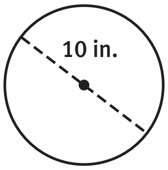
ACTIVITY 16 PRACTICE

Write your answers on a separate piece of paper. Show your work.

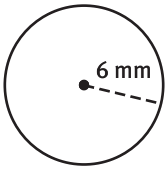
Lesson 16-1

1. Find the circumference of each circle below. Use 3.14 for π.

a.



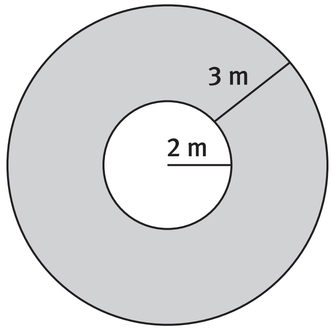
b.



2. The diameter of a pizza is 14 inches. What is the circumference of the pizza? Tell what value you used for π.
3. The radius of a circular mirror is 4 centimeters. What is the circumference of the mirror? Tell what value you used for π.
4. The radius of a circular garden is 28 feet. What is the circumference of the garden? Tell what value you used for π.
5. Find the diameter of a circle if $C = 78.5$ feet. Use 3.14 for π.
6. Find the radius of a circle if $C = 88$ yards. Use $\frac{22}{7}$ for π.
7. Multiple Choice. A standard circus ring has a radius of 6.5 meters. Which of the following is the approximate circumference of the circus ring?
   A. 13 meters
   B. 20.4 meters
   C. 40.8 meters
   D. 132.7 meters

Lesson 16-2

8. What is the area of a pizza with a diameter of 12 inches?
9. A circle has circumference 28.26 cm. What is the area of the circle? Use 3.14 for π.
10. Find the area of the shaded region. Use 3.14 for π.



11. Multiple Choice. The circular base of a traditional tepee has a diameter of about 15 feet. Which of the following is the approximate area of the base of the tepee?
    A. 23.6 square feet
    B. 47.1 square feet
    C. 176.6 square feet
    D. 706.5 square feet

12. A window is shaped like a semicircle. The base of the window has a diameter of $3\frac{1}{2}$ feet. Find the area of the window to the nearest tenth of a foot. Explain how you found the answer.

13. A circle has a diameter of 5 meters and a square has a side length of 5 meters.
   a. Which has the greater perimeter? How much greater?
   b. Which has the greater area? How much greater?

14. A circle with center at $(1, -1)$ passes through the point $(1, 2)$. Find the radius and then the area of the circle. Use 3.14 for π. Make a sketch on graph paper if it is helpful.

15. A quiche with a diameter of 12 inches can feed 6 people. Can a quiche with a diameter of 10 inches feed 4 people, assuming the same serving size? Explain your thinking.

16. A pizza with a diameter of 8 inches costs \$10. A pizza with a diameter of 14 inches costs \$16. Which is the better buy? Explain your thinking.

17. The radius of a circle is doubled.
   a. How does the circumference change?
   b. How does the area change?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

18. Is it possible for a circle to have the same numerical value for its circumference and area? Explain your reasoning.

Composite Area

ACTIVITY 17

Tile Designs

Lesson 17-1 Area of Composite Figures

My Notes

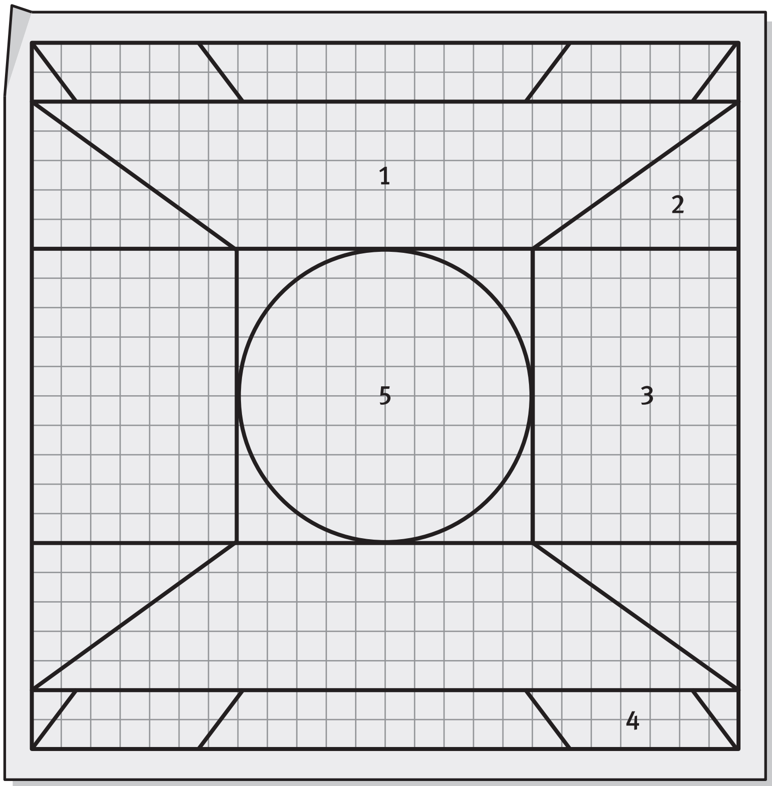
Learning Targets:

- Determine the area of geometric figures.
- Determine the area of composite figures.

SUGGESTED LEARNING STRATEGIES: Create Representations, Discussion Groups, Identify a Subtask, Think-Pair-Share, Visualization

Each year the students in Ms. Tessera's classes create a design for a stained-glass window. They draw two-dimensional figures on grid paper to create the design for their stained-glass windows.

1. This drawing shows the design for one of the projects done last year.



a. What is the area of the entire stained-glass window? Explain.

b. What is the most precise geometric name for each of the numbered shapes in the design?

My Notes

MATH TIP

Recall the following area formulas.

Triangle: $A = \frac{1}{2}bh$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Circle: $A = \pi r^2$

ACADEMIC VOCABULARY

In geometry, when you divide a **composite figure** into smaller figures, you ***decompose*** the figure.

c. Find the area of each numbered shape.

Figure 1 Figure 2 Figure 3

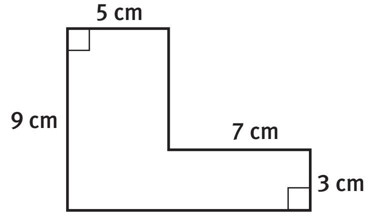
Figure 4 Figure 5

d. Explain why the students might be interested in finding the areas of the different shapes used in the design of the stained-glass window.

Some geometric figures have formulas that can be used to find the area of the figure. Other shapes do not have their own formulas.

A **composite figure** is made up of two or more geometric figures. You can find the area of a composite figure by dividing, or ***decomposing***, it into simpler geometric shapes with known formulas.

2. The composite figure below can be divided into two rectangles.



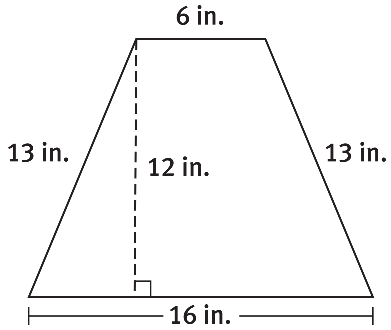
a. Draw a line segment on the diagram to divide the figure into two rectangles. Label the length and width of each rectangle.

b. Find the total area of the composite figure. Show your work. Label your answer with the appropriate unit of measure.

c. Find the perimeter of the composite figure. Show your work. Label your answer with the appropriate unit of measure.

Check Your Understanding

3. Use the trapezoid shown.



a. Explain how to find the area of the trapezoid by dividing it into simpler geometric shapes.

b. Find the area of the trapezoid using the simpler geometric shapes you found in part a.

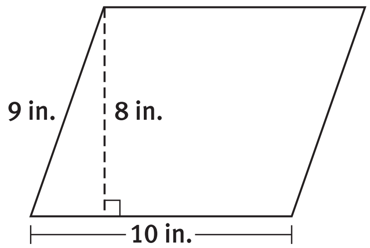
c. Use the formula for the area of a trapezoid to find the area. Compare this area to the area you found in Part b.

4. Construct viable arguments. When dividing a composite figure into simpler geometric shapes to find the area, explain why the simpler figures cannot overlap or have gaps.

LESSON 17-1 PRACTICE

For Items 5–10, find the perimeter and area of each figure.

5.



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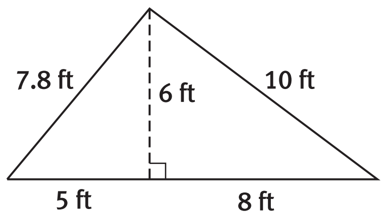

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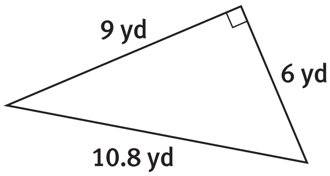

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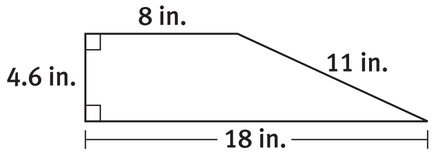

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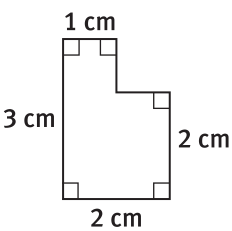
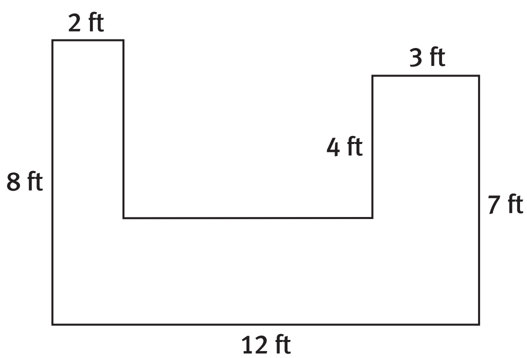
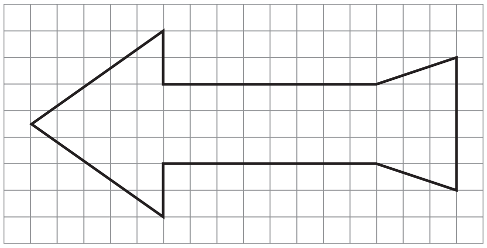


10.



My Notes

11. What is the area of the arrow in square units? Justify your answer.



12. Make sense of problems. A 4-inch-wide by 6-inch-long picture is placed on a solid mat that forms a frame around it. The mat is 8 inches long. The mat and the picture are similar rectangles. What is the area of the mat?

13. Make use of structure. Parallelogram 1 has a base of 26 centimeters and a height of 15 centimeters. Parallelogram 2 is identical to it. A triangle with a base of 8 centimeters is cut from parallelogram 2 and placed so its base rests on the top of the original figure. How does the area of the resulting composite figure compare to the area of parallelogram 1? Explain.

14. Reason quantitatively. A kite is formed by connecting the bases of two triangular frames. The height of the top frame of the kite is 8 inches. The height of the lower section is 14 inches. The bases of the frames are 10 inches long. What is the least amount of material needed to make two kites?

My Notes

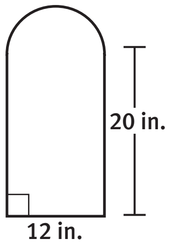
Learning Targets:

- Determine the area of composite figures.
- Solve problems involving area.

SUGGESTED LEARNING STRATEGIES: Chunking the Activity, Group Presentation, Summarize/Paraphrase/Retell, Identify a Subtask, Visualization

Composite figures may contain parts of circles. To find the area of these figures, it is necessary to identify the radius or the diameter of the circle.

1. The composite figure shown can be divided into a rectangle and a semicircle.



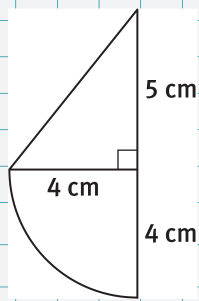
a. What is the diameter of the **semicircle**?

b. Find the total area of the figure. Use $\pi = 3.14$. Show your work.

c. Find the distance around the figure. Show your work.

MATH TERMS

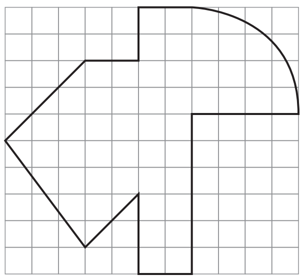
A **semicircle** is an arc whose measure is half of a circle. The area of a semicircle is half of the area of a circle with the same radius.



2. The figure in the My Notes column is divided into a right triangle and a quarter-circle. Find the area of the composite figure. Use $\pi = 3.14$.

3. A student dropped a piece of stained glass. A fragment has the shape shown below.

a. Divide the fragment into smaller shapes you can use to find its total area.

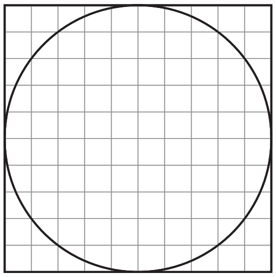


My Notes

b. Find the area of the fragment if each square on the grid represents 1 cm^2. Use $\pi = 3.14$. Show the calculations that led to your answer.

You can break a composite figure into geometric shapes and add the areas of the shapes to find the area. You can also subtract to find the area of part of a composite figure.

4. The stained-glass design from on page 179 contains a circle **inscribed** in a square, as shown.



a. Shade the region that is inside the square but outside the circle.

b. Describe a method for finding the area of the shaded region.

c. Find the area of the shaded region if each square on the grid represents 1 cm^2. Use $\pi = 3.14$. Show your work.

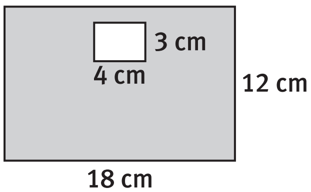
MATH TERMS

An **inscribed figure** is a figure that is placed inside another figure so that it fits exactly.

For example, in Item 4 to the right, the circle inscribed in the square fits so that the diameter of the circle and the side of the square have the same measure.

Check Your Understanding

5. What is the area of the shaded region? Explain your thinking.

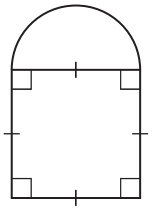


6. Construct viable arguments. Explain how to find the area of a composite figure composed of simpler geometric shapes.

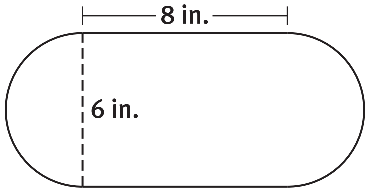
LESSON 17-2 PRACTICE

Find the area of each figure. Use $\pi = 3.14$.

7.

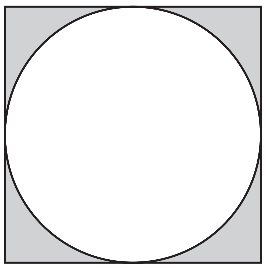


8.

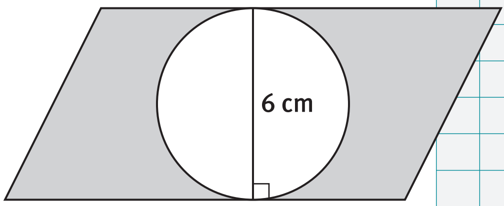


Find the area of the shaded region. Use $\pi = 3.14$.

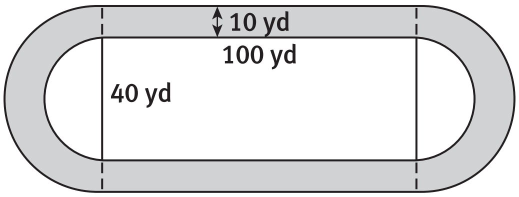
9.



10.



11. An athletic field has the shape of a 40-yard-by-100-yard rectangle with a semicircle at each end. A running track that is 10 yards wide surrounds the field. Use this information to answer the questions below. Use $\pi = 3.14$.



a. Find the area of the athletic field, without the track.
b. Find the area of the athletic field with the track included.
c. Find the area of the track, the shaded portion of the diagram.
d. Suppose a rectangular fence of 180 yards by 80 yards encloses the athletic field and running track. How much of the fenced area is not a part of the field and track?

12. Make sense of problems. A circular plate has a diameter of 6 inches. A pancake in the center of the plate has a radius of 2 inches. How much of the plate is not covered by the pancake? Use $\pi = 3.14$.

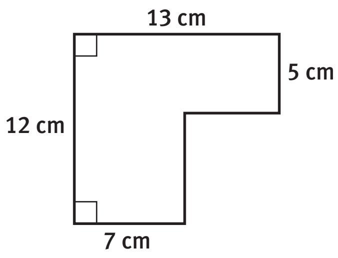
13. Reason quantitatively. A section of stained glass is made by placing a circle at the top of a triangle with a base of 10 centimeters and a height of 8 centimeters. The diameter of the circle is equal to the height of the triangle. What is the area of the section of stained glass? Show your work. Use $\pi = 3.14$.

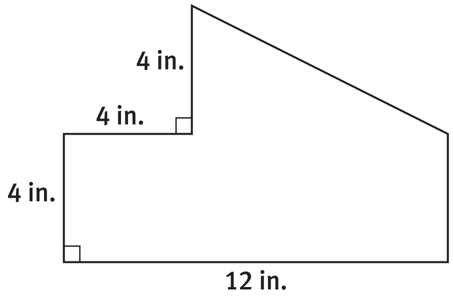
ACTIVITY 17 PRACTICE

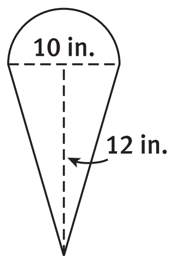
Write your answers on a separate piece of paper. Show your work.

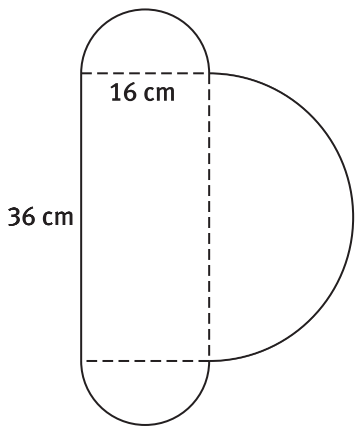
Lesson 17-1

For Items 1–4, find the area of the figure. Use $\pi = 3.14$.

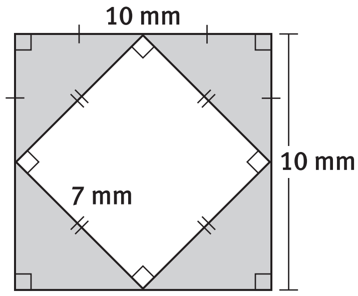
1. 

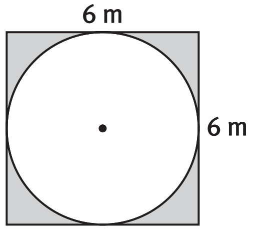
2. 

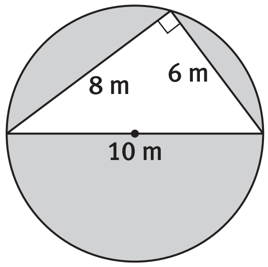
3. 

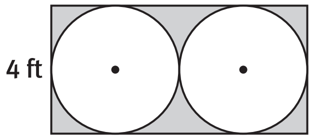
4. 

For Items 5–8, find the area of the shaded region. Use $\pi = 3.14$.

5. 

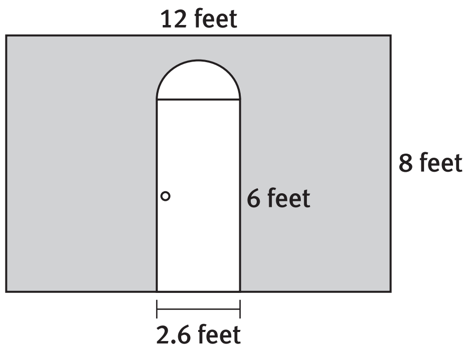
6. 

7. 

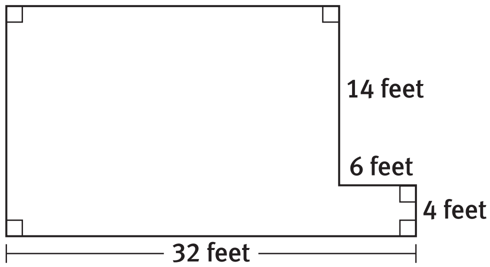
8. 

Lesson 17-2

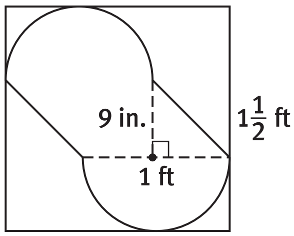
9. Sue wants to paint the wall shown. What is the area of the wall to the nearest tenth? Use $\pi = 3.14$.
A. 72.2 ft^2
B. 75.1 ft^2
C. 77.7 ft^2
D. 80.4 ft^2



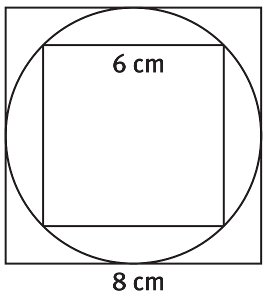
10. A room with the dimensions shown needs carpet. How much carpet is needed to cover the entire floor of the room?
A. 576 sq ft
B. 492 sq ft
C. 472 sq ft
D. 388 sq ft



11. A square blanket has a design on it as shown. Find each of the following in square inches and in square feet. Use $\pi = 3.14$.
a. area of the design
b. area of the blanket without the design



12. Each square section of a quilt has the design shown. Use $\pi = 3.14$.
a. What is the area of the circular section between the two squares?
b. What is the area of the four corner sections?

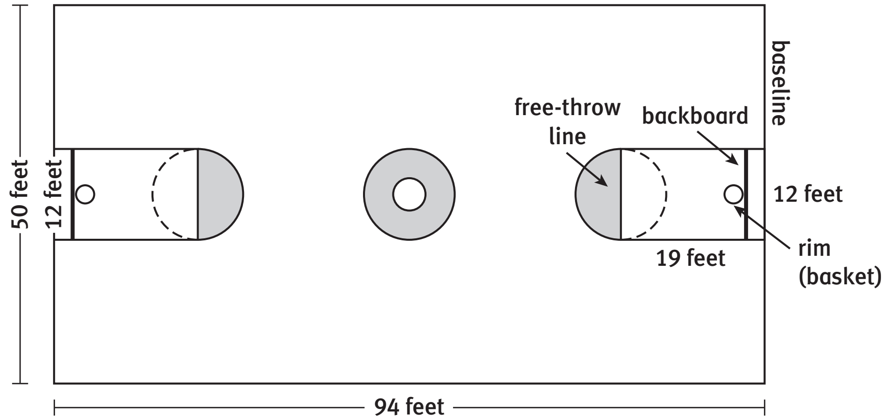


MATHEMATICAL PRACTICES
Look for and Make Use of Structure

13. How does knowing the area formulas of simple geometric shapes help you find the area of composite figures?

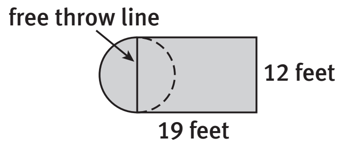
Circumference and Area
IN THE PAINT

An NBA basketball court is 94 feet long and 50 feet wide. It contains three circles, each with a diameter of 12 feet. Two of these circles are located at the free-throw lines, and the third circle is at the center of the court. Within the third circle is another circle with a radius of 2 feet.



1. One gallon of paint will cover 110 square feet. How many gallons of paint will be needed to paint the shaded regions on the court? Use $\pi \approx 3.14$. Explain your thinking.

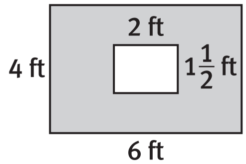
2. The region including the circle at the free-throw line to the baseline is shown. Find the area of this region. Use $\pi \approx 3.14$.



3. The key is the rectangular region on the basketball court from the free-throw line to the backboard. The backboard is 4 feet from the baseline.
   a. Is the key similar to the basketball court? Explain.
   b. Is the inner circle similar to the entire circle in the center of the court? Explain.

Circumference and Area
IN THE PAINT

A vertical *backboard* located 4 feet from the baseline supports the rim of the basketball net. The backboard measures 6 feet wide and 4 feet high. The shooter's square is a white box above the rim of the basket. It must measure $1\frac{1}{2}$ feet high and 2 feet wide, as shown at right.

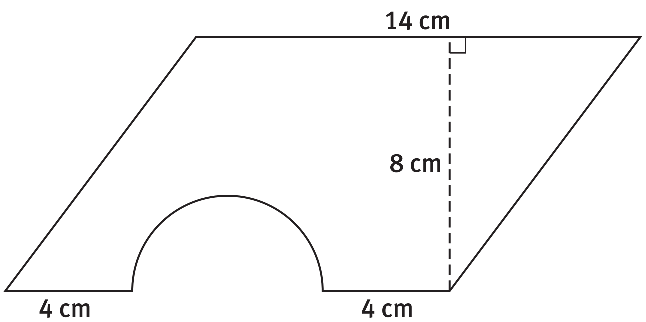


4. What is the area of the portion of the backboard that is NOT white?

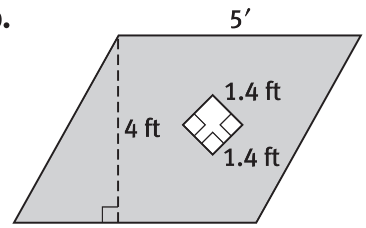
5. The rim of the basket has a radius of 9 inches.
   a. What is the approximate circumference of the basket? Use $\pi \approx 3.14$.
   b. Explain why 3.14 is used when finding the circumference of circles.

6. The design of a basketball team's logo sometimes includes geometric designs. The shapes below are from the logos of two teams. Find the area of each shape.

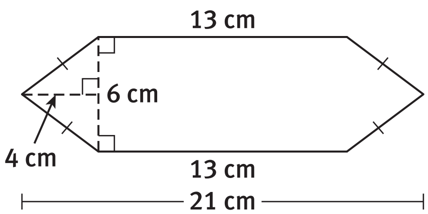
a.



b.



7. Michael claims he can find the area of the composite shape shown by inscribing it in a rectangle and subtracting. Devora claims that to find the area you need to use addition. Which student is correct? Justify your answer.



Circumference and Area
IN THE PAINT

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3a-b, 4, 5a-b, 6a-b, 7)	• Accurately and efficiently finding the circumference and area of circles and the area of composite figures.	• Finding the circumference and area of circles and the area of composite figures.	• Difficulty finding the circumference and area of circles and the area of composite figures.	• No understanding of finding the circumference and area of circles and the area of composite figures.
Problem Solving (Items 1, 2, 4, 5a, 6a-b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 3a-b, 7)	• Clear and accurate understanding of similar figures. • Solving composite figures by adding or subtracting.	• An understanding of similar figures. • Recognizing that composite figures are made up of simpler figures.	• Difficulty recognizing similar figures. • Difficulty in working with composite figures.	• No understanding of similar figures. • No understanding of composite figures.
Reasoning and Communication (Items 1, 3a-b, 5b, 7)	• Precise use of appropriate terms to explain similar figures, finding area, and π.	• An adequate explanation of similar figures, finding area, and π.	• A partially correct explanation of similar figures, finding area, and π.	• An incomplete or inaccurate explanation of similar figures, finding area, and π.

Sketching Solids

Putt-Putt Perspective

Lesson 18-1 Shapes That Result from Slicing Solids

My Notes

Learning Targets:

- Draw different views of three-dimensional solids.
- Identify cross sections and other views of pyramids and prisms.

SUGGESTED LEARNING STRATEGIES: Visualization, Look for a Pattern, Use Manipulatives, Create Representations

The Service Club at Park Middle School is creating a miniature golf course to raise funds for a food bank. The theme is interesting structures around the world. Buildings that will be included are the Washington Monument, the Flatiron Building, the Louvre Pyramid, and the Pentagon Building.

CONNECT TO GEOGRAPHY

The Washington Monument is located in Washington, DC. The Pentagon is nearby in Arlington County, VA. The Flatiron Building is in New York, NY. The Louvre Pyramid is the entrance to the Louvre Museum in Paris, France.

1. Compare and contrast the two- and three-dimensional shapes in these four buildings.

My Notes

MATH TERMS

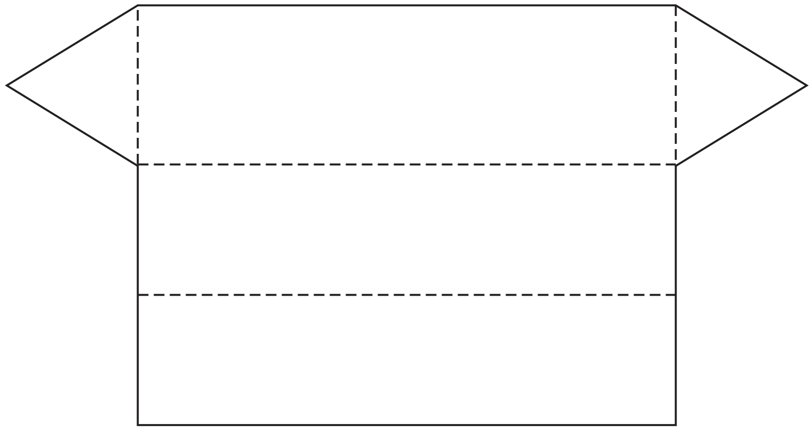
A **net** is a a two-dimensional pattern that can be folded to form a solid.

To prepare for drawing and building the structures for the mini-golf course, the students explore relationships between shapes, views of shapes, and drawings of shapes.

A **prism** is a solid with two parallel congruent bases that are both polygons.

2. The **net** shows a two-dimensional pattern for a prism.

Figure 1



a. Cut out Figure 1 on the next page. Fold it along the dashed lines to form a prism.

b. A prism is named using the shape of its bases. Name the solid formed by the net in Figure 1.

c. The **faces** of a prism are the sides that are not the bases. What is the shape of the three faces?

d. **Reason abstractly.** Imagine making a slice through the prism parallel to the bases. What is the shape of the two-dimensional slice?

e. **Reason abstractly.** Imagine making a slice through the prism perpendicular to the bases. What is the shape of the two-dimensional slice?

My Notes

Figure 1

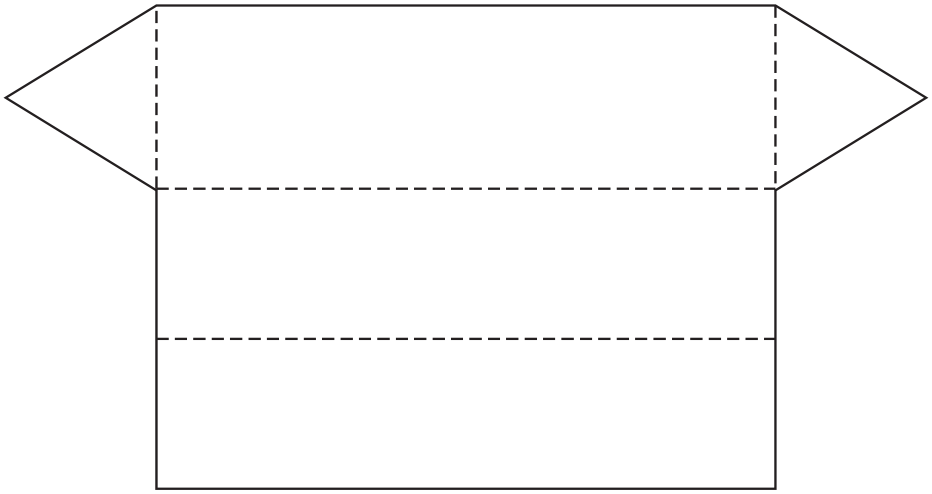
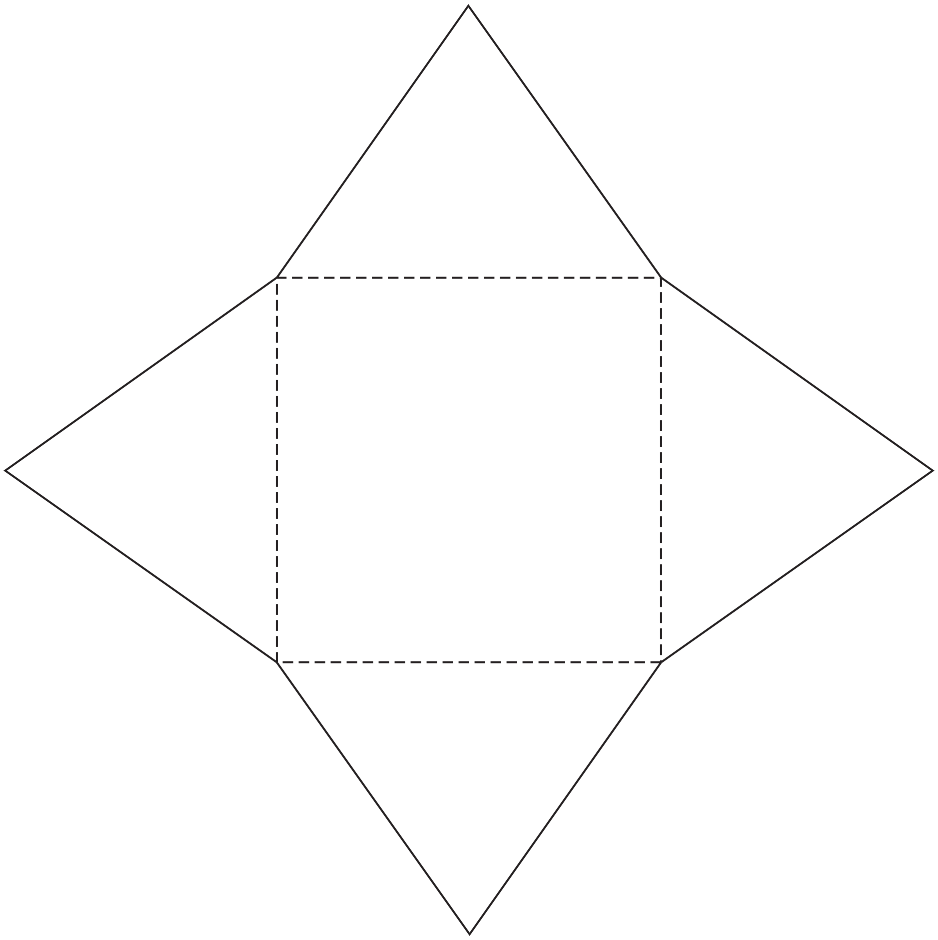


Figure 2



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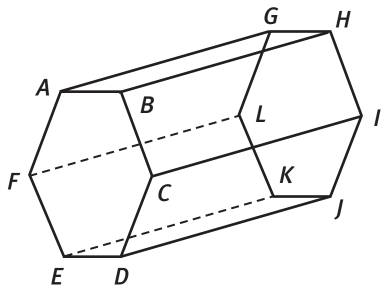
My Notes

3. Consider the hexagonal prism shown on the right.



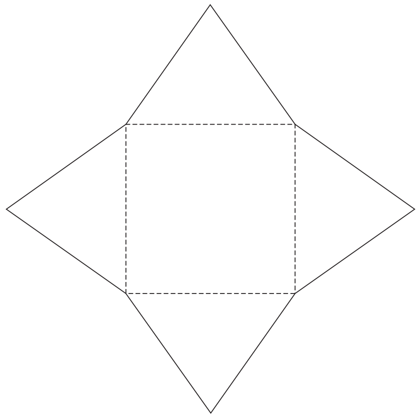
a. Why do you think some of the lines are dotted?

b. Imagine making a slice that goes through points *B*, *D*, *J*, and *H* of the prism. What is the shape of the two-dimensional slice?

A **pyramid** is a solid that has only one base. That base is a polygon. The faces of a pyramid are all triangles.

4. The net shows a pattern for a pyramid.

Figure 2



a. Cut out Figure 2 on page 193. Fold it to form a pyramid.

b. A pyramid is named using the shape of its base. What is the name of the solid formed by Figure 2?

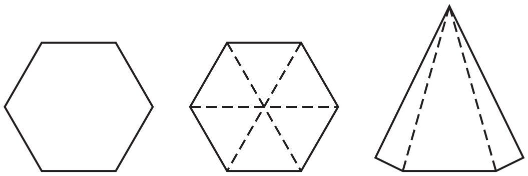
c. **Reason abstractly.** Imagine making a slice through the pyramid parallel to the base. What is the shape of the two-dimensional slice?

d. **Model with mathematics.** Sketch and label the view of the base, side, and top of the pyramid.

My Notes

5. Three views of a hexagonal pyramid are shown.

a. Label each view as *side*, *top*, or *base*.



b. Explain the significance of the dashed segments in the second view.

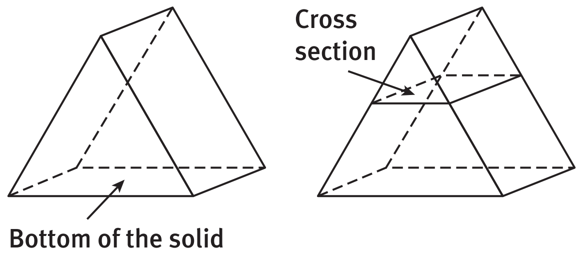
6. Model with mathematics. Sketch and label the bottom, top, and side views of the Pentagon Building.

My Notes

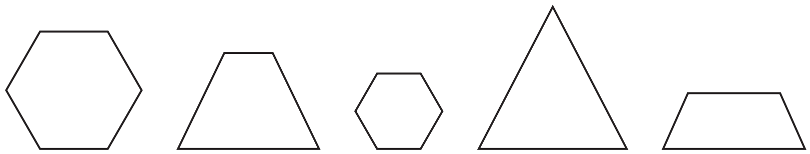
MATH TIP

You can think of a cross section as a slice of a solid that is parallel or perpendicular to the base of the solid.

A **cross section** of a solid figure is the intersection of that figure and a plane.



7. Reason abstractly. Several cross sections of a hexagonal pyramid are shown. Label each cross section as *parallel* or *perpendicular* to the base of the pyramid.



Check Your Understanding

Write your answers on a separate piece of paper.
For Items 8–10, consider a rectangular pyramid.

8. Describe the shape of the base and the faces of the pyramid.

9. a. What shapes are formed by cross sections parallel to the base? Explain your thinking.

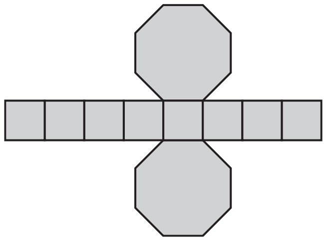
b. Are all of the cross sections parallel to the base the same size?

10. Construct viable arguments. Are all of the cross sections perpendicular to the base the same shape and size? Justify your answer.

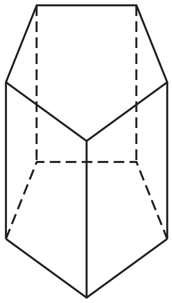
My Notes

LESSON 18-1 PRACTICE

11. What is the name of the solid formed by the net?



Use the solid to the right for Items 12–14.

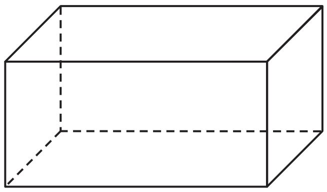


12. What is the name of the solid?

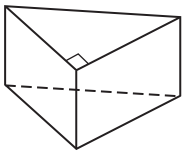
13. **Reason abstractly.** Imagine making slices through the solid parallel to the bases. What two-dimensional shapes are formed?

14. **Reason abstractly.** Imagine making slices through the solid perpendicular to the bases. What two-dimensional shapes are formed?

15. **Model with mathematics.** Sketch and label the bottom, top, and side views of the rectangular prism.



Use the solid to the right for Items 16 and 17.



16. Sketch the cross section that is parallel to the bases.

17. Sketch three different cross sections that are perpendicular to the bases.

For items 18 and 19, consider an octagonal pyramid.

18. Sketch two different cross sections that are parallel to the base of the pyramid.

19. Sketch three different cross sections that are perpendicular to the base of the pyramid.

20. **Construct viable arguments.** Can the cross section of a solid ever be a point? Explain your thinking.

21. **Reason abstractly.** How can the name of a prism or a pyramid help you visualize the cross sections of the solid?

My Notes

Learning Targets:

- Calculate the lateral and total surface area of prisms.

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations, Summarize/Paraphrase/Retell, Think-Pair-Share, Visualization, Discussion Groups

The students in the service club will paint the structures in the golf course. They first investigate how to find the surface area of prisms.

1. Work with your group. Look at Net 1, Net 2, and Net 3 on pages 203 and 204. What solids do the nets form?

A **lateral face** of a solid is a face that is not a base. A **right prism** is a prism on which the bases are directly above each other, making the lateral faces perpendicular to the bases. As a result, all the lateral faces are rectangles. The **lateral area** of a solid is the sum of the areas of the lateral faces.

2. For Nets 1–3, what are the shapes of the lateral faces of the figures? Explain.

3. **Attend to precision.** Find the area of each lateral face and the lateral area of each prism.

	Face 1	Face 2	Face 3	Face 4	Face 5	Lateral Area
Net 1						
Net 2						
Net 3						

GROUP DISCUSSION TIP

With your group, read the text carefully. Reread definitions of terms as needed to help you comprehend the meanings or words, or ask your teacher to clarify vocabulary terms.

MATH TIP

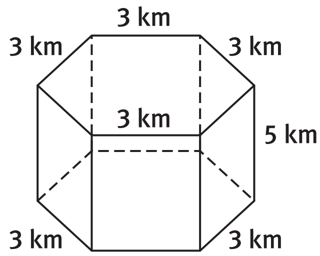
Net 1: Use the square units on the figure to find the lateral areas.

Net 2: Use the given measurements to find the lateral areas.

Net 3: Measure the side lengths to the nearest whole centimeter. Then find the lateral areas.

My Notes

4. Examine the right hexagonal prism.



a. Find the area of each lateral face. Show your work.

b. Find the lateral area of the solid. Show all your work.

c. Determine the perimeter of the base, P.

d. Multiply the perimeter of the base, P, times the height of the prism, h.

e. Compare your responses to Parts b and d. What do you notice?

My Notes

The following formula can be used to find the lateral area of a prism: $L = P \times h$, where L represents the lateral area, P represents the perimeter of the base, and h represents the height of the prism.

5. Use the formula to find the lateral area of the prisms in Item 24 for Nets 1 and 2. Are the lateral areas the same as the ones you recorded in the table?
Net 1:

Net 2:

The **surface area** of a prism is the sum of the areas of the lateral faces and the areas of the bases.

6. **Attend to precision.** Describe the relationship between the lateral area and the surface area of a prism.

7. **Reason quantitatively.** Find the surface area of each of the prisms in Item 24. Explain your thinking.

Net 1:

Net 2:

Net 3:

Check Your Understanding

Write your answers on a separate piece of paper.

8. **Make use of structure.** Why do you think that the lateral area of a prism is equal to the product of the perimeter and the height of the prism?

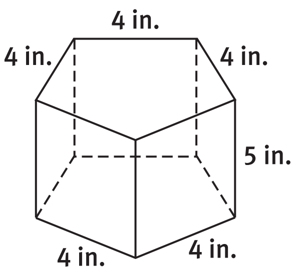
9. **Construct viable arguments.** Explain how to use a net to find the lateral and total surface area of a prism.

My Notes

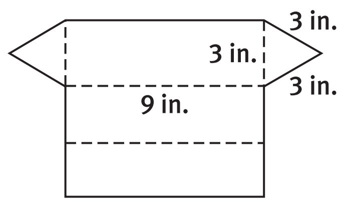
LESSON 18-2 PRACTICE

Find the lateral area of the prisms in Items 10 and 11.

10.



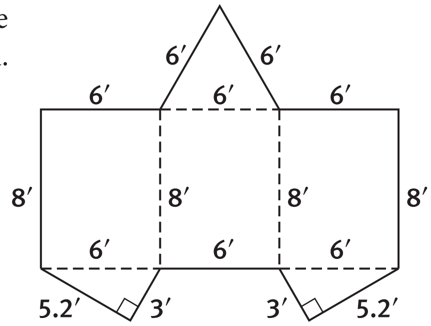
11.



Use the net to the right for Items 12 and 13.

12. Reason quantitatively. Find the lateral area of the triangular prism.

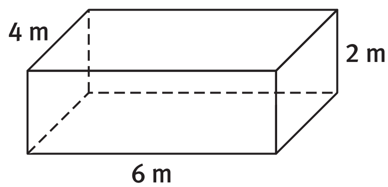
13. Find the surface area of the triangular prism.



Use the prism to the right for Items 14 and 15.

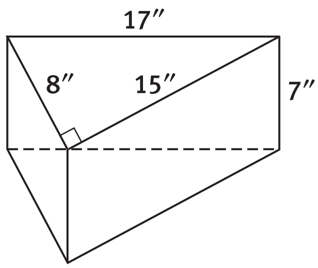
14. Model with mathematics. Draw a net to find the lateral area of the prism.

15. Find the surface area of the prism.



Use the prism for Items 16–18.

16. A display case is shaped like the prism shown. The case needs to be covered with a plastic film. How much film is needed to cover the lateral area?

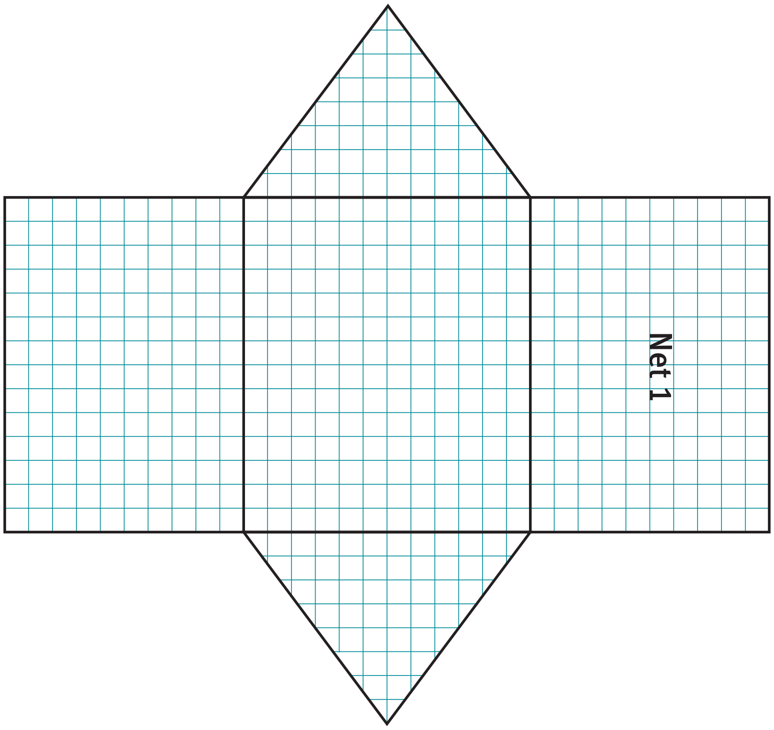


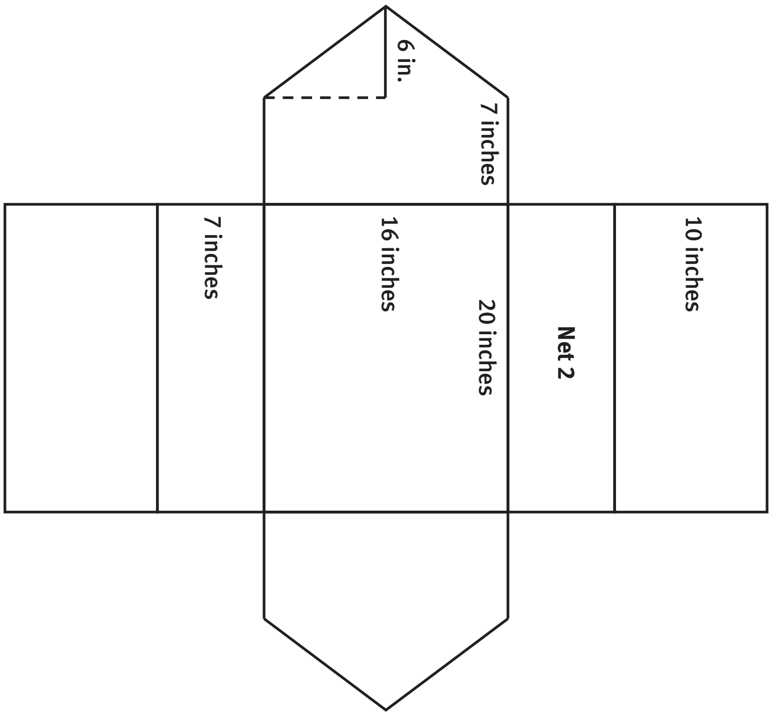
17. How much film is needed to cover the surface area of the display case?

18. Make sense of problems. How much film is needed to cover all but the face the case rests on?

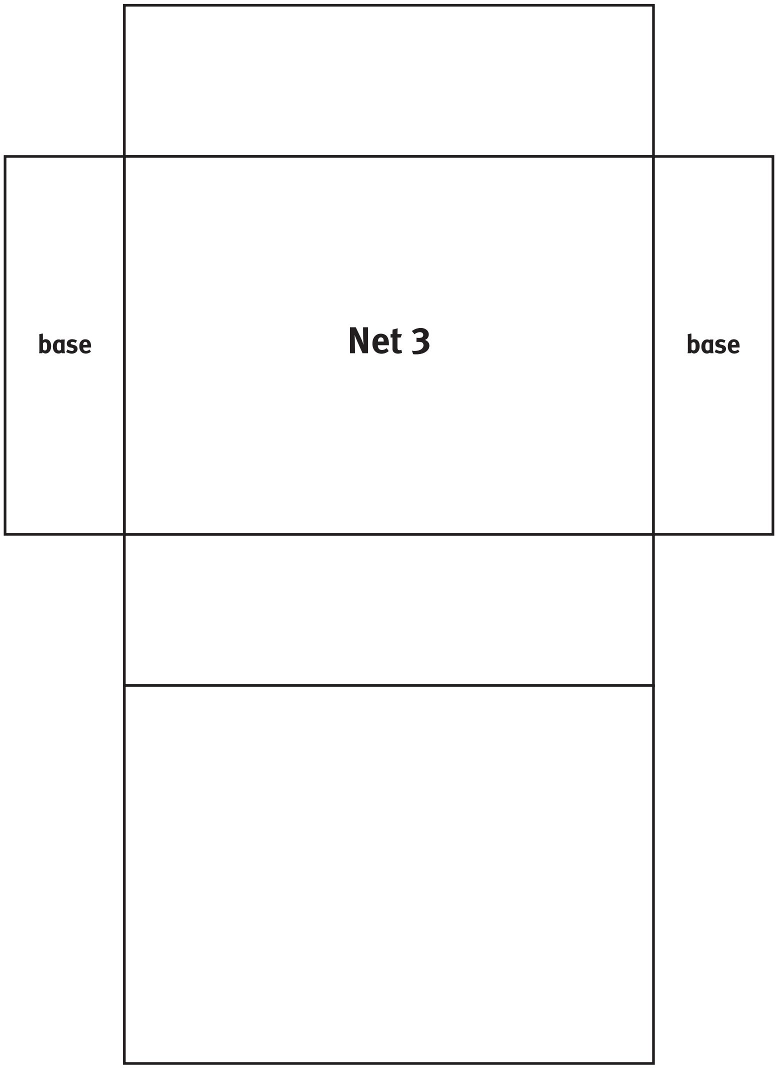
19. Attend to precision. A cube-shaped block has edges that are 3 inches long. A larger block has edges that are twice as long. Compare the surface area of the smaller block to the surface area of the larger block. Support your answer.

My Notes





My Notes



My Notes

Learning Targets:

- Calculate the lateral and total surface area of pyramids.

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation, Marking the Text, Use Manipulatives, Visualization, Vocabulary Organizer

The students in the service club also investigate how to find the surface area of pyramids.

Two nets of pyramids the students use are on page 210.

The lateral area of a pyramid is the combined area of the faces. The height of a triangular face is the **slant height** of the pyramid.

1. Use Net 4, the net of the square pyramid.
  a. Draw the slant height on the net.

  b. Why do you think it is called the slant height?

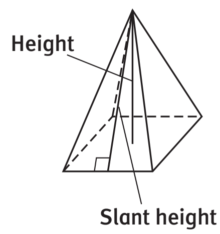
2. Use Net 4 to find the lateral area of the square pyramid. Explain your thinking.

The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base.

3. Use Net 4 to find the surface area of the square pyramid. Explain your thinking.

MATH TIP

Be sure you do not confuse the *slant height* of a pyramid with its *height*. Slant height is a measure along a triangular face. Height is an internal measure from the vertex to the base.



My Notes

CONNECT TO History

The triangular pyramid that can be created using Net 5 has four identical faces. It is an example of a Platonic solid. The Platonic solids are sometimes also called "cosmic figures." There are only five Platonic solids: cube, tetrahedron, dodecahedron, octahedron, and icosahedron.

4. **Attend to precision.** Use a centimeter ruler to measure Net 5. Then find the surface area of the triangular pyramid.

5. **Model with mathematics.** Consider a square pyramid with base edges 12 cm and slant height 10 cm.
   a. Draw a net to represent the pyramid and label the dimensions of the base edges and the slant height.

   b. Students found the lateral area of the square pyramid using the method shown below. Explain the steps of the method.

Step 1: $\frac{1}{2}\times 12\times 10+\frac{1}{2}\times 12\times 10+\frac{1}{2}\times 12\times 10+\frac{1}{2}\times 12\times 10$

Step 2: $=\frac{1}{2}\times(12+12+12+12)\times 10$

Step 3: $=\frac{1}{2}\times(4\times 12)\times 10$

Step 4: $=\frac{1}{2}\times 48\times 10$

Step 5: $=240$ cm2

The following formula can be used to find the lateral area of a regular pyramid:

$L = \frac{1}{2}P \times \ell$, where L represents the lateral area, P represents the perimeter of the base, and ℓ represents the slant height of the pyramid.

6. **Construct viable arguments.** A student says that the above formula can be used to find the lateral area of a rectangular pyramid. Is the student correct? Explain your reasoning.

7. **Make sense of problems.** The base of a regular triangular pyramid has sides that are 8 meters long and a height of 6.9 meters. The slant height of the pyramid is 6.9 meters.
   a. Find the lateral area of the pyramid. Explain your thinking.

   b. Find the surface area of the pyramid. Explain your thinking.

Check Your Understanding

Write your answers on a separate piece of paper.

8. Why do you need to know the slant height, rather than the height, of a regular pyramid to find the surface area of the pyramid?
9. **Attend to precision.** Explain how to use a net to find the lateral and the total surface area of a pyramid.

MATH TERMS

A **regular polygon** is a polygon with congruent sides and congruent angles.

A **regular pyramid**, also called a right regular pyramid, is a pyramid with a base that is a regular polygon, and all the lateral faces are congruent.

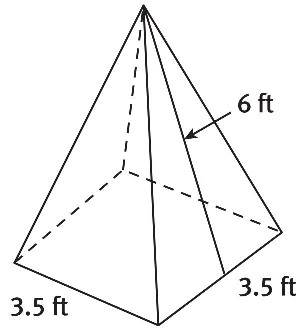
My Notes

LESSON 18-3 PRACTICE

Use the square pyramid for Items 10 and 11.

10. **Model with mathematics.** Use a net to find the lateral area of the pyramid.

11. Find the surface area of the pyramid.



Use the following information for Items 12 and 13.

A regular triangular pyramid has a base length of 4.6 centimeters and a slant height of 4 centimeters.

12. Use a net to find the lateral area of the pyramid.

13. If the height of the triangular base is approximately 4 cm, what is the surface area of the pyramid?

Use the following information for Items 14 and 15.

The Louvre Pyramid has a square base with sides that are 35 meters long. The slant height of each triangular face of the pyramid is 27.84 meters.

14. What is the lateral area of the Louvre Pyramid?

15. What is the surface area of the Louvre Pyramid?

My Notes

16. A square pyramid has a slant height of 5 meters. The perimeter of the base is 32 meters. Find the surface area of the pyramid.

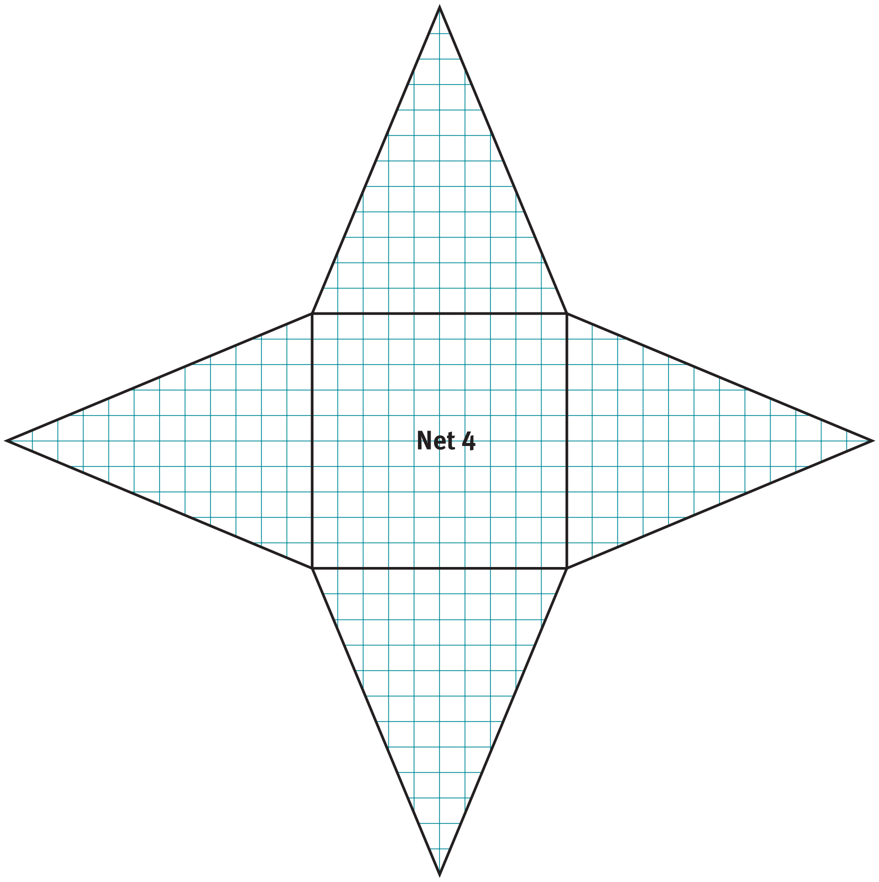
17. Construct viable arguments. A regular triangular pyramid has a base length of 3.5 meters, a height of 1.3 meters, and a surface area of 21 square meters. What is the approximate slant height of the pyramid? Justify your answer by showing how you found it.

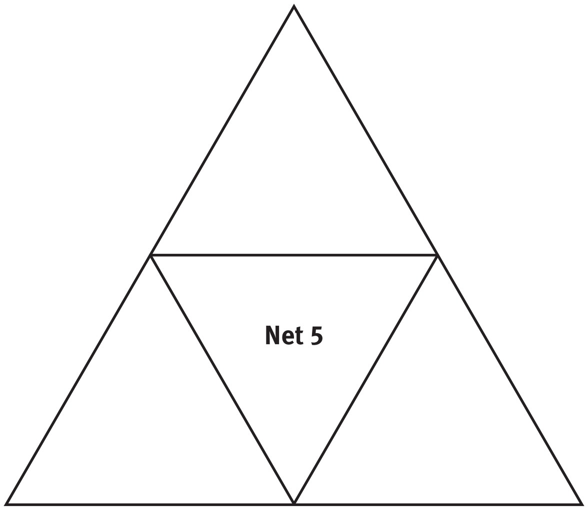
18. The pyramid of Khufu in Giza, Egypt, is a square pyramid with a base length of 756 feet. The slant height of this great pyramid is 612 feet. What is the lateral area of the pyramid of Khufu?

19. Make use of structure. A party favor is made from two square pyramids joined at their bases. Each edge of the square base is 3 centimeters. The slant height of the triangular faces is 4 centimeters. What is the surface area of the party favor?

20. Make sense of problems. A model of a Mayan pyramid has a square base with sides that are 1.3 meters long. The slant height of the pyramid is 0.8 meter. It costs $4.59 per square meter to paint the pyramid. How much will it cost to paint the lateral area of the model?

My Notes



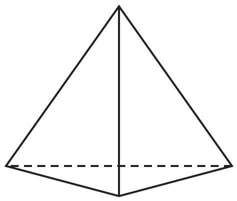


ACTIVITY 18 PRACTICE

Write your answers on a separate piece of paper. Show your work.

Lesson 18-1

Use the solid for Items 1 and 2.

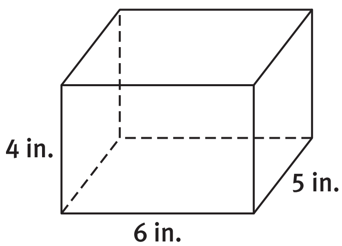


1. Imagine making slices through the solid parallel to the base. What two-dimensional shapes are formed?
2. Imagine making slices through the solid perpendicular to the base. What two-dimensional shapes are formed?

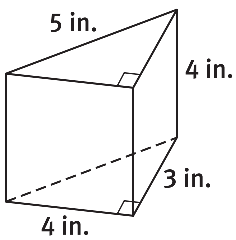
Lesson 18-2

Find the lateral and surface area of the figures in Items 3–5.

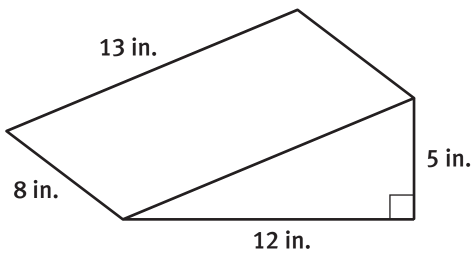
3.



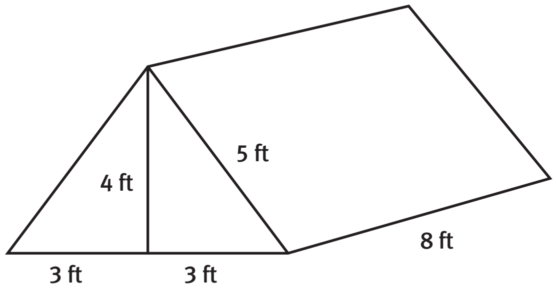
4.



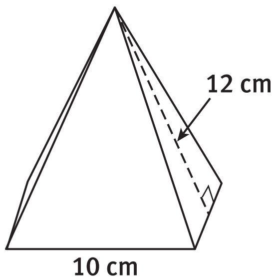
5.



6. A tent with canvas sides and a floor is shown. How much canvas is used to make the sides and floor of the tent?

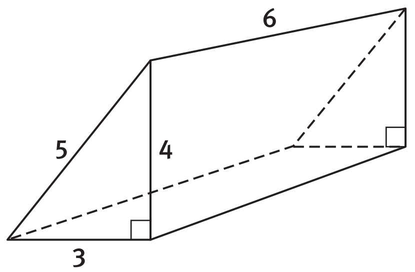


7. A rectangular prism is 10 meters tall. It has a square base with sides that are 4 meters long. What is the surface area of the prism?
8. Find the lateral and the surface area of the square pyramid.



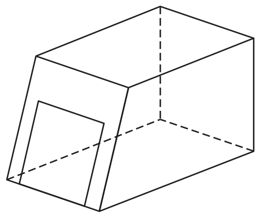
Lesson 18-3

9. The diagram shows the dimensions of a wooden block. The block will be covered with a reflective film. How much of the film is needed to cover the entire block?



A. 72 cm^2
B. 84 cm^2
C. 96 cm^2
D. 108 cm^2

Use the information and the drawing for Items 10 and 11. The Pup Company has a new model of dog kennel. The left and right sides are trapezoids and all other faces are shown in the diagram.



10. Sketch the top, front, and side views of the kennel.

11. Which cross sections of the kennel described below will be congruent?
A. all cross sections that are perpendicular to the bottom and parallel to the front and back faces
B. all cross sections that are perpendicular to the bottom and parallel to the left and right faces
C. all cross sections that are parallel to the top and bottom
D. none of the above

12. A cardboard box is 32 centimeters long, 15 centimeters wide, and 6 centimeters tall. The box does not have a top. How much cardboard was used to make the box?

13. The length of a side of the base of a square pyramid is 15 inches. The pyramid has a lateral area of 270 square inches. What is the slant height of the pyramid?

14. Three identical boxes are stacked by placing the bases on top of each other. Each box has a base that is 18 inches by 9 inches and is 4 inches tall. The stack of boxes will be shrink-wrapped with plastic. How much shrink-wrap is needed to cover the boxes?

15. A shed has the shape of a cube with edges that are 6 feet long. The top of the shed is a square pyramid that fits on top of the cube. The slant height of the faces is 5 feet. The shed has a single rectangular door that is 5 feet tall by 4 feet wide. All but the door and the bottom of the shed need to be painted. What is the area of the surface that needs to be painted?

MATHEMATICAL PRACTICES
Attend to Precision

16. Describe the similarities and differences in finding the lateral areas of a prism and a pyramid that have congruent bases.

Volume—Prisms and Pyramids

ACTIVITY 19

Berneen Wick's Candles

Lesson 19-1 Find the Volume of Prisms

My Notes

Learning Targets:

- Calculate the volume of prisms.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Look for a Pattern, Predict and Confirm, Use Manipulatives

Berneen makes all the candles that she sells in her shop, Wick's Candles. The supplies for each candle cost $0.10 per cubic inch. Berneen wants to find the volume of every type of candle she makes to determine the cost for making the candles.

Volume measures the space occupied by a solid. It is measured in cubic units.

1. Berneen uses unit cubes as models of 1-inch cubes.

a. Use unit cubes to build models of 2-inch cubes and 3-inch cubes. Then complete the table.

Length of Edge (in.)	Area of Face (in.^{2})	Volume of Cube (in.^{3})
1		
2		
3		

b. Make use of structure. Describe any relationships you see in the data in the table.

MATH TIP

Cubes are named by the lengths of their edges. A 1-inch cube is a cube with edges that are 1 inch in length. A 2-inch cube is a cube with edges that are 2 inches in length.

Cubes of any size can be used to build larger cubes.

My Notes

The formula for the volume, V, of a cube, with edge length e, is $V = e^3$.

2. Reason quantitatively. One of Berneen's candles is a cube with sides that are 1.5 inches long.

a. Use the formula to find the volume of this candle. Show your work.
$V = e^3 = 1.5 \times 1.5 \times 1.5 =$

b. Recall that supplies for each candle cost \$0.10 per cubic inch. How much does it cost to make this candle, to the nearest cent?

Most of the candles Berneen makes are in the shape of rectangular prisms.

3. The formula for the volume of a cube is also equal to the area of the base times the height of the cube.

a. Consider this relationship to help complete the table. Use cubes to build the prisms as needed.

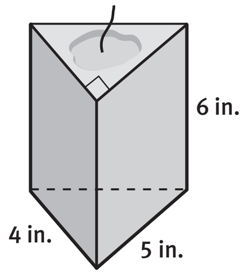
Dimensions of Candle (in.)	Area of Base (in.^{2})	Candle Height (in.)	Candle Volume (in.^{3})
$l = 4$ $w = 2$ $h = 1$			
$l = 4$ $w = 2$ $h = 2$			
$l = 4$ $w = 2$ $h = 3$			
$l = 5$ $w = 3$ $h = 1$			
$l = 5$ $w = 3$ $h = 2$			

b. Describe any pattern you see for finding volume.

My Notes

The volume, *V*, of a prism is the area of the base, *B*, times the height, *h*: $V = B \times h$.

4. Berneen makes a candle in the shape of a triangular prism as shown. The candle is very popular with many customers because of its interesting shape.



a. What is the volume of the candle? Explain your thinking.

b. Make sense of problems. Remember that the cost of the supplies for each candle is \$0.10 per cubic inch. How much profit will Berneen make if she sells this candle for \$8.99? Show your work.

Check Your Understanding

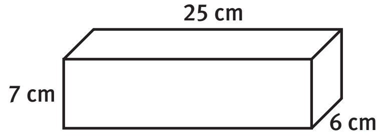
5. Construct viable arguments. Can a rectangular prism and a cube have the same volume? Support your opinion with an example or counterexample.

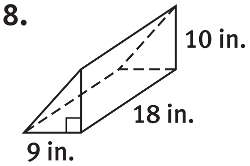
6. Make use of structure. How does the volume of a prism change when one dimension is doubled? When two dimensions are doubled? When three dimensions are doubled? Explain your thinking.

My Notes

LESSON 19-1 PRACTICE

Find the volume of each figure.

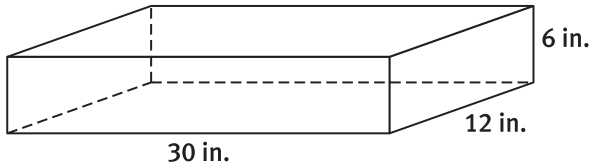
7. 

8. 

9. A cube with edge length 7 centimeters.

10. A cube that has a face area of 25 square inches.

Use the prism for Items 11 and 12.



11. How many cubes with a side length of 3 inches will fit into the rectangular prism? Explain.

12. Find the volume of the prism.

13. A small refrigerator has a square base with sides that are 3 feet long. The refrigerator has a capacity of 40.5 cubic feet. How tall is the refrigerator?

14. **Reason quantitatively.** How many cubic feet are equivalent to 1 cubic yard? Explain.

15. **Make sense of problems.** The Gray family is putting in a pool in the shape of a rectangular prism. The first plan shows a pool that is 15 feet long, 12 feet wide, and 5 feet deep. The second plan shows a pool with the same length and width, but a depth of 7 feet. How much more water is needed to fill the second pool if both pools are filled to the top?

16. **Attend to precision.** A student says that the volume of a triangular prism with a base area of 12 meters and a height of 5 meters is 60 square meters. Is the student correct? If not, what is wrong with the student's statement?

My Notes

Learning Targets:

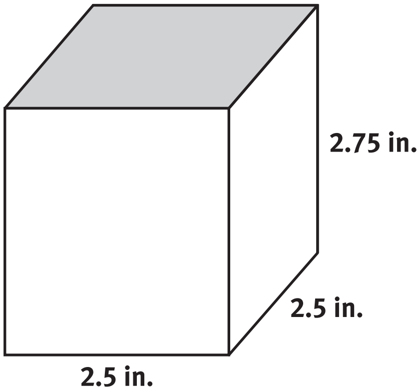
- Calculate the volume of pyramids.
- Calculate the volume of complex solids.
- Understand the relationship between the volume of a prism and the volume of a pyramid.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Predict and Confirm, Think-Pair-Share, Use Manipulatives

1. Other candles in Wick's Candles are in the shape of pyramids. To find the volumes, Berneen makes models to look for a relationship between the volume of a prism and the volume of a pyramid.

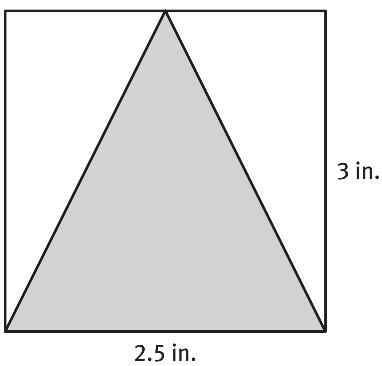
a. Make a model of the prism candle mold.

- Use index cards or card stock to cut out 1 square with side length 2.5 inches and 4 rectangles with length 2.5 inches and width 2.75 inches.
- Tape them together to form a net for a rectangular prism with no top. Then fold the net and tape it together to form a rectangular prism with no top as shown.

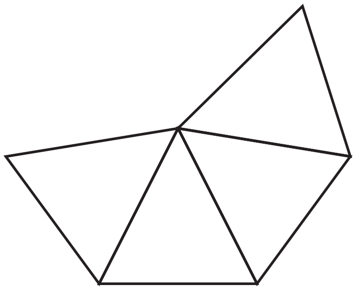


b. Make a model of the pyramid candle mold.

- Use index cards or card stock to cut out 4 isosceles triangles with the dimensions shown in the diagram.



- Tape the triangles together along their congruent sides to form a net for a square pyramid, as shown.
- Tape the net together to form a square pyramid.



My Notes

2. Compare the dimensions of the prism and the square pyramid you built in Item 1. What relationships do you notice?

3. Using the material your teacher distributes, fill your pyramid to the top. Predict the number of times you can fill and empty the pyramid into the rectangular prism to fill the prism to the top. Confirm your prediction by filling and emptying the pyramid into the prism.

The volume, V, of a pyramid is one-third the area of the base, B, times the height, h:

$$V = \frac{1}{3} \times B \times h$$

4. Reason quantitatively. Two of Berneen's candles have congruent rectangular bases. One candle is shaped like a rectangular prism, while the other is shaped like a rectangular pyramid. Both candles are 5 inches tall. What is the relationship between the volumes of the two candles? Explain.

5. Make sense of problems. A candle in the shape of a square pyramid has a base edge of 6 inches and a height of 6 inches.

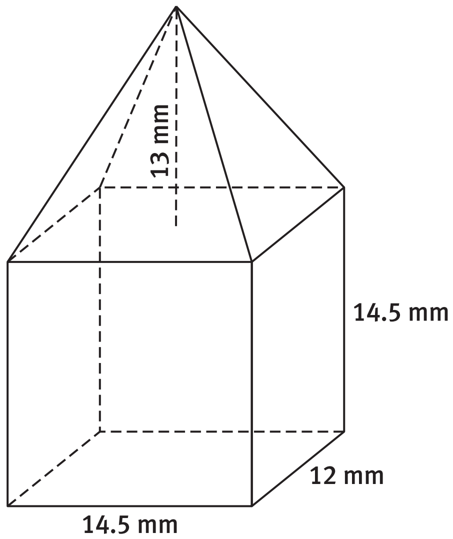
a. What is the volume of the candle?

b. How much does it cost Berneen to make the candle? Show your work.

My Notes

A **complex solid** is formed when two or more solids are put together. The volume of the complex solid is the sum of the volumes of the smaller solids.

6. Consider the complex solid shown.



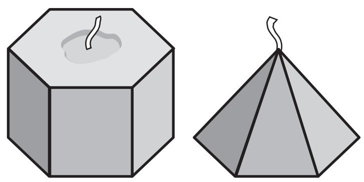
a. Find the volume of the rectangular prism.

b. Find the volume of the rectangular pyramid.

c. Find the volume of the complex solid.

Check Your Understanding

Use the candles for Items 7 and 8.



7. Express regularity in repeated reasoning. The candles shown have congruent bases and heights. What is true about the relationship between the volumes of the candles?

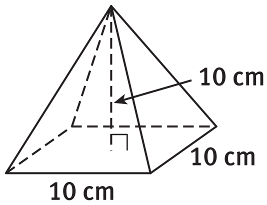
8. Model with mathematics. Suppose Berneen makes a candle by setting the pyramid on top of the prism. Write a formula for the volume of this candle.

My Notes

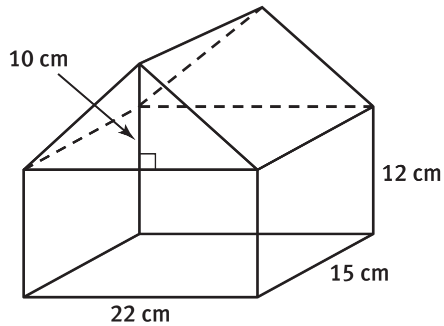
LESSON 19-2 PRACTICE

Find the volume of each figure in Items 9–11.

9.



10.



11. A triangular pyramid with a base area of 18 square inches and a height of 15 inches.

12. How does the volume of the triangular pyramid in item 11 compare with the volume of a triangular prism with a base area of 18 square inches and a height of 15 inches? Use words and symbols to explain.

13. The area of the base of a pyramid is 85 square centimeters. If its volume is 255 cubic centimeters, find the height of the pyramid.

14. A square pyramid 9 feet tall has a volume of 507 cubic feet. How long is each side of the base of the pyramid?

15. Two square pyramids are joined at their bases. Each base is 30 millimeters long. The distance between the vertices of the combined pyramids is 28 millimeters. What is the volume of the solid formed?

16. Reason quantitatively. A square pyramid with base lengths of 6 inches is 14 inches tall. The top part of the pyramid is cut off to form a smaller pyramid with base lengths that are 3 inches long and a height of 7 inches. How many square inches greater was the volume of the larger pyramid than that of the new smaller pyramid?

17. Make sense of problems. The Pyramid of Cestius in Rome today stands about 27 meters tall with a square base whose sides are about 22 meters long. The pyramid was based on ancient Nubian pyramids. These pyramids average a base area of 25.5 square meters and a height of 13.5 meters. How does the volume of the Pyramid of Cestius compare to the volume of an average Nubian pyramid?

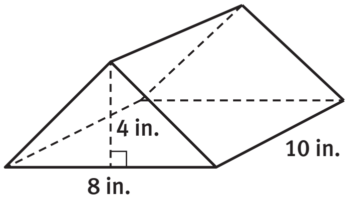
ACTIVITY 19 PRACTICE

Write your answers on a separate piece of paper. Show your work.

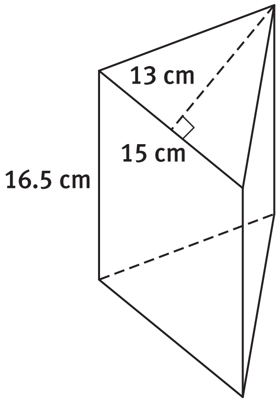
Lesson 19-1

For Items 1–4, find the volume of each figure.

1.



2.



3. A cube with edge length 8 inches.

4. A rectangular prism with sides that are 1.2, 1.8, and 2.5 meters long.

5. A rectangular prism with a square base is 6.4 meters tall. The prism has a volume of 409.6 cubic meters. What are the dimensions of the base of the prism?

6. Mariah is filling a terrarium in the shape of a rectangular prism with sand for her tarantula. The sand will be one-quarter of the way to the top. If the length of the terrarium is 17 inches, the width 12 inches, and the height 12 inches, what is the volume of the sand she uses?

Lesson 19-2

7. A container in the shape of a rectangular prism has a base that measures 20 centimeters by 30 centimeters and a height of 15 centimeters. The container is partially filled with water. A student adds more water to the container and notes that the water level rises 2.5 centimeters. What is the volume of the added water?

A. 1,500 cm^3
B. 3,600 cm^3
C. 4,500 cm^3
D. 9,000 cm^3

For Items 8–11, find the volume of the figure described.

8. A triangular pyramid with a base area of 43.3 meters and a height of 12 meters.

9. A square pyramid with base edge 10 centimeters and height 12 centimeters.

10. A triangular pyramid with a base length of 9 inches, a base height of 10 inches, and a height of 32 inches.

11. A square pyramid with a base length of 4 centimeters and a height of 6 centimeters resting on top of a 4-centimeter cube.

12. The area of the base of a triangular pyramid is 42 square feet. The volume is 1,197 cubic feet. Find the height of the pyramid.

13. The square pyramid at the entrance to the Louvre Museum in Paris, France, is 35.42 meters wide and 21.64 meters tall. Find the volume of the Louvre Pyramid.

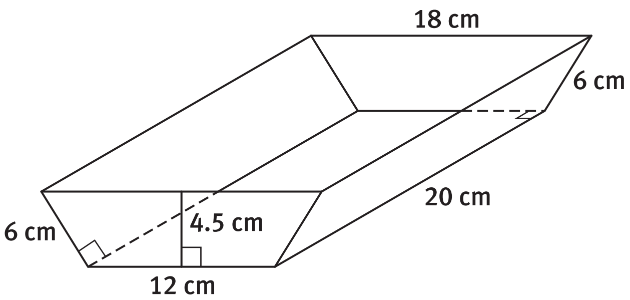
14. For prisms and pyramids, how are the area of the base of the solid *and* the shape of the solid related to the volume?

15. a. A triangular prism and a triangular pyramid have congruent bases and heights. What is the relationship between the volumes of the two figures? Explain in words using an example.

b. Explain the relationship between the volumes using their formulas.

16. A plastic tray is shown, with the dimensions labeled. The bottom and two of the sides are rectangles. The other two sides are congruent isosceles trapezoids. What is the volume of the tray?

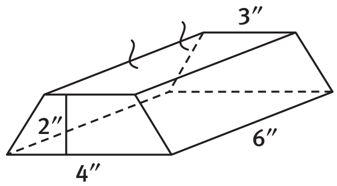
A. 1,350 cm^3
B. 1,080 cm^3
C. 1,440 cm^3
D. 1,620 cm^3



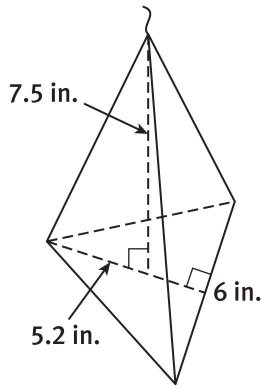
MATHEMATICAL PRACTICES
Attend to Precision

17. Berneen Wick wants to offer a gift set containing the three candles shown. Remember: The cost per cubic inch of a candle is $0.10. Prepare a report for Berneen in which you provide her with:

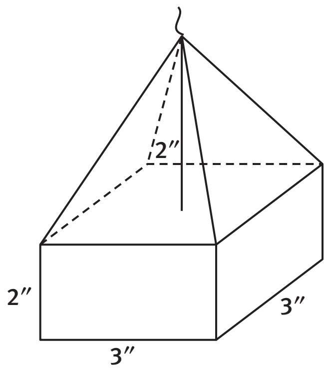
- a name and a cost for each candle and the method of calculating each cost
- your recommendation for the price of the gift set
- your reasons for the recommendation



Candle 1



Candle 2



Candle 3

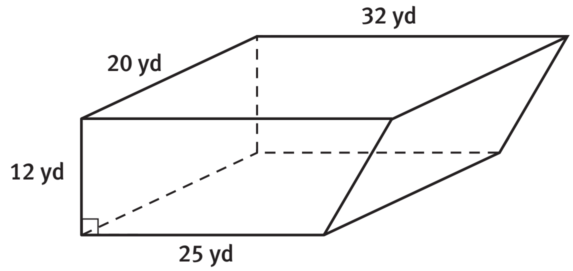
Surface Area and Volume
UNDER THE SEA

Embedded Assessment 3
Use after Activity 19

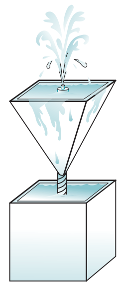
Mackeral “Mack” Finney is designing a new aquarium called Under the Sea. He plans to include several different types of saltwater tanks to house the aquatic life.

1. Mack begins by designing the smallest fish tank. This tank is a rectangular prism with dimensions 4 feet by 2 feet by 3 feet.
   a. Draw and label a net to represent the aquarium.
   b. The tank will have a glass covering on all six sides. Find the surface area of the tank. Explain your reasoning.
   c. Find the volume of the tank. Show your work.

2. Near the main entrance to the aquarium, Mack has decided to put a larger pool for four dolphins. Its shape is the trapezoidal prism shown.
   a. Sketch and label the dimensions of a cross section parallel to the bases of the prism.
   b. Find the amount of water needed to fill the pool. Explain your thinking.

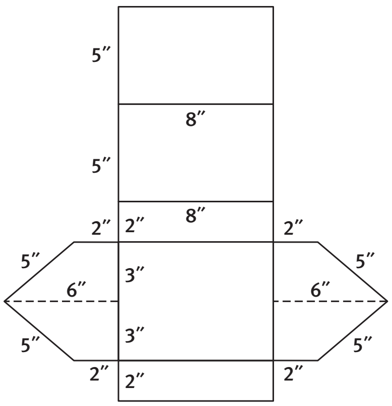


3. Mack designed a water fountain with a square pyramid flowing into a cube, as shown at right. The edges of the bases of the pyramid and the cube have the same length and the heights of the pyramid and the cube are the same. Describe the relationship between the volume of the cube and the volume of the pyramid.

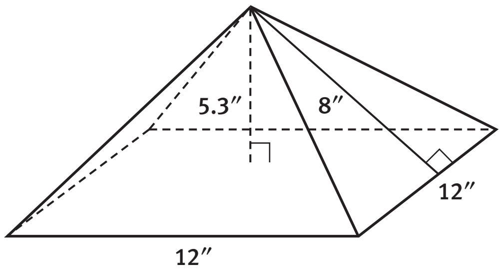


In addition to tanks for the aquatic life, Mack designs some hanging birdhouses for the trees around the aquarium.

4. The net for one birdhouse is shown below. What is the total surface area of the solid? Show your work.



5. Another birdhouse design is in the shape of a square pyramid, as shown below. Find the surface area and volume of the birdhouse.



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-c, 2b, 3, 4, 5)	• Accurately and efficiently finding the surface area and volume of prisms and pyramids.	• Finding the surface area and volume of prisms and pyramids.	• Difficulty finding the surface area and volume of prisms and pyramids.	• No understanding of finding the surface area and volume of prisms and pyramids.
Problem Solving (Items 1b-c, 2b, 4, 5)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-b, 2a, 4, 5)	• Clear and accurate understanding of how a net represents a three-dimensional figure.	• Relating a net to the surfaces of a three-dimensional figure.	• Difficulty recognizing how a net represents a three-dimensional figure.	• No understanding of how a net represents a three-dimensional figure.
Reasoning and Communication (Items 1b, 2b, 3)	• Precise use of appropriate terms to explain finding surface area and volume of solids. • A precise and accurate description of the relationship between the volume of a pyramid and a cube.	• An adequate explanation of finding surface area and volume of solids. • A basically correct description of the relationship between the volume of a pyramid and a cube.	• A partially correct explanation of finding surface area and volume of solids. • A partial description of the relationship between the volume of a pyramid and a cube.	• An incomplete or inaccurate explanation of finding surface area and volume of solids. • A partial description of the relationship between the volume of a pyramid and a cube.

Probability

5

Unit Overview

In this unit, you will begin your study of probability. You will learn how to interpret probabilities and how to calculate probabilities in a variety of settings. You will also learn several ways to estimate probabilities.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- predict
- simulation

Math Terms

- probability experiment
- probability
- equally likely outcomes
- selected at random
- event
- complement
- theoretical probability
- estimated probability
- sample space
- tree diagram
- random digits

ESSENTIAL QUESTIONS

? How is probability used to make decisions in everyday situations?

? How can a probability be estimated?

EMBEDDED ASSESSMENTS

These assessments, following activities 21 and 23, will give you an opportunity to demonstrate your understanding of probability and your ability to calculate and estimate probabilities.

UNIT 5

Getting Ready

Write your answers on notebook paper. Show your work.

1. Express each of the following as a fraction in lowest terms.

a. $\frac{3}{6}$

b. $\frac{2}{8}$

c. $\frac{12}{20}$

d. $\frac{5}{8}$

2. Express each fraction as a decimal. (Where applicable, round your answers to the nearest hundredth.)

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{1}{8}$

d. $\frac{5}{7}$

e. $\frac{5}{8}$

f. $\frac{12}{15}$

g. $\frac{22}{40}$

h. $\frac{37}{50}$

3. Express each decimal as a percentage.

a. 0.19

b. 0.47

c. 0.032

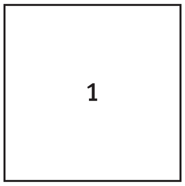
d. 0.05

e. 0.169

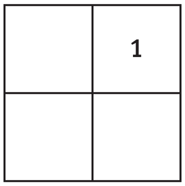
f. 0.95

4. What fraction of the area of the squares shown below corresponds to the section labeled 1?

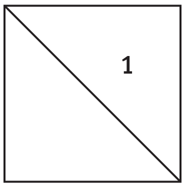
a.



b.



c.



Exploring Probability

Spinner Games

Lesson 20-1 Making Predictions

My Notes

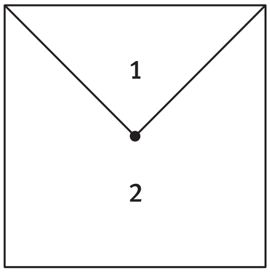
Learning Targets:

- Reason about the likelihood of winning a game based on a probability experiment.
- Provide support for winning strategies of a game based on a probability experiment.

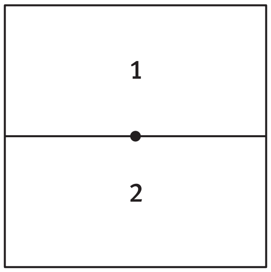
SUGGESTED LEARNING STRATEGIES: Mark the Text, Think-Pair-Share, Create Representations, Predict and Confirm, Look for a Pattern, Summarizing, Paraphrasing

Suppose that you and a partner play a game using the two spinners below.

Spinner 1



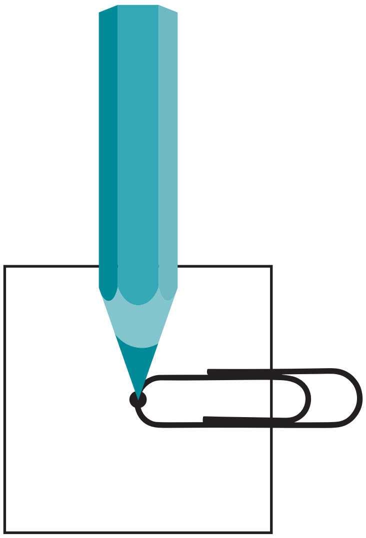
Spinner 2



Use a paper clip and a pencil to make the spinner work, by following these steps:

- Place the paper clip on the spinner so that the center dot is just inside the long loop of the paper clip.
- Put the pencil point on the dot.
- Flick the paper clip with your finger.
- After the paper clip has stopped spinning, look to see where the center of the outer loop of the paper clip falls and note the number for this section. If the center of the outer loop of the paper clip falls on a boundary line, spin again.

Try this a few times with each of Spinners 1 and 2.



My Notes

For this game, you and your partner will each spin one of the spinners. Whoever spins the greater number wins. If you both spin the same number, you each spin again until someone wins.

1. If you could decide which spinner you will use and which spinner your partner will use, how would you assign the spinners?

My spinner will be Spinner ________.

My partner's spinner will be Spinner ________.

2. Explain your reasoning in assigning the spinners. Why did you assign them this way?

3. If you play the game once using the spinner you chose in Item 1, are you sure to win?

4. Based on your answer to Item 3, do you want to change your spinner? Explain your reasoning.

ACADEMIC VOCABULARY

The word ***predict*** means to make a reasonable guess about something that will happen.

5. **Reason abstractly.** If you play the game 20 times using the spinner you chose in Item 1, which of the following would you ***predict***?

a. You would win more often than you would lose.
b. You would win and lose about the same percentage of the time.
c. You would lose more often than you would win.

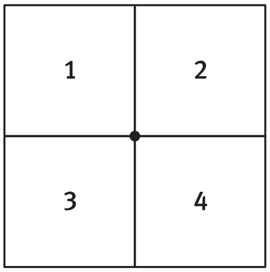
Explain why you made this prediction.

6. Suppose you play the game 20 times using the spinner you chose in Item 1, about how many times do you think you would win? Explain your answer.

My Notes

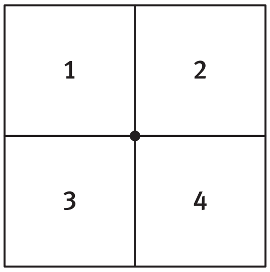
Check Your Understanding

7. You play a game in which two players each spin the spinner below. The player who lands on the greatest number wins. This game is an example of a *fair* game. Why do you think this is called a fair game?

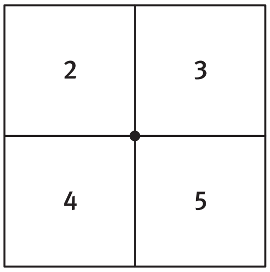


8. Construct viable arguments. You will play a game where two players each spin one of the two spinners below. The player who lands on the greatest number wins. Which spinner would you choose and why?

Spinner 1



Spinner 2

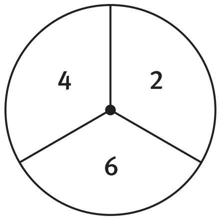


9. Describe a situation in which the person using Spinner 1 would win over the person using Spinner 2.

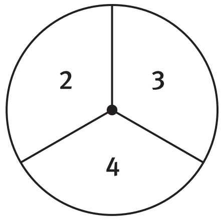
My Notes

LESSON 20-1 PRACTICE

Spinner A



Spinner B



You and a friend are about to play a game in which each player spins one of the spinners above. The player who lands on the greater number wins.

10. Which spinner would you choose and why?

11. List all of the possible outcomes for spinning a number on each spinner. List the outcomes as ordered pairs, where the first number is the result from Spinner A and the second is the result from Spinner B.

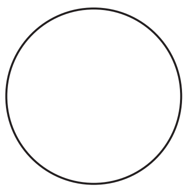
12. Play the game. Fill in the table below with W if Spinner A wins, T if the two spinners show the same number, and L if Spinner B wins.

		Spinner B Results		
		2	3	4
Spinner A Results	2			
	4			
	6			

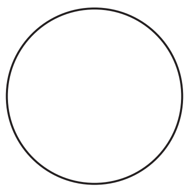
Do the results in the table support your answers to Items 10 and 11? Explain.

13. **Model with mathematics.** Create two spinners to play a game, using the circles below. If you use the same rules as those above, explain whether either spinner gives a player a winning advantage and why.

Spinner C



Spinner D



My Notes

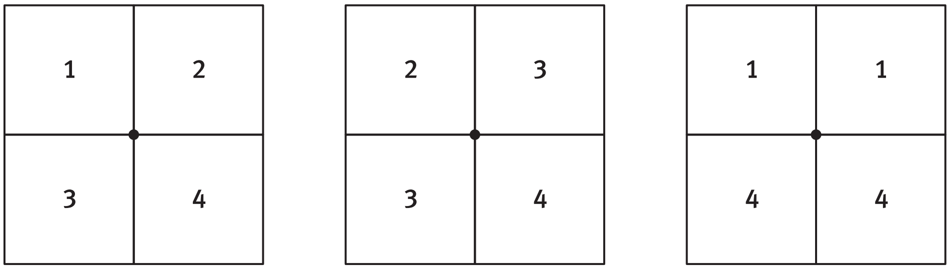
Learning Targets:

- Collect data about chance processes in frequency tables or lists.
- Determine probabilities for outcomes in a probability experiment.
- Describe the results of an investigation and support the conclusions.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Look for a Pattern, Interactive Word Wall

Suppose you and a partner plan to play a game using two of the three spinners shown below. Each person will spin his or her spinner, and the person that lands on the greater number will win.

Spinner 3 **Spinner 4** **Spinner 5**



1. Your partner picks a spinner first and picks Spinner 3. Which would you then choose, Spinner 4 or Spinner 5? Explain the reason for your choice.

2. Use appropriate tools strategically. If you could choose first, which of the three spinners would you choose, and why? As you justify your choice of spinner, use precise mathematical language to describe your reasoning.

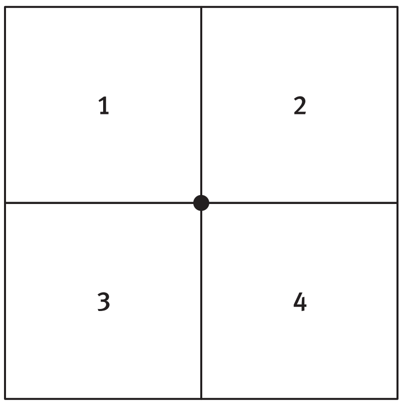
My Notes

3. Now let's do some spinning. Spin Spinner 6 below 20 times, and record the results in the table. Remember that if the paper clip lands on a line, you should just spin again.

Spin	Spin Result
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Spin	Spin Result
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Spinner 6



4. Count the number of spins that resulted in each number. Record the results in the table below.

Spin Result	Count
1	
2	
3	
4	

MATH TIP

To complete the table, count the number of 1's in the table above, and record the number under *Count*. Then do the same for the number of 2's, the number of 3's, and the number of 4's.

My Notes

5. Do you have exactly five 1's, five 2's, five 3's, and five 4's? Does your answer surprise you?

6. Compare your counts from Item 4 to the counts of another student in your class. Did you both get the same counts? Does this surprise you?

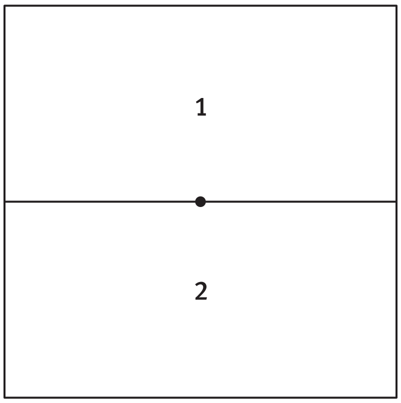
Spinning a spinner and observing the result is an example of a **probability experiment**. Even though you may be able to say what the possible outcomes are (such as 1, 2, 3, or 4 for Spinner 6), you can't be sure which outcome will actually occur on any spin.

MATH TERMS

A **probability experiment** is the process of observing an outcome when there is chance involved. This means that before you do the experiment, you can't be sure what the outcome will be.

7. Think about the probability experiment of spinning Spinner 7, shown below. What are the possible outcomes of this experiment?

Spinner 7



8. Even though you can't be certain what the outcome of any particular spin will be, if you were to spin Spinner 7 many times, about what percentage of the time do you think the outcome 1 would occur? Explain your reasoning.

My Notes

MATH TERMS

The **probability** of an outcome of a probability experiment is a number between 0 and 1 that tells you what fraction of the time you would expect the outcome to occur.

WRITING MATH

Probability Notation
The probability of an outcome is written in symbols as

P(outcome), where the *P* stands for probability.

For example, the probability that the outcome of a spin is 1 can be written as

P(spin results in a 1), or simply, as *P*(1).

9. The **probability** of spinning a 1 with Spinner 7 is $\frac{1}{2}$, or 0.5, when written as a decimal. How does this probability relate to your answer to Item 8?

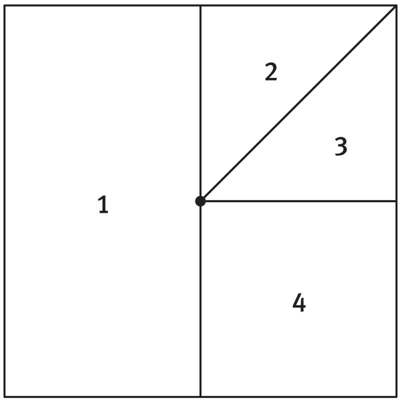
10. For Spinner 7, what is $P(2)$?

11. **Reason abstractly.** Since $P(1) = \frac{1}{2}$ for Spinner 7, if you spin this spinner many times you would expect $\frac{1}{2}$ or 50% of the spin results to be 1. Does this mean that if you spin ten times, you will get exactly five 1's? Explain why or why not.

12. Now think about the probability experiment of spinning Spinner 8 below. For Spinner 8, what are the following probabilities? Use clear and precise mathematical language to explain your reasoning. Remember to use complete sentences, including transitions and words such as *and*, *or*, *since*, *for example*, *therefore*, *because of*, to make connections between your thoughts.

$P(1) =$ $P(2) =$ $P(3) =$ $P(4) =$

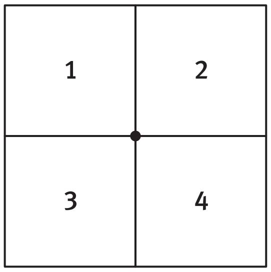
Spinner 8



My Notes

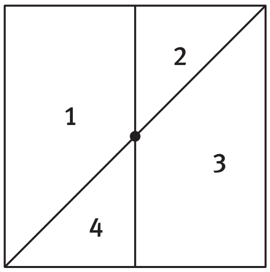
Check Your Understanding

13. You and your friend are playing a game with the spinner below. Here are the rules:
- The first player spins. Then the second player spins.
- The second player wins if he or she lands on the same number as the first player did. If not, the first player wins.



a. If your friend spins first and spins a 1, is it more likely that you will win? Why or why not?
b. Which is more likely in this game, landing on the same number as the first player or not landing on the same number?
c. Who has the better chance of winning the game?

14. Consider spinning the following spinner.

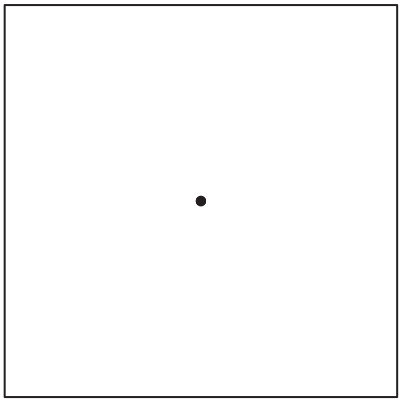


For this spinner, what are the following probabilities?
a. $P(1)$
b. $P(2)$
c. $P(3)$
d. $P(4)$

My Notes

15. Using the blank spinner, design a spinner for which

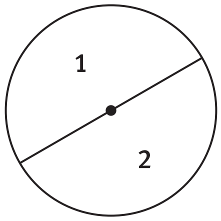
$P(1) = 25\%$
$P(2) = 12.5\%$
$P(3) = 12.5\%$
$P(4) = 25\%$
$P(5) = 25\%$



LESSON 20-2 PRACTICE

16. You and your friend are playing a game with the spinner below.

- The first player spins. Then the second player spins.
- The second player wins if he or she lands on the same number as the first player does. If not, the first player wins.



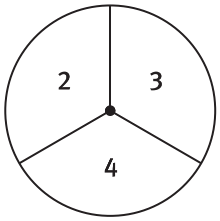
a. If your friend spins a 2, is it more likely that your spin will match your friend's spin, or is it more likely that your spin will not match your friend's spin?

b. When you play this game, is *a match* or *not a match* more likely?

c. **Make sense of problems.** Is this considered to be a fair game? Why or why not?

My Notes

17. You and your friend are playing a game with the spinner below.
- The first player spins. Then the second player spins.
- The second player wins if he or she lands on the same number as the first player does. If not, the first player wins.



a. List the possible outcomes using ordered pairs. The first number in the ordered pair is the number the first player spins.

b. Complete the table below. Fill in the ordered pairs for the spin results in the first column. In the second column, write *M* if the numbers match and *N* if the numbers do not match.

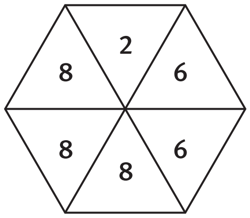
Ordered Pairs	M or N

c. **Construct viable arguments.** If you want to win this game, should you play so that the spins will match or so that the spins will not match? How do you know?

d. Is this a fair game? Why or why not?

My Notes

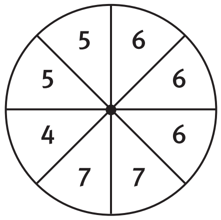
18. Consider the following spinner.



For this spinner, what are the following probabilities?

$P(2) =$
$P(5) =$
$P(6) =$
$P(8) =$

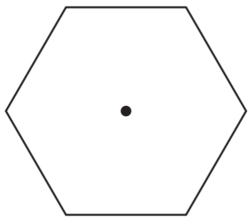
19. Consider the following spinner.



For this spinner, what are the following probabilities?

$P(4) =$
$P(5) =$
$P(6) =$
$P(7) =$
$P(\text{even number}) =$

20. Using the blank spinner, design a spinner for which:



$P(1) = \frac{1}{6}$

$P(3) = \frac{1}{3}$

$P(7) = \frac{1}{12}$

$P(\text{odd number}) = 1$

My Notes

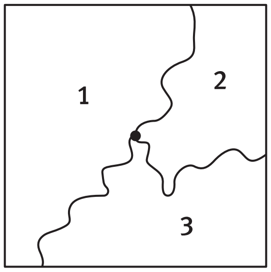
Learning Targets:

- Interpret a probability as the fraction of the number of times that an outcome occurs when a probability experiment is repeated many times.
- Estimate probabilities of outcomes in probability experiments.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Predict and Confirm, Look for a Pattern

1. Below is a "crazy" spinner. What can you tell about $P(1)$ just by looking at the spinner? Explain.

Spinner 9



2. Do you think that $P(1)$ is closer to 0, 0.25, 0.50, 0.75, or 1.0? Explain your answer.

3. Make sense of problems. Even though you don't know the exact value of $P(1)$ for crazy Spinner 9, can you think of a way to estimate this probability?

CONNECT TO GEOMETRY

In geometry, you learned to find the area of different shapes. You can use areas to determine probability. The greater the area you want, the greater the chance (or probability) of landing in that area. Think about how the area you want compares to the entire area on the spinner.

My Notes

4. Spin crazy Spinner 9 twenty times and record the results in the table below.

Spin	Spin Result
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Spin	Spin Result
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

5. How many of your 20 spins resulted in a 1? What fraction of the spins resulted in a 1?

6. If you had to estimate $P(1)$ for crazy Spinner 9 based on your twenty spins, what would your estimate be?

Estimate of $P(1) =$

7. Combine your spin results with those of another student in your class so that you now have outcomes from 40 spins. Based on the 40 spins, estimate $P(1)$ for crazy Spinner 9.

Estimate of $P(1) =$

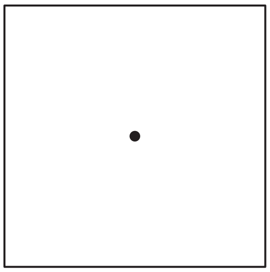
My Notes

8. Which of the two probability estimates do you think is a better estimate of $P(1)$ for crazy Spinner 9, the one from Item 6 or the one from Item 7? Explain your choice.

9. How could you get an even better estimate of this probability?

Check Your Understanding

10. Using the blank spinner below, design a “crazy” spinner with four sections. Label the four sections 2, 4, 6 and 8. (By a “crazy” spinner, we mean one for which the probabilities of the different possible spinner results are not obvious from looking at the spinner.)



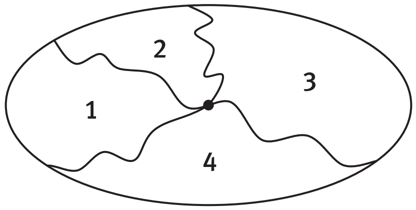
11. Spin your spinner 25 times, and use the results to estimate the following probabilities:

Estimate of P(spin result is 2)
Estimate of P(spin result is 8)

My Notes

LESSON 20-3 PRACTICE

12. Consider the crazy spinner below.



What can you tell about *P*(1) and *P*(2) just by looking at the spinner? Explain your reasoning.

13. Should a reasonable estimate of the value of *P*(4) be closest to 0, 0.25, 0.50, 0.75, or 1. Why do you think this?

14. Suppose Jamie spins the crazy spinner from Item 12 sixteen times and records the results as shown in the table below. How many of the spins resulted in a 4? What fraction of the spins resulted in a 4?

Spin	Spin Result
1	1
2	2
3	4
4	1
5	1
6	3
7	2
8	3

Spin	Spin Result
9	3
10	3
11	1
12	2
13	3
14	1
15	3
16	4

15. Suppose Ibdi had seven 4's in 16 spins and Dar had six 4's in 16 spins. Estimate *P*(4) based on the combined results of Jamie, Ibdi, and Dar.

16. What do you feel is the best estimate for *P*(4)? How could you produce a better estimate of this probability?

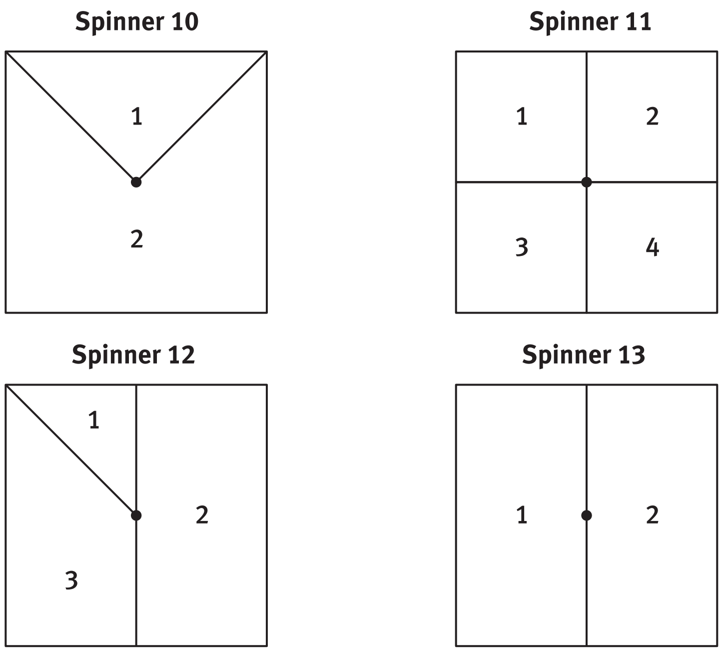
My Notes

Learning Targets:

- Make decisions based on probabilities.
- Expect variation in results from chance processes.
- Write about chance processes and justify conclusions based on probability experiments.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Predict and Confirm, Look for a Pattern

Consider the four spinners below.



1. Order the four spinners starting with the one that has the greatest probability of spinning 1, and ending with the one that has the least probability of spinning 1.

Greatest probability Spinner ______
↓ Spinner ______
↓ Spinner ______
Least probability Spinner ______

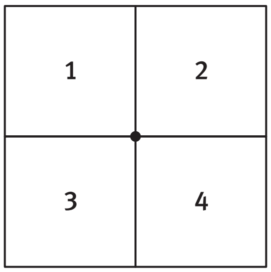
My Notes

2. Suppose you spin Spinner 12 twenty times and Spinner 13 twenty times. Would it surprise you if you got more 1's with Spinner 12 than with Spinner 13? Explain your thinking.

3. Suppose you spin Spinner 10 twenty times and Spinner 11 twenty times. Would it surprise you if you got more 1's with Spinner 10 than with Spinner 11? Explain your thinking.

4. Now consider Spinner 14 shown below.

Spinner 14



Only one of the following sequences resulted from actually spinning Spinner 14 twenty times. Which sequence do you think this was?

Sequence 1: 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
Sequence 2: 2 3 1 3 1 1 3 2 2 3 1 1 3 3 3 2 3 1 3 3
Sequence 3: 1 1 1 3 2 4 3 4 4 3 3 2 4 2 1 4 3 1 4 3

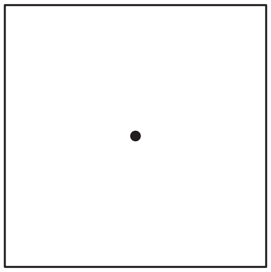
5. For the two choices you did not select in Item 4, explain why you ruled out that choice.

My Notes

6. The following were the outcomes from 20 spins of a spinner.

1 1 2 4 1 4 1 3 2 1 4 4 1 2 1 4 3 1 4 1

Using the blank spinner below, design a spinner that you think might have generated this sequence of outcomes. Explain your reasoning in designing this spinner.



My Notes

MATH TIP

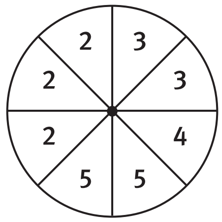
It is often easier to compare decimals than to compare fractions. You can write probabilities in decimal form to compare them.

For example, if the probability of spinning 3 is $\frac{3}{8}$ on one spinner and $\frac{2}{5}$ on another spinner, you can compare the probabilities in decimal form. Since $\frac{3}{8} = 0.375$ and $\frac{2}{5} = 0.4$, compare 0.4 and 0.375 to see that $\frac{2}{5}$ is greater.

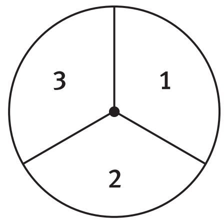
Check Your Understanding

Consider the following spinners.

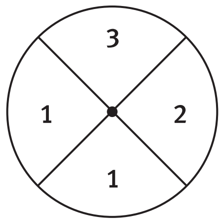
Spinner A



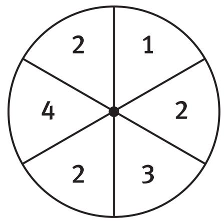
Spinner B



Spinner C



Spinner D



7. Order these four spinners starting with the one that has the greatest probability of spinning 2 and ending with the one that has the least probability of spinning 2.

______ ______ ______ ______

Greatest probability → → → Least probability

8. Order these four spinners starting with the one that has the greatest probability of spinning 1 and ending with the one that has the least probability of spinning 1.

______ ______ ______ ______

Greatest probability → → → Least probability

9. Suppose you spin Spinner B twenty times and Spinner C twenty times. Would it surprise you if you got more 2's with Spinner C than with Spinner B? Explain your thinking.

10. Make sense of problems. One of the following sequences resulted from spinning Spinner D twenty times. Which sequence do you think that this was? Explain your reasoning.

Sequence 1: 2 2 2 3 1 4 3 4 4 3 3 1 4 1 2 4 3 2 4 3
Sequence 2: 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
Sequence 3: 1 2 2 4 2 4 2 1 1 3 2 2 3 3 3 1 4 2 3 3

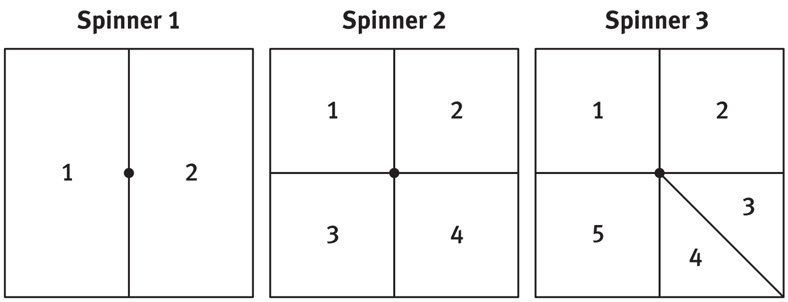
11. The following numbers represent the outcomes for 20 spins of a spinner. On a separate piece of paper, design a spinner that you think might have generated this sequence of outcomes. Explain your reasoning in designing the spinner.

1 2 3 1 4 1 2 1 2 1 1 4 4 1 1 1 1 3 1 2

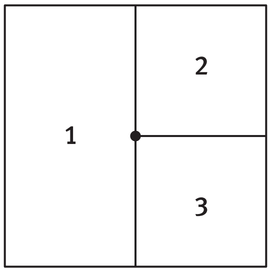
My Notes

LESSON 20-4 PRACTICE

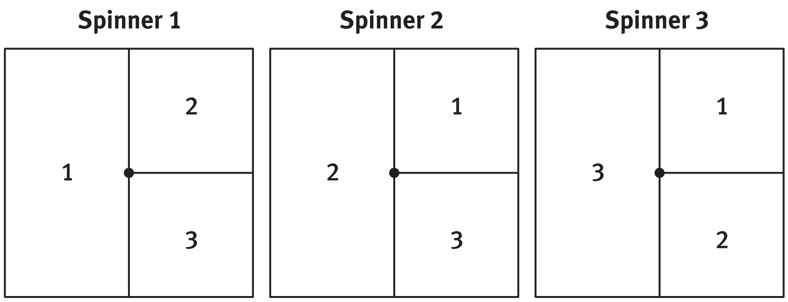
12. For which of the following three spinners is $P(4)$ the greatest?



13. The spinner below is one of the spinners used in a two-spinner game.



If the greater number wins, which of the following spinners is most likely to win when playing against the spinner above?



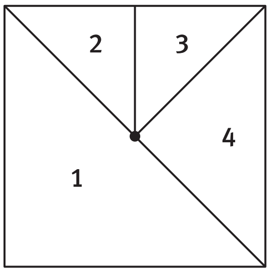
14. What are the following probabilities for the spinner shown below?

a. $P(1)$

b. $P(2)$

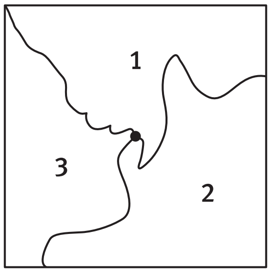
c. $P(3)$

d. $P(4)$



My Notes

15. Estimate $P(1)$ for the spinner below. Explain how you arrived at your estimate.



16. Only one of the following sequences resulted from actually spinning the spinner below twenty times. Which sequence do you think this was?

Sequence 1: 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4

Sequence 2: 2 4 3 1 4 3 4 2 2 3 1 3 2 2 3 2 2 4 1 1

Sequence 3: 1 1 1 2 4 1 3 1 1 1 4 4 3 1 2 1 1 4 1 4

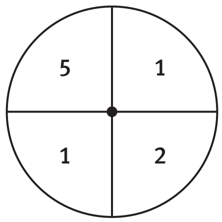
17. **Reason abstractly and quantitatively.** For each choice you did not select in Item 12, explain why you ruled out that choice.

ACTIVITY 20 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 20-1

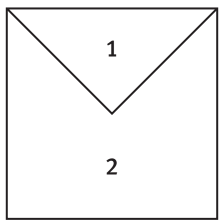
1. You plan to play "The Sum Game" in which two players each spin the spinner below one time. Player One wins if the sum of the two numbers is even. Player Two wins if the sum of the two numbers is odd. Is this game an example of a "fair" game? Explain your reasoning.



2. Consider "The Sum Game" in Item 1.
   a. Will spinning first give a player an advantage in winning? Explain.
   b. If "The Sum Game" is played twice, will each player win exactly once? Explain.
   c. If "The Sum Game" is played 100 times, how many wins would you predict for Player One and Player Two? Explain why you think this.

Lesson 20-2

3. Consider the spinner shown below. Each of the following two statements is *incorrect*. For each statement, explain why the statement is incorrect.



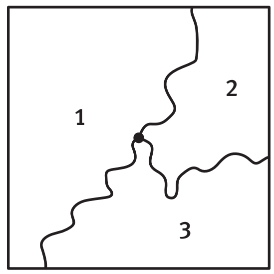
Incorrect Statement 1: If we spin this spinner many times, we would expect the result to be 1 about half the time and 2 about half the time.

Incorrect Statement 2: If we spin this spinner 20 times, we will see five 1's and fifteen 2's.

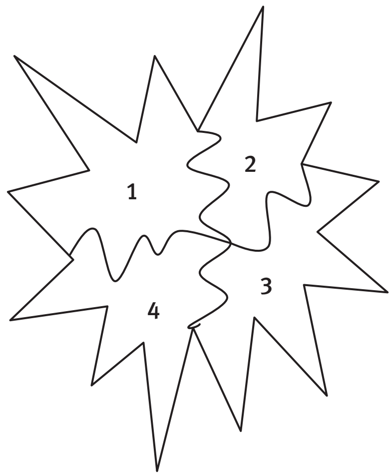
Lesson 20-3

4. Below is a "crazy" spinner.

   Even though you cannot tell the exact value of $P(2)$, do you think that this probability is closest to 0, 0.25, 0.5, 0.75 or 1? Explain your reasoning.



5. Consider Spinner B below.

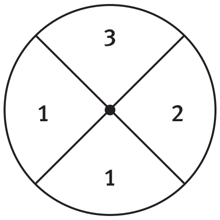


   a. Spin Spinner B twenty times and record the results in a frequency table.
   b. Estimate $P(2)$ for Spinner B. Explain your reasoning.
   c. How could you improve your estimate for $P(2)$?

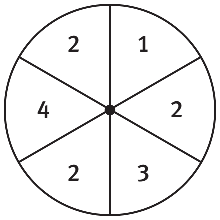
Lesson 20-4

6. Consider the following spinners.

Spinner C



Spinner D



You plan to play the spinner game where each player chooses a spinner and the player who spins the greater number wins.

a. Which spinner would you choose to play? Explain why.

b. Play the spinner game 25 times. Keep track of how many times Spinners C and D win and how many ties occur.

c. Should you change your spinner based on these results? Explain.

d. Give the possible outcomes for these two spinners as ordered pairs with the results from Spinner C given first. (Spinner C, Spinner D)

e. Find *P*(Spinner C) and *P*(Spinner D). Explain how you know.

f. Is this a "fair game?" Justify your answer.

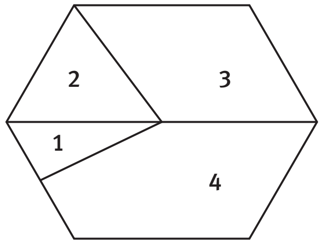
g. Which of the three lists below is most likely to represent 27 results of the game for Spinner C versus Spinner D? Justify your choice. (W means that C wins. L means that D wins. T means a tie between C and D.)

List 1: T W L W L W W T W L T W W W T W W L L T W W W W L W W

List 2: T W W W W L L L L T W W W W L L L L T W W W W L L L L

List 3: W W W L L L T W W W L L L T W W W L L L T W W W L L L

7. Consider Spinner E below.



The following numbers represent the results for fifty spins of Spinner E.

4 3 3 2 3   2 1 2 3 1   4 4 4 3 2   1 3 4 1 1

1 3 4 3 2   3 1 4 3 4   4 4 4 2 2   3 3 3 1 4

2 3 1 2 4   4 4 4 4 4

a. What fraction of the time would you expect each of the different results to occur?

b. Create a frequency table for the results of the fifty spins of Spinner E.

c. Calculate the probability for each of the different results based on frequency table.

d. Which result has the greater likelihood when using Spinner E, getting a 1 or getting a 2? Explain your reasoning.

MATHEMATICAL PRACTICES
Construct Viable Arguments

8. Consider the following:

A spinner has the probability $P(3) = \frac{2}{3}$. If this spinner is spun 30 times, 20 of these spins will definitely result in a 3.

Is this correct? Explain why or why not.

Probability

Probability Two Ways

Lesson 21-1 Equally Likely Outcomes

ACTIVITY 21

Learning Targets:

- Recognize when a probability experiment has outcomes that are equally likely.
- Calculate probabilities for a probability experiment with equally likely outcomes.
- Know what “selected at random” means.

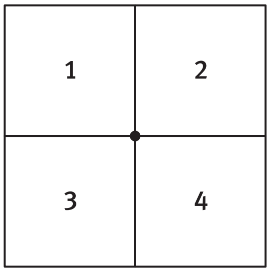
SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

In the previous activity, you learned about probability experiments in the context of spinner games. In this activity, you will see other examples of probability experiments.

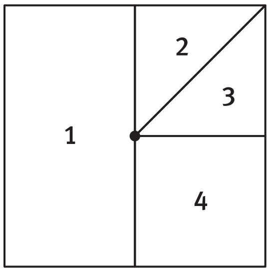
Some probability experiments have outcomes that are **equally likely**.

1. Each of the following spinners has four possible outcomes. Which of these spinners has equally likely outcomes?

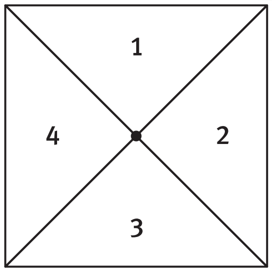
Spinner 1



Spinner 2



Spinner 3



2. What are the different possible outcomes for Spinner 1?

3. For Spinner 3, what are the following probabilities?

$P(1) =$

$P(2) =$

$P(3) =$

$P(4) =$

My Notes

MATH TERMS

A probability experiment has **equally likely outcomes** if every different outcome has the same chance of occurring.

My Notes

4. What do you notice about the probabilities for the possible outcomes for Spinner 3? How is this probability related to the number of possible outcomes?

5. If a probability experiment has 7 different equally likely outcomes, what is the probability of each outcome?

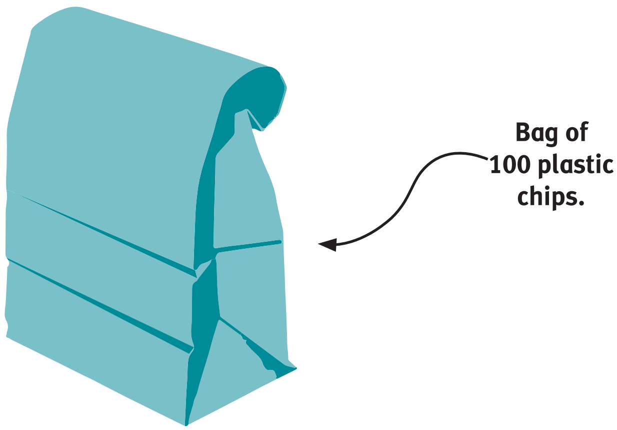
6. Suppose that a probability experiment has k different equally likely outcomes. Write an expression for the probability of each outcome.

One type of probability experiment that has equally likely outcomes is **selecting at random** from a group of objects or people. For example, suppose that a brown paper bag holds 100 plastic chips and that each chip has a different number on it. The chips are numbered from 1 to 100.

MATH TERMS

Selected at random means choosing in a way that gives each member of the group the same chance of being chosen.

For example, if you shake a bag containing 100 chips to mix up all of the chips and then put your hand in the bag and pull out a chip without looking in the bag, that would be one way of selecting a chip at random from the bag.



Suppose that you can win points in a game based on the number on the chip when you select a chip at random from the bag.

7. In the probability experiment that consists of selecting a chip at random from the bag of 100 numbered chips, what are the different possible outcomes for the number you might get? Are these outcomes equally likely?

8. What is the probability of each of the different outcomes?

My Notes

9. What is the probability that you get the chip numbered 61 when you select a chip at random from the bag?

10. Suppose that you can earn points if you get an even number when you select a chip at random from the bag of 100 chips. What do you think the probability of getting an even number is? Explain how you arrived at this probability.

When all of the outcomes are equally likely, the probability of an event *A* is

$$P(A) = \frac{\text{number of outcomes in the event}}{\text{number of possible outcomes}}$$

Use this result to answer the following items.

11. Express regularity in repeated reasoning. If the outcomes of a probability experiment are equally likely, it is easy to calculate the probability of an **event**, such as the event that you select an even number. For the probability experiment of selecting a chip at random from the bag of 100 numbered chips, calculate the following probabilities:

a. The probability that the number selected is less than 10.

b. The probability that the number selected is greater than 75.

c. The probability that the number selected is equal to your age.

d. The probability that the number selected is **not** equal to your age.

e. The probability that the number selected is 75 or less.

MATH TERMS

An **event** in math is a collection of outcomes in a probability experiment.

MATH TIP

For a probability experiment with equally likely outcomes, the probability of an event *E*, written *P*(*E*), is calculated by dividing the number of outcomes in *E* by the total number of possible outcomes.

Only use this method to calculate a probability if the outcomes are equally likely!

My Notes

MATH TERMS

The **complement** of an event *A* represents all the outcomes from a probability experiment not included in the outcomes of the event *A*. The *complement of event A* is written as *A′*.

Consider each pair of events below.

The number selected is equal to your age and the number selected is **not** equal to your age.

The number selected is greater than 75 and the number is 75 or less.

These paired events are **complements** of one another since when considered together they represent every possible outcome of the probability experiment.

12. Give the complement of each event and find its probability.

a. The number selected is an even number.

b. The number selected is less than 10.

Suppose that you can earn game points that can be redeemed for prizes if any of the following events occur when you select a chip at random.

Event	Description
E	The selected number is even.
T	The selected number is less than 10.
S	The selected number is greater than 75.
A	The selected number is equal to your age.

The number of points earned might be 10, 20, 30 or 40.

13. Do you think the greatest number of points should be assigned to the event with the greatest probability or the event with the least probability? Explain why you think this.

My Notes

14. If 10 points are assigned to the event with the greatest probability, 20 points to the event with the next greatest probability and so on, how many points would be assigned to each of the four events?

Event	Description	Points Assigned
E	The selected number is even.	
T	The selected number is less than 10.	
S	The selected number is greater than 75.	
A	The selected number is equal to your age.	

15. Suppose you will select one chip at random.

a. What is the probability that you will win 10 points?

b. **Make sense of problems.** If two of the events happen at the same time, then you get the points assigned to *both* events. For example, if you pick 76, then S and E both happen, and so you get 30 points. What is the probability that you will win 40 points? (*Hint:* There is more than one way to win 40 points.)

c. List all the outcomes that would result in you winning 30 points. What is the probability that you will win 30 points?

d. Is there a way for someone in your class to win 50 points? If so, how could this happen? If not, explain why this is not possible.

My Notes

Check Your Understanding

16. If a probability experiment has 12 equally likely outcomes, what is the probability of each outcome?

17. A probability experiment will consist of selecting one student at random from your math class. Is the probability that the selected student is female greater than, equal to, or less than the probability that the selected student is male? Explain your reasoning.

18. A brown paper bag contains 50 plastic chips. Each chip has a different number and the chips are numbered from 1 to 50.

a. For a probability experiment where one chip is selected at random from the bag, what is the probability of each of the following events?

Event	Description
B	The selected number is between 20 and 35 (not including 20 and not including 35).
L	The selected number is less than your age on your last birthday.
T	The selected number is a multiple of 3.

$P(B) =$
$P(L) =$
$P(T) =$

b. Write a description of each of the complementary events.

B'
L'
T'

c. Find the probability for each of the complementary events.

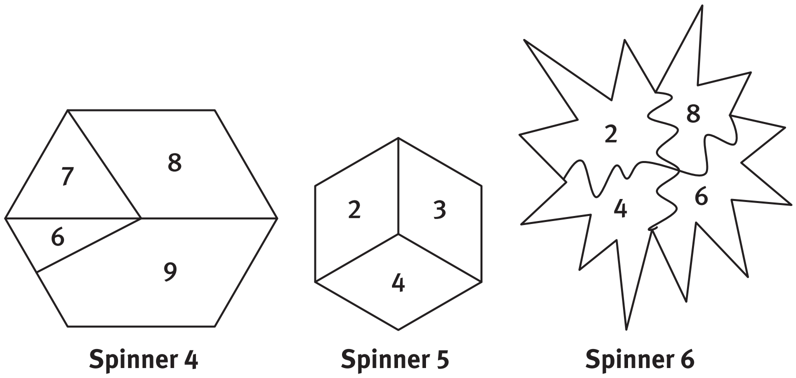
$P(B') =$
$P(L') =$
$P(T') =$

d. Write a statement about the relationship between the probability of an event and the probability of the complement of the event. Give examples to support your statement.

My Notes

LESSON 21-1 PRACTICE

19. Consider the spinners below.



a. Which spinner has equally likely outcomes?

b. If a spinner has six equally likely outcomes, what is the probability of each outcome?

Consider a bag of white marbles, each with a number from 1 to 80 printed on it. After selecting a marble from the bag and replacing it, the bag is shaken to mix up the marbles so that each time a marble is pulled from the bag it is selected at random.

20. Suppose that you win points in a game based on the number of the marble that is chosen.

a. What are the possible outcomes for the number you might get?

b. Are the outcomes equally likely? If so, why is this important? If not, is there a way to estimate the probabilities?

c. What is the probability of each of these different outcomes?

d. What is the probability that a number greater than 60 will be selected?

21. Suppose that you earn points based on the ones digit that appears on the marble chosen at random.

a. What are the possible outcomes for this probability experiment?

b. Are the outcomes equally likely? If so, what is the probability of each outcome? If not, which outcomes are most likely to occur?

My Notes

22. Consider the following events.

Event	Description
O	The selected number is odd.
S	The selected number is more than seventy.
D	The selected number is equal to today's date.
T	The selected number is less than twenty.

a. If the number of points to be earned for each event is 10, 20, 30, 40, with the least points being assigned to the event with the greatest probability, then how should the points be assigned?

O ______ S ______ D ______ T ______

b. What is the probability that you will win 20 points?

23. Suppose a game is to be played with the marble bag where points are awarded for the following events.

a. If two of the events happen at the same time, then you get the points assigned to both events. What is the probability that you win 40 points?

b. List the possible outcomes for earning 25 points.

c. What is the greatest number of points that can be earned for the selection of a single number and what selection would allow that to happen?

d. State the complement of events *O* and *S*.

e. Find the $P(D')$ and $P(T')$.

My Notes

Learning Targets:

- Calculate theoretical probabilities for a probability experiment.
- Estimate probabilities by observing outcomes of a probability experiment.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Now think about a different probability experiment. Suppose that a brown paper bag contains 40 chocolate candies that are all the same size and shape. Thirty of these candies are milk chocolate and 10 are dark chocolate.

1. Explain what it means when we say that a candy will be selected from the bag at random.

2. For this probability experiment, what is the probability of the event M, where M is the event that a milk chocolate candy is selected? How did you calculate this probability?

This probability is an example of a **theoretical probability**.

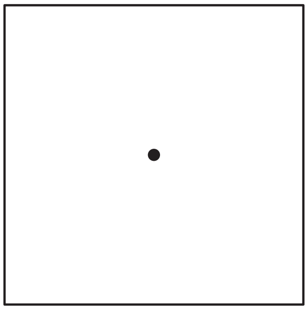
3. If we are just interested in whether the selected candy is milk chocolate or dark chocolate, are the outcomes milk chocolate and dark chocolate equally likely?

MATH TERMS

A **theoretical probability** is one that you can calculate from the description of a probability experiment.

My Notes

4. Use the blank spinner below to design a spinner with two regions that could represent the probability experiment of selecting a candy at random from the bag and noting if it is milk chocolate or dark chocolate. Label the two regions *Milk* and *Dark*.



5. With a partner, discuss how spinning this spinner is like selecting a candy at random from the bag and noting whether the candy is milk chocolate or dark chocolate. Then write a few sentences explaining how spinning is like selecting from the bag.

6. Using a paper clip, as you did in the previous activity, record the outcomes (milk or dark) for 20 spins of the spinner. This is like making 20 selections from the bag.

Spin outcomes:

7. If someone didn't know what the spinner looks like, but saw your outcomes from Item 6, what would that person estimate $P(M)$ to be? How does this **estimated probability** compare to the actual (theoretical) probability calculated in Item 2?

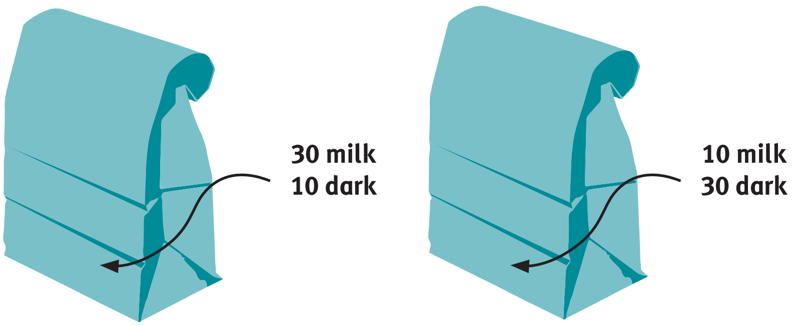
8. Combine your spin results with those of another student in your class so that you now have outcomes from 40 random selections. What is the estimated value of $P(M)$ for these combined results?

Total number of milk chocolate out of 40:

Estimated $P(M) =$

Is this estimated probability closer to the actual probability of 0.75 than your earlier estimate?

9. Suppose there are two paper bags and that each bag has 40 chocolate candies. One bag has 30 milk chocolate candies and 10 dark chocolate candies, and the other bag has 10 milk chocolate candies and 30 dark chocolate candies.



Someone selects one of the bags above at random and gives it to you, but you don't know which bag you have been given.

Bag?

My Notes

MATH TERMS

An ***estimated probability*** is one that is calculated by observing the outcome of a probability experiment many times.

Estimated probabilities are sometimes also called empirical probabilities.

My Notes

You select one candy at random from this bag, and it is a milk chocolate candy.

a. If you had to guess if this bag is Bag 1 or Bag 2, what would you say? Why did you pick the bag you did?

b. How likely do you think it is that you picked the correct bag?

1. There is about a 50-50 chance.
2. There is better than a 50-50 chance.
3. It is certain that I picked the correct bag.

c. Explain your reasoning.

d. Discuss your reponses to parts a-c in pairs or small groups. As you share ideas in your group, ask group members or your teacher for clarification of any language, terms, or concepts you do not understand.

10. Suppose that you put the selected candy back in the bag, mix up the candies in the bag, and select a candy at random. This candy is milk chocolate. How would you answer the two items above (part a and part b from Item 9) in light of this additional information?

My Notes

Check Your Understanding

A brown paper bag contains 100 plastic chips. Of these chips, 30 are red, 50 are green, and 20 are blue. A chip will be selected at random from this bag. Use this information to answer Items 11–14 below.

11. Calculate the probability that the selected chip is green.

Is the probability that you just calculated a theoretical probability or an estimated probability?

12. A friend reasons that because there are 3 different colors of chips in the bag, the probability of selecting a blue chip is $\frac{1}{3}$. Explain why this is incorrect.

13. Make sense of problems. Suppose you can add blue chips to the bag. If you wanted the probability of selecting a blue chip to be $\frac{1}{3}$, how many blue chips should you add to the bag?

14. Once you have added enough blue chips to the bag so that the probability of selecting a blue chip is $\frac{1}{3}$, explain why the three events described below are still not equally likely.

Event	Description
R	The selected chip is red.
G	The selected chip is green.
B	The selected chip is blue.

My Notes

LESSON 21-2 PRACTICE

A paper bag contains 100 chips. Some of these chips are labeled Win and the others are labeled Lose. You will select a chip at random from this bag and note whether the chip says win or lose. Then, put that chip back in the bag, mix up the chips, and select at random from the bag again. Repeat 30 times.

15. Suppose the results of these 30 selections are shown in the table below, in which W represents Win and L represents Lose.

L	L	W	L	W	W	L	L	L	L
W	L	L	L	L	W	L	W	W	L
L	L	W	L	L	L	L	L	L	L

a. Use the 30 observed outcomes to estimate the probability of selecting a chip that says Win. (Write your answer as a decimal, rounded to two decimal places.)

b. Is the probability you just calculated a theoretical probability or an estimated probability?

c. The actual number of Win chips in the bag is a multiple of 5. How many Win chips do you think are in the bag? Explain how you arrived at your answer.

16. Design and draw a spinner that has two regions labeled Win and Lose. Create the spinner to represent the probability experiment of selecting a chip at random from the bag and noting if it says Win or Lose.

17. Use the spinner you drew to generate 30 outcomes. Record them and produce totals.

18. Calculate $P(\text{W})$ based on your outcomes. Is the probability that you calculated an estimated or theoretical probability?

19. **Construct viable arguments.** Do you think that the spinner generates results similar to selecting chips at random from the bag? Provide reasoning and evidence to support your answer.

My Notes

Learning Targets:

- Compare theoretical probabilities and estimated probabilities.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing

The students in Ms. Bailey's seventh grade class completed a survey with these five questions:

a. Are you male or female?
b. Are you right-handed or left-handed?
c. About how many text messages did you send yesterday?
d. What is your favorite subject in school?
e. If you could have one super power, which of the following would you choose?

Invisibility   Super strength   Able to read minds
Able to fly   Able to freeze time

Review the survey questions to ensure that you understand their meaning. A summary of the survey results is on page 268. Remove that page and use it to answer the following items.

1. In the probability experiment that consists of selecting a student at random from Ms. Bailey's class, how many possible outcomes are there?

2. Are the possible outcomes equally likely? Explain why or why not.

3. If a student is selected at random from this class, what is the probability that the student selected is female? How did you calculate this probability?

4. Is the probability you calculated in Item 3 a theoretical probability or an estimated probability?

My Notes

WRITING MATH

You could use the notation of events and probability to write the probability notation for events in these questions.

For example,

If *F* is the event that the selected student is female in Item 3, you would write *P*(*F*) for the probability.

If *S* is the event that the selected student's favorite subject is science, then Item 5 asks you to calculate *P*(*F* and *S*).

5. If a student is selected at random from this class, what is the probability that the student selected is female and a student whose favorite subject is science? How did you calculate this probability?

6. Is the probability that the selected student is a female whose favorite subject is science less than, equal to, or greater than the probability that the selected student is female? Explain why this makes sense.

7. If a student is selected at random from this class, what is the probability that the student selected sent more than 20 text messages yesterday? How did you calculate this probability?

8. If a student is selected at random from this class, what is the probability that the student selected is a right-handed male? How did you calculate this probability?

9. Answer this question without calculating the probability that the randomly chosen student is male: Which is greater, the probability that the randomly chosen student is male, or the probability that the randomly chosen student is a right-handed male? Explain your reasoning.

10. **Make sense of problems.** Write Item 7 using events and probability notation.

Check Your Understanding

11. Write three probability questions for the probability experiment of selecting a student at random from Ms. Bailey's class that could be answered using the survey data provided.

Question 1:

Question 2:

Question 3:

12. Trade these probability questions with another student in your class. Answer the questions that the other student wrote while that person answers your questions. Then, working together, check each other's work to make sure the probabilities calculated are correct.

13. Considering the results for the thirty-four students on the survey and what you have learned about calculating probabilities, predict (without calculating) how the following probabilities will compare using $<, >, =$.

a. Compare *P*(Male and Flying) with *P*(Flying)

b. Compare *P*(English) with *P*(English and less than 20 texts)

c. Compare *P*(Male and Left) with *P*(Male and Left and Math)

d. Compare *P*(Female and Right) with *P*(Right and Female)

14. Express regularity in repeated reasoning. Calculate the probabilities for the comparisons in the previous item to confirm your predictions. Write a conclusion based on patterns you have noticed that would help you when making future comparisons of this type.

LESSON 21-3 PRACTICE

Recall the probability experiment of selecting a student at random from Ms. Bailey's seventh grade class. Some events are defined below.

Event	Description
M	Student selected is male
R	Student selected is right-handed
S	Student selected chose science as favorite subject
T	Student selected sent more than thirty text messages
F	Student selected chose freeze time as super power

Use the survey data to find the following probabilities.

15. $P(R) =$

16. $P(S) =$

17. $P(T) =$

18. $P(F) =$

19. $P(S') =$

20. $P(T') =$

21. $P(F') =$

22. $P(M \text{ and } S) =$

23. $P(R \text{ and } S) =$

24. $P(M \text{ and } T) =$

25. $P(S \text{ and } T) =$

26. $P(R \text{ and } F) =$

27. $P(T \text{ and } F) =$

ACTIVITY 21 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 21-1

1. Suppose that a person has a bag of marbles. Explain what you could conclude about what is in the bag for each of the following statements.
   a. The probability of selecting a green marble when a marble is selected at random is 1.
   b. The probability of selecting a green marble when a marble is selected at random is 0.

Use the information below to answer Items 2–4.

A box contains 50 balls that are numbered from 1 to 50 and each ball has a different number. The balls numbered 1 to 22 are red. The balls numbered 23 to 40 are white. The balls numbered 41 to 50 are blue and white striped. One ball will be selected at random from this box.

2. For the probability experiment of selecting a ball at random from this box, how many different outcomes are there?
3. Are the outcomes equally likely? How do you know this?
4. Find the following probabilities:
   a. *P*(striped)
   b. *P*(a number greater than 27)
   c. *P*(red and has an even number)
   d. Are the probabilities you calculated theoretical probabilities or estimated probabilities?

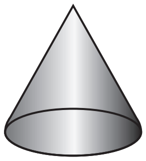
5. From 1999 to 2008, the United States Mint produced commemorative quarters for each state in the United States. Each year, quarters for five different states were circulated to the public, and many people collected the coins. Steven wanted to collect these coins, so his uncle gave him a handful of the state quarters. Steven made this list of the coins that he was given.

State	Year
Tennessee	2002
Tennessee	2002
Ohio	2002
Alabama	2003
Maine	2003
Florida	2004
Florida	2004
Florida	2004
Texas	2004
Texas	2004
California	2005
California	2005
Kansas	2005

Steven will put all the coins in his pocket and will pull one quarter out at random. Find the following probabilities.
   a. The probability that he selects a Texas quarter
   b. The probability that he selects a quarter minted in 2002
   c. The probability that he selects a quarter that has a state name that begins with a letter that is in the first half of the alphabet

Lesson 21-2

6. Sam tossed a cone 25 times and recorded whether it landed on its base or on its side.



On its base	On its side
𝍸 |||	𝍸 𝍸 𝍸 ||

What is the estimated probability that the cone will land on its base when tossed?

7. A bag contains 80 marbles. If the probability of drawing a yellow marble is $\frac{3}{5}$ and the probability of drawing a red marble is $\frac{2}{5}$, how many marbles of each color are in the bag?

8. A bag contains 100 chips. Each of the chips in the bag is either purple or yellow. Suppose you select a chip from the bag at random and record the color. Then you put the chip back in the bag and select again. You repeat this until you have observed 50 outcomes. Of these 50 outcomes, 17 were yellow and 33 were purple.

a. What is the estimated probability of selecting a purple chip when a chip is selected at random from this bag?

b. Suppose that you know that the bag was one of two bags—Bag 1 or Bag 2. Bag 1 has 30 yellow chips and 70 purple chips. Bag 2 has 50 yellow chips and 50 purple chips. Based on the outcomes you observed, which bag do you think you were selecting from—Bag 1 or Bag 2? Explain your reasoning.

Lesson 21-3

Recall the probability experiment of selecting a student at random from Ms. Bailey's seventh grade class. Use the survey data sheet to help you answer Items 9 and 10.

9. What is an example of an event that would have a probability greater than 0.5?

10. What is an example of an event that would have a probability less than 0.25?

MATHEMATICAL PRACTICES
Attend to Precision

11. Explain the difference between a theoretical probability and an estimated probability. Select a probability experiment to provide examples to support your answer.

Survey Data from Ms. Bailey's Seventh Grade Class

Student	Male or Female?	Right or Left-Handed?	Favorite Subject?	Texts Sent?	Superpower?
1	Male	Right	Math	34	Read minds
2	Female	Left	English	20	Fly
3	Female	Right	English	0	Fly
4	Female	Right	Art	1	Fly
5	Male	Right	Science	34	Read minds
6	Female	Right	English	3	Fly
7	Male	Right	History	0	Freeze time
8	Female	Right	English	0	Freeze time
9	Female	Right	Science	300	Fly
10	Male	Right	PE	0	Read minds
11	Male	Right	English	34	Read minds
12	Male	Right	Music	237	Fly
13	Male	Right	PE	100	Fly
14	Male	Right	Science	0	Freeze time
15	Female	Left	English	200	Read minds
16	Female	Left	English	40	Fly
17	Male	Right	Math	34	Read minds
18	Female	Right	English	30	Invisibility
19	Female	Right	Science	200	Fly
20	Female	Right	English	0	Fly
21	Male	Right	History	34	Read minds
22	Male	Left	PE	94	Fly
23	Female	Right	English	2	Fly
24	Male	Right	English	35	Freeze time
25	Female	Right	Science	10	Fly
26	Female	Right	English	15	Freeze time
27	Female	Right	English	20	Fly
28	Female	Right	Art	2	Fly
29	Female	Right	English	0	Freeze time
30	Female	Right	Science	2	Read minds
31	Male	Right	Science	34	Read minds
32	Female	Right	Math	0	Fly
33	Female	Right	Art	0	Fly
34	Female	Left	English	8	Fly

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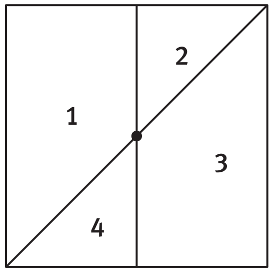
Finding Probabilities
SPINNING SPINNERS AND RANDOM PICKS

Write your answers on notebook paper or grid paper. Show your work.

1. Only one of the following sequences resulted from actually spinning the spinner below 20 times. Which sequence do you think this was?



Sequence 1: 4 3 2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3 2 1

Sequence 2: 4 1 3 4 2 3 4 3 1 4 1 4 1 2 2 2 4 2 1 3

Sequence 3: 4 3 3 3 2 1 1 1 4 3 3 3 2 1 1 1 4 3 3 3

Sequence 4: 1 4 1 3 3 3 1 3 2 2 3 4 1 1 4 1 1 3 3 3

2. For each sequence you did not select in Item 1, explain why you ruled out that sequence.

3. The following were the outcomes from 20 spins of a spinner. Design a spinner that you think might have generated this sequence of outcomes. Explain your reasoning in designing the spinner.

3 4 4 4 4 4 2 4 1 4 4 2 4 4 4 3 1 4 4 2

4. Mr. Lund's science class has 30 students. There are 20 students in this class who do not plan to enter a project in the upcoming school science fair.

a. If you were to select a student at random from this class, what is the probability that the selected student is planning to enter the science fair?

b. Is the probability you calculated in part a a theoretical probability or an estimated probability?

c. Could the probability that a student selected at random from this class is a female who plans to enter a project in the science fair be greater than the probability you calculated in part a? Explain why or why not.

d. Could the probability that a student selected at random from this class is a female be greater than the probability you calculated in part a? Explain why or why not.

5. A paper bag contains 100 plastic chips. Each chip has a different number and the chips are numbered from 1 to 100. One chip will be selected at random from the bag.

a. Which of the following events has the greatest probability? Which has the smallest probability?

Event	Description of Event
A	Selected number is an odd number greater than 50
B	Selected number is an odd number less than 20
C	Selected number is even

b. Give an example of an event that would have a probability that is greater than any of the events listed in part a.

c. Give an example of an event that would have a probability that is less than any of the events listed in part a.

6. Give an example of an event and its complement. What do you know about the sum of their probabilities?

7. Give an example of a probability experiment and a probability question that would involve calculating a theoretical probability.

8. A box contains a large number of plastic balls. Some of the balls are red and the rest are green. One ball will be selected at random from the box.

a. Using only the information you have been given, can you conclude that the probability of getting a red ball is $\frac{1}{2}$? Explain why or why not.

b. Based on the description of the probability experiment, can you calculate the theoretical probability that a red ball is selected? Explain why or why not.

c. What would you have to do in order to find an estimated probability of selecting a red ball?

9. If two people each correctly calculate the theoretical probability of an event, will they get the same answer? Explain why or why not.

10. If two people each use a correct method to find an estimated probability of an event, will they always get the same answer? Explain why or why not.

Finding Probabilities
SPINNING SPINNERS AND RANDOM PICKS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4a-d, 5a-c, 6, 7, 8a-c, 9, 10)	• Clear and accurate understanding of calculating estimated and theoretical probabilities.	• Calculating estimated and theoretical probabilities with few if any errors.	• Difficulty calculating estimated and theoretical probabilities.	• Incorrect or incomplete calculation of estimated and theoretical probabilities.
Problem Solving (Items 4a, 5a)	• Accurately interpreting possible outcomes as probability.	• Interpreting possible outcomes as probability.	• Difficulty interpreting outcomes as probability.	• No clear understanding of interpreting outcomes.
Mathematical Modeling / Representations (Items 1, 3. 5b-c)	• Accurately using theoretical probability to model outcomes of events.	• Using theoretical probability to model outcomes of events.	• Errors in using theoretical probability to model outcomes of events.	• Inaccurate or incomplete use of theoretical probability to model outcomes.
Reasoning and Communication (Items 2, 3, 4c-d, 5b-c, 6, 7, 8a-c, 9, 10)	• Clear and accurate explanation of estimated and theoretical probabilities.	• An adequate explanation of estimated and theoretical probability.	• Difficulty in explaining estimated and theoretical probability.	• An inaccurate explanation of estimated and theoretical probability.

Games and Probability

Rock, Paper, Scissors . . . and Other Games

Lesson 22-1 Rock, Paper, Scissors

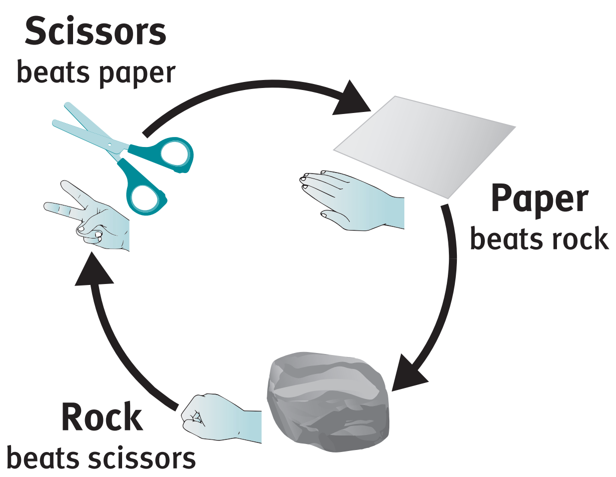
My Notes

Learning Targets:

- Use observed outcomes to estimate probabilities.
- Use tables to represent the possible outcomes of a probability experiment.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Rock, Paper, Scissors (RPS) is a fun two-person game. Some sources say that more people have played Rock, Paper, Scissors than any other game in the world.



To play RPS, do the following:

- Each player taps a fist in his or her palm twice.
- Then, both players simultaneously extend their hands in the shape of a rock, piece of paper, or pair of scissors. This is called a throw.

The winner of a round is decided based on the following rules:

- Rock beats scissors
- Scissors beats paper
- Paper beats rock

If both players show the same shape, the round results in a tie.

1. With a partner, play 10 rounds of RPS. For each round, record whether you won, you lost, or the round resulted in a tie.

Round	1	2	3	4	5	6	7	8	9	10
Result										

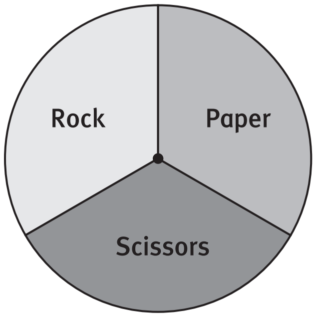
2. Did one player win more often? What fraction of the rounds resulted in a tie?

My Notes

3. **Reason abstractly.** If both players in a RPS game choose between rock, paper, and scissors at random, which do you think would occur most often: Player 1 wins, Player 2 wins, or the round results in a tie? Explain your reasoning.

4. Do you think players really pick at random each time they play RPS or do you think some players tend to favor some moves (rock, paper or scissors) over others?

5. To ensure that a throw is selected at random, use the spinner below to first determine a throw for Player 1. Then spin again to determine a throw for Player 2. (Use a paper clip, just as you did in the previous two activities.) Do this a total of 20 times, filling in the first two columns of the table on the next page.



My Notes

Round	Player 1	Player 2	Winner
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

6. Complete the last column of the table above by determining the winner for each round. If the round was a tie, enter Tie.

How many times did Player 1 win?

How many times did Player 2 win?

How many rounds were ties?

7. Combine your counts from Item 6 with those of two other students so that you have the outcomes of 60 rounds of play.

What fraction of the time did Player 1 win?

What fraction of the time did Player 2 win?

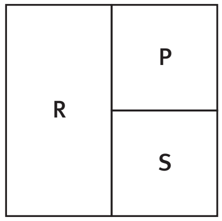
What fraction of the rounds were ties?

8. Based on the data you have generated from the class, state a hypothesis about the probability of a student choosing rock, paper, or scissors. Do you think the data collected is the same as expected? Why or why not? Find the probability of each.

My Notes

Check Your Understanding

9. RPS is a fair game if each player chooses a throw at random. Suppose a player used the following spinner to choose a throw.



Is this equivalent to selecting a throw at random? Explain why or why not.

10. In a random choice game of RPS, how often do you expect to lose a round of play on average?

Suppose you are playing a game of RPS in which the other player is using the spinner in Item 9 above to select throws. Can you think of a strategy in which you could minimize how often you expect to lose a round? Explain.

LESSON 22-1 PRACTICE

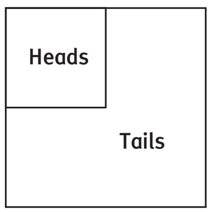
Sam and Jo are playing a coin game. If both coins land on either heads or tails, Sam wins. If one coin lands on heads and the other on tails, Jo wins.

11. The results of fifteen games are shown below. Determine which games were won by Sam and by Jo. (H for heads; T for tails)

Coin 1	H	T	T	T	T	T	T	H	T	T	T	T	T	T	H
Coin 2	H	H	H	H	H	T	H	H	T	T	H	H	H	H	H
Winner															

12. What fraction of the time did Sam win? What fraction of time did Jo win?

The match game is a fair game when played with fair coins. Suppose Sam and Jo use the spinner below to determine whether they get heads or tails.



13. **Make sense of problems.** Make a table to show Sam's and Jo's results using the spinner. What percent of the time should Sam win? What percent of the time should Jo win? Explain.

My Notes

Learning Targets:

- Use tables to represent the possible outcomes of a probability experiment.
- Assign probabilities to outcomes in a sample space.
- Use probabilities assigned to outcomes in a sample space to compute event probabilities.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

1. One possible outcome of a round of RPS is that Player 1 throws a rock and Player 2 throws scissors. This could be written in the form of a (Player 1, Player 2) pair as (R, S).

Make a list of all the possible outcomes for a round of RPS. (*Hint:* There are 9 possible outcomes, including (R, S).)

The collection of all the possible outcomes of a probability experiment is the **sample space**. For the probability experiment that is one round of RPS, the sample space consists of the nine outcomes from Item 9.

MATH TERMS

The **sample space** for a probability experiment is the set of all possible outcomes for that experiment.

Sometimes it helps to have a strategy that will help you find the sample space for a probability experiment.

One strategy is to make a list. For example, for the RPS game, you could begin with one possibility for Player 1—for example, rock. Then you could list rock with each of the different possibilities for Player 2:

(R, R), (R, P), (R, S)

Next, you could list all of the outcomes that have paper for Player 1, and then all the outcomes that have scissors for Player 1.

My Notes

Another possibility is to construct a table like the one below.

		Player 2 Throws		
		R	P	S
Player 1 Throws	R	(R, R)	(R, P)	
	P			
	S			

Each cell in the table above corresponds to one possible outcome.

2. Complete the rest of the table so that it includes all of the possible outcomes.

MATH TERMS

An **event** is a collection of outcomes of a probability experiment. It is a part of the sample space.

Recall that an **event** is a collection of outcomes from a probability experiment.

3. For the RPS game, what outcomes are in the event that Player 1 wins?

4. For the RPS game, what outcomes are in the event that Player 2 wins?

5. For the RPS game, what outcomes are in the event that a round results in a tie?

6. Look for and express regularity

a. If players are picking throws at random, would any of the nine possible outcomes be more likely to occur than the others?

b. For the random RPS strategy, what probability would you assign to each outcome in the sample space?

7. Calculate the following probabilities for a round of RPS where players are selecting throws at random.

   P(Player 1 throws a rock)

   P(Player 1 wins)

   P(Player 1 throws a rock and wins)

   P(The round results in a tie)

8. Suppose two players were selecting throws at random and play 100 rounds of RPS.
   a. About how many of them would result in a tie?

   b. About how many of them would result in a win for Player 2?

Play the game again, but this time, do not use the spinner to determine the throws. Instead, choose your throws as you would in a real game of RPS.

9. With a partner, play 20 rounds, recording your throw in the table on the next page after each round. Note that for this part of the activity, you only need to record your throw, and you do not need to record your partner's throw.

My Notes

MATH TIP

To find the probability of an event, you can add the probabilities for each outcome in the event.

If the outcomes in the sample space are equally likely, the probability can also be found by calculating

$$\frac{\left(\text{number of outcomes in the event}\right)}{\left(\text{number of outcomes in the sample space}\right)}$$

My Notes

Round	My Throw (R, P or S)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

10. Count how many times you threw each of rock, paper and scissors and record your counts here:

Number of rocks:

Number of papers:

Number of scissors:

Add your counts to a class poster that your teacher will provide by adding the appropriate tally marks. (You will be adding a total of 20 tally marks to the poster.)

11. **Reason abstractly.** Are these counts consistent with what you would expect if you and your classmates were selecting throws at random? Explain why or why not.

MATH TIP

Add tally marks in groups of fives (using four strokes and a fifth stroke across the first four.)

For example:

Class Throws

Rock: 𝍸 𝍸 ||| …

Paper: 𝍸 |||| …

Scissors: 𝍸 𝍸 | …

Make sure to continue the groups of five where the last student left off.

My Notes

Your teacher will also provide two more posters—one for girls and one for boys. Add your counts by adding tally marks to the appropriate poster.

12. Some people believe that boys are more likely than girls to throw a rock when playing RPS.

a. Based on the two posters your class just made, do you think this is the case? Explain why or why not.

b. Determine the theoretical probabilities from the class poster data.

Check Your Understanding

A variation of RPS is played in some countries, which adds a fourth throw called a Well. In this game:

- Well beats Rock
- Well loses to Paper
- Well beats Scissors

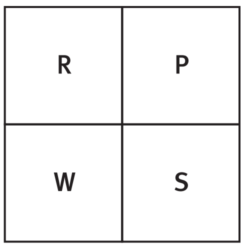
13. Construct a table (like the one in Item 9) that shows the different possible outcomes for a RPSW round.

14. a. What outcomes for RPSW result in a win for Player 1?

b. What outcomes result in a win for Player 2?

c. Is the probability of a win the same for Player 1 and for Player 2?

15. RPSW is a fair game if both players select one of the four possible throws at random. Suppose you know that your opponent is selecting throws at random using this spinner:



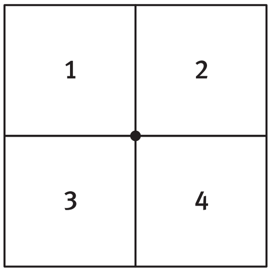
Can you think of a strategy in which you would expect to lose only about $\frac{1}{4}$ of the rounds that you play? Explain why you think this strategy will work.

My Notes

LESSON 22-2 PRACTICE

The following are rules for the High Roller Game, using the spinner below:

- Each player spins, and the player with the greater number wins.
- If both players spin the same number, then it's a tie for that round.



16. Give the sample space for this game, written as ordered pairs (first player, second player).

17. Play the game 20 times, recording the following information:

Round	Player 1's Number	Player 2's Number	Winner

18. Using the table, calculate the following probabilities:
a. *P*(Player 1 wins)
b. *P*(tie)
c. *P*(Player 2 wins)

Instead of spinning a spinner, each player writes down the numbers 1 to 20, representing the rounds of play, and then places five 1's, five 2's, five 3's, and five 4's into the twenty rounds. Next, the players compare numbers to determine the winner of each round. A sample is shown below.

Round	1	2	3	4	5	6	7	8	9	10
Person A	1	2	3	4	1	2	3	4	1	2
Person B	1	1	1	1	1	2	2	2	2	2
Winner										

Round	11	12	13	14	15	16	17	18	19	20
Person A	3	4	1	2	3	4	1	2	3	4
Person B	3	3	3	3	3	4	4	4	4	4
Winner										

19. Determine the winner each time. Then calculate the following:
a. *P*(Person A wins) **b.** *P*(Person B wins) **c.** *P*(tie)

20. Reason quantitatively. If you were going to place five 1's, five 2's, five 3's, and five 4's in twenty rounds of the High Roller game, what strategy would you use to maximize your wins?

My Notes

Learning Targets:

- Use observed outcomes to estimate probabilities.
- Use tables and tree diagrams to represent the possible outcomes of a probability experiment.
- Calculate the probabilities of events for a probability experiment with equally likely outcomes.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

A New Game . . .

Now let's think about a different game. Suppose there are two boxes—one big box and one small box. Each box has two drawers.

There is a prize hidden in one of these drawers.

To play the game, you first pick a box, and then you pick either the top drawer or the bottom drawer.

1. Choose a size and then either top or bottom, and then share with your class. Using the class data, estimate the probability that you would win the prize.

2. If you think of this as a probability experiment, there are four possible outcomes. One is (Big, Top), which stands for picking the big box and the top drawer.

   List all four possible outcomes for this probability experiment.

   (Big, Top)

MATH TIP

You can try a systematic approach to list possible outcomes.

For example, starting with one box and listing the different possibilities is a systematic approach.

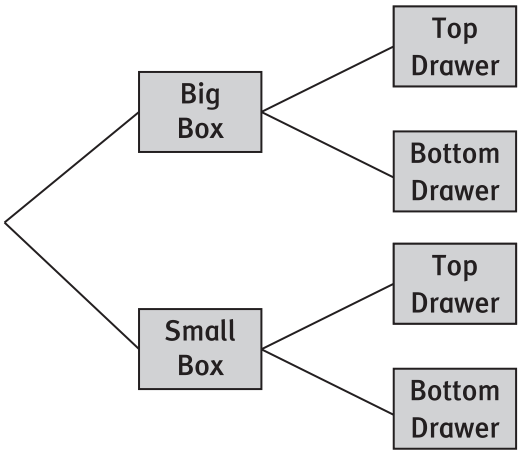
My Notes

3. If you make your selections (box and then drawer) at random, are the four outcomes equally likely? What is the probability of each of these outcomes?

MATH TERMS

A **tree diagram** is a diagram that represents the total possible outcomes.

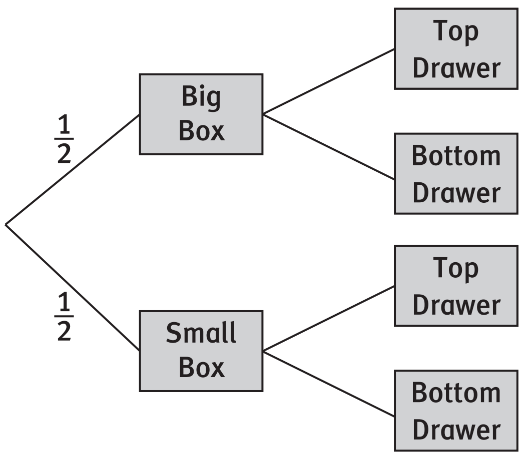
When a probability experiment can be carried out in steps, a common way to organize the outcomes is to construct a **tree diagram**. The tree diagram for the probability experiment just described looks like this:



Outcome	Probability
(Big, Top)	$\frac{1}{4}$
(Big, Bottom)	$\frac{1}{4}$
(Small, Top)	$\frac{1}{4}$
(Small, Bottom)	$\frac{1}{4}$

4. If you select a box at random, what is the probability that you select the big box? What is the probability that you select the small box?

We sometimes write the probabilities on the branches of the tree diagram, as shown below. This indicates that the probability of selecting the big box is $\frac{1}{2}$, and the probability of selecting the small box is $\frac{1}{2}$.

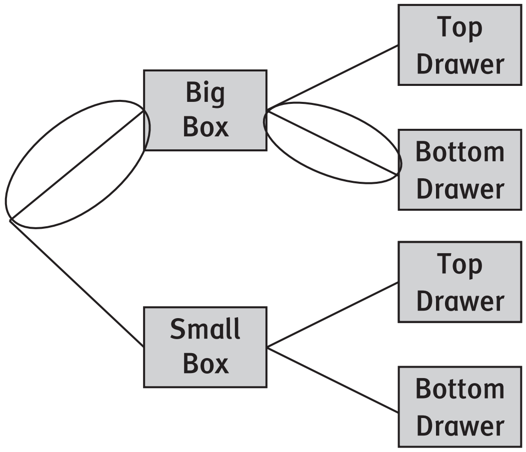


My Notes

5. If you select a drawer at random, what is the probability that you select the top drawer? The bottom drawer?

Label the remaining four branches in the tree diagram on the previous page with the appropriate probabilities.

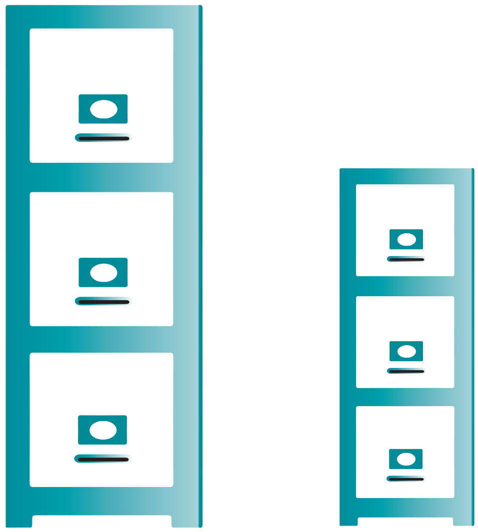
One way to arrive at a possible outcome is to follow branches of the tree starting at the left. For example, the outcome (Big, Bottom) could be arrived at by following the two circled branches:



6. Reason quantitatively. Because the four possible outcomes of the probability experiment are equally likely, once you know how many outcomes there are, you know what probability to assign to each outcome (in this case, $\frac{1}{4}$). What is another way to calculate the probability of an outcome that uses the probabilities on the branches that lead to that outcome?

My Notes

Suppose each box now has three drawers as shown below.



7. Compare the probability from the class data and the probability from Item 6. Explain your comparison.

8. If you pick a box at random and then pick a drawer at random, how many different outcomes are there?

9. Construct a tree diagram that shows the possible outcomes.

10. Put probabilities on the branches of your tree diagram in Item 7. Pick an outcome and multiply the probabilities on the branches that lead to that outcome. What do you notice about the product?

MATH TIP

To find the probability of an outcome in a tree diagram with probabilities on the branches, multiply the probabilities on the branches that lead to that outcome.

My Notes

11. **Express regularity in repeated reasoning.** When there were two boxes and two drawers, there were four possible outcomes. When there were two boxes and three drawers, there were six possible outcomes. How many outcomes would there be if there were four boxes that each had two drawers? How did you find your answer?

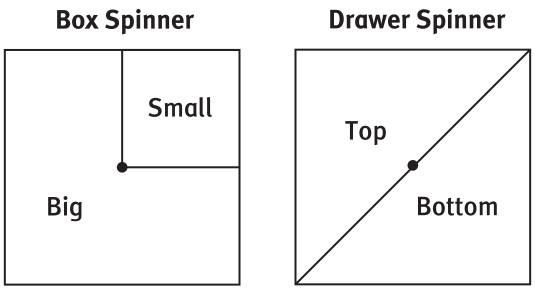
12. Use a table or a tree diagram to represent all of the possible outcomes when there are four boxes and each box has two drawers. Was your answer in Item 9 correct?

13. How many possible outcomes are there with 20 boxes if each box has four drawers? If you were to pick a box at random and then pick a drawer at random, what is the probability of each of the outcomes?

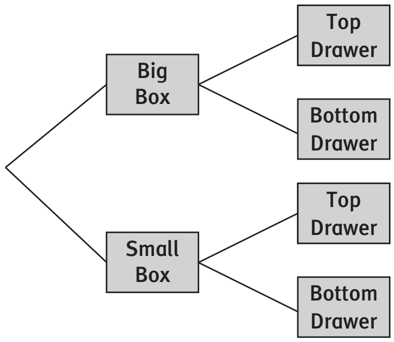
My Notes

Check Your Understanding

Consider the game in which there are two boxes and each box has two drawers. Instead of picking a box at random and then picking a drawer at random, you decide to use the following two spinners to make your selections.



14. Put the appropriate probabilities on the branches of the tree diagram below.



15. Consider the probability experiment that consists of choosing a box and then choosing a drawer using the spinners above. One possible outcome is (Big, Top).

What is the sample space for this experiment?

LESSON 22-3 PRACTICE

Using the spinner in Check Your Understanding, complete Items 14–18.

16. **Make sense of problems.** The four outcomes in the sample space are not equally likely. How do you know?

17. If a prize is hidden in the top drawer of the big box, what is the probability that you will win this prize?

18. If a prize is hidden in the top drawer of the small box, what is the probability that you will win this prize?

19. Explain why the probability for a prize in the top drawer of the big box is different from the probability for the prize hidden in the top drawer of the small box.

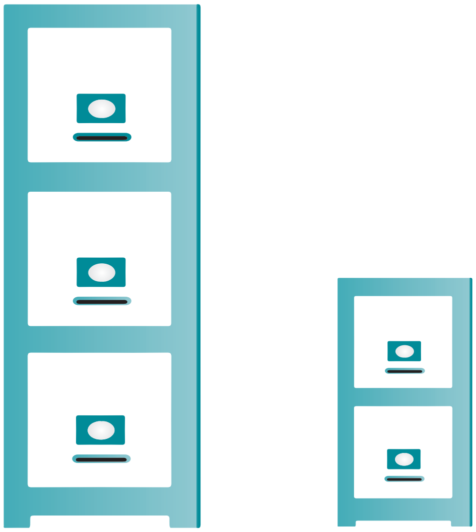
My Notes

Learning Targets:

- Use observed outcomes to estimate probabilities.
- Use tables and tree diagrams to represent the possible outcomes of a probability experiment.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

For the rest of this activity, consider a game with the big boxes and small boxes below. The big box has three drawers (top, middle, and bottom) and the small box has two drawers.



You will pick a box at random and then pick a drawer at random from that box.

1. Draw a tree diagram that shows the possible outcomes.

My Notes

2. Put probabilities on the branches of your tree diagram and multiply across branches to get a probability for each outcome.

3. Reason abstractly. Are all of the outcomes equally likely? Explain.

4. If there is one prize and it is hidden in the top drawer of the small box, what is the probability that you select the drawer with the prize?

5. Suppose that there are two prizes and they are hidden in the top drawer of the big box and in the bottom drawer of the small box. What is the probability that you select a drawer that has a prize? How did you arrive at this probability?

6. Suppose that there are three prizes and they are hidden in the top drawer of the big box, the middle drawer of the big box and the top drawer of the small box. What is the probability that you select a drawer that has a prize? How did you arrive at this probability?

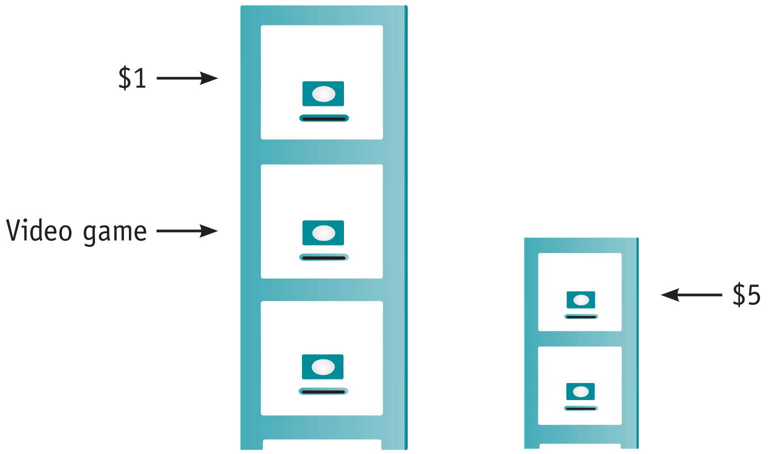
My Notes

Suppose that there are prizes in three different drawers, and you are going to select a box at random and then a drawer at random from that box.

7. Construct viable arguments. Is the probability of getting a prize greater if all three prizes are in the big box or if one prize is in the big box and two prizes are in the small box? Support your answer with probabilities.

Check Your Understanding

Suppose that three different prizes are placed in different drawers as shown here.



8. Make a tree diagram for selecting a box at random and then selecting a drawer at random. Write the probabilities along the branches of the tree diagram. Calculate each of the five possible outcomes.

9. If you select a box at random and then a drawer at random, what are the following probabilities?

a. P(win video game) =
b. P(do not win a prize) =
c. P(win a prize) =
d. P(win $5) =
e. P(win money) =
f. P(win a prize that is not money) =
g. P(win a prize that is worth more than $1) =
h. P(win the video game or $5) =

MATH TIP

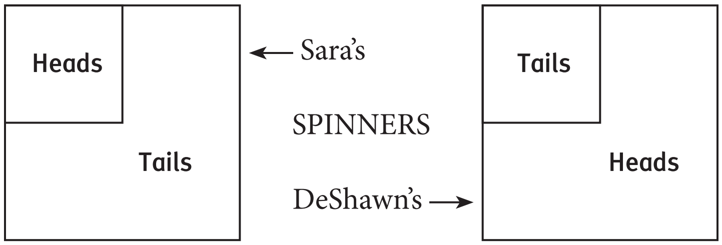
Since a tree diagram shows all of the outcomes, you can use a tree diagram to help you determine probabilities.

For example, you can use the tree diagram you drew in Item 8 to help you determine the probabilities in Item 9.

My Notes

LESSON 22-4 PRACTICE

Sara and DeShawn are playing the Match Game with two spinners.



Both players spin at the same time.

10. Model with mathematics. Draw a tree diagram with probabilities to summarize the outcomes of the Match Game.

11. Write the sample space of the two spinners as ordered pairs (Sara's result; DeShawn's result).

12. What is the probability that DeShawn's spinner will result in tails?

13. What is the probability that both spinners will show heads?

14. If you were advising Sara, should she play to win if the results of both spinners are the same (match) or if they are different? Explain.

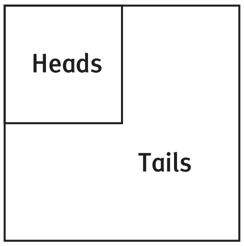
ACTIVITY 22 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 22-1

Use the following situation to answer Items 1–3.

Tia and Li are playing a coin game using the spinner below. If both spins land on either heads or tails, Tia wins. If one spin lands on heads and the other on tails, Li wins.



1. Complete the table for ten games. Determine which games were won by Tia and Li.

Coin 1										
Coin 2										
Winner										

2. What fraction of the time did Tia win?
3. What fraction did Li win?

Lesson 22-2

4. Calculate the following probabilities for a round of the Rock, Paper, Scissors game where players are selecting throws at random.
   a. *P*(Player 2 throws paper)
   b. *P*(Player 2 wins)
   c. *P*(Player 2 throws paper and wins)

Lesson 22-3

Use the following situation to answer Items 5–9.

Three bags each contain four balls. Bag 1 has three red balls and one blue ball. Bag 2 has four red balls. Bag 3 has two red balls and two blue balls.

A probability experiment consists of selecting a bag at random and then selecting a ball at random from that bag, and noting its color.

5. One possible outcome for this probability experiment is (Bag 1, Red).
   Draw a tree diagram that shows all of the possible outcomes for this probability experiment.
6. How many different outcomes are there?
7. Put appropriate probabilities on the branches of the tree diagram. Then for each outcome, multiply across the branches to obtain the probability of the outcome.
8. Are the outcomes equally likely? If not, give an example of two outcomes that do not have the same probability.
9. Calculate the following probabilities.
   a. *P*(ball is from Bag 1 and is red)
   b. *P*(ball is from Bag 2 and is blue)
   c. *P*(ball is from Bag 1)
   d. *P*(ball is red)
   e. *P*(ball is blue)

Lesson 22-4

Use the information below to answer Items 10 and 11.

A game has three boxes. Box 1 has one drawer, Box 2 has two drawers, and Box 3 has three drawers.

You will pick a box at random and then pick a drawer from that box at random.

10. A prize is hidden in the top drawer of Box 3. What is the probability that you select this drawer?

11. A prize is hidden in the only drawer of Box 1. What is the probability that you select this drawer?

12. Make up a box game like the ones in this activity that has two boxes and seven possible outcomes. How many drawers are in each box?

13. Consider a box game with two boxes. Box 1 has one drawer and Box 2 has 5 drawers. A player will select a box at random and then pick a drawer from that box at random. You have two prizes to place in the drawers. Where should you place them to create the greatest probability that someone who plays the game will win a prize? Explain your reasoning.

14. Suppose you perform a probability experiment in which you first toss a fair coin and then roll a fair number cube with the faces labeled with the numbers 1 through 6.

a. Draw a tree diagram to show all of the possible outcomes for this experiment. How many outcomes are in the sample space?

b. Calculate the probability that the coin will show heads and the number cube will show a three.

c. Calculate the probability that the coin will show tails and the number cube will show an odd number.

d. Are all the outcomes equally likely? Explain.

15. Consider a two-player game called Otto-Mamu based on the tree diagram you created with the following rules:

- An Otto occurs whenever (heads and an odd number) or (tails and an even number) results.
- A Mamu occurs for all other throws.

a. Is the Otto-Mamu game a fair game? Explain your reasoning.

b. If it is not a fair game, what strategy should a player choose to gain the winning advantage?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

16. When is knowing how many outcomes are in the sample space enough to tell you what probability should be assigned to each outcome?

Probability

Estimating Probabilities Using Simulation

Lesson 23-1 What is Simulation?

My Notes

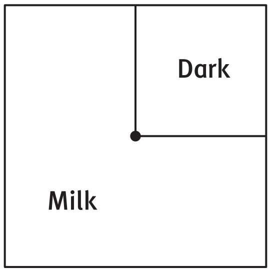
Learning Targets:

- Use artificial processes to simulate outcomes.
- Assign random digits to outcomes.
- Carry out a simulation using random digits.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

In Activity 20, you learned that it is possible to estimate probabilities by observing many outcomes of a probability experiment. In Activity 21, you learned that it is sometimes possible to generate outcomes by using an artificial process (like spinning a spinner) in place of actually carrying out the probability experiment.

As an example, the spinner below was used as a substitute for selecting a candy at random from a bag of 40 candies in which 30 were milk chocolate and 10 were dark chocolate.



In this activity, you will see how **random digits** can be used as a substitute for spinners to observe the outcome of a probability experiment.

The Random Digit Table on page 316 shows 30 rows of random digits. Each row has 25 random digits organized in groups of five. Carefully tear out that page so that you can use it as you answer the items in this activity.

We will start with a well known probability experiment—tossing a fair coin. It is easier to develop some important ideas in a simple setting. Then we will move on to more complex experiments.

1. What are the two possible outcomes that make up the sample space when you toss a coin?

MATH TERMS

Random digits are digits (0, 1, 2, . . . , 8, 9) arranged in a random order.

Because the order is random, it is not possible to predict what the next digit will be from knowing the digits that come before it.

You can think of a list of random digits as having been generated by spinning a spinner with ten equal sections labeled 0 through 9.

My Notes

2. Toss a penny 10 times and record the outcomes in the table below. Then estimate the probability of getting heads using the fraction of the 10 tosses that resulted in heads.

Toss	1	2	3	4	5	6	7	8	9	10
Outcome										

Estimated probability of tossing heads:

3. What is the theoretical probability of getting heads when a fair coin is tossed? How does the estimated probability from Item 2 compare to the theoretical probability?

4. Combine your toss results with those of four other students so that you know how many heads were observed in a total of 50 tosses. Use these 50 tosses to estimate the probability of heads. Is this estimated probability closer to the theoretical probability of heads than your estimate from Item 2?

ACADEMIC VOCABULARY

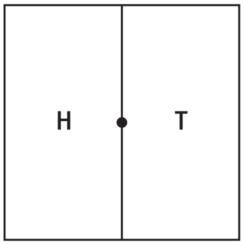
A ***simulation*** is when an artificial process for generating an outcome is substituted for a probability experiment.

It is important that the artificial process generates outcomes that are like outcomes from the actual experiment.

The estimated probability in Item 4 was based on actually carrying out the probability experiment of tossing a coin many times. Instead of actually tossing a coin, we could substitute an artificial process that behaves in a similar way and use the artificial process to generate outcomes. This is called carrying out a ***simulation.***

My Notes

5. A simple artificial process that could be substituted for tossing a fair coin is spinning a spinner like the one shown below. Explain why using this spinner to generate outcomes is like tossing a fair coin.



Even though spinning this spinner could be substituted for tossing a fair coin, it is not an improvement over actually tossing the coin. In fact, it may even take longer to spin the spinner and observe an outcome than it would take to toss a coin! Instead, look at how random digits can be used to simulate the process.

Example A

Random integers can be chosen using a table of values or by using a calculator.
Suppose we use a random digit to represent the outcome of a coin toss with even digits (0, 2, 4, 6, and 8) representing heads and odd digits (1, 3, 5, 7, and 9) representing tails.

A sequence of 10 random digits written in row 1 of the table below could be interpreted as ten tosses.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome	T	T	T	T	H	H	H	H	H	T

Try These A

a. Explain why using even digits to represent heads and odd digits to represent tails produces outcomes that are like tossing a fair coin.

b. Should every group of ten random digits produce five heads and five tails? Explain.

TECHNOLOGY TIP

For this example, use a graphing calculator to simulate random number generation. Choose the probability simulation and enter the starting number, the ending number, and how many numbers you want.

My Notes

6. Can we use random digits to represent the outcome of a fair coin toss if we use 0, 1 and 2 to represent heads and 3, 4, 5, 6, 7, 8, and 9 to represent tails? Explain why or why not.

7. Make sense of problems. What assignment of digits different than the one already given would produce results similar to tossing a fair coin? Explain your reasoning.

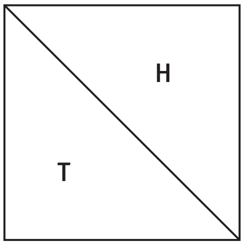
8. Use the digits you chose in Item 7 to determine heads and tails for the digits in the chart. Did you find the same number of heads as tails? Explain how this could occur.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome										

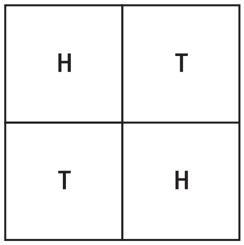
Check Your Understanding

9. Which of the following spinners could be substituted for tossing a fair coin?

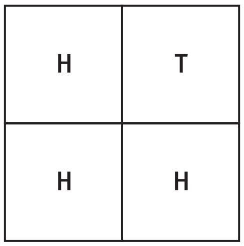
Spinner 1



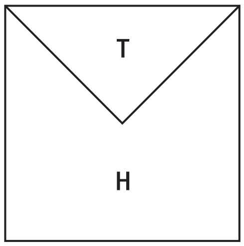
Spinner 2



Spinner 3



Spinner 4



10. Suppose you wanted to simulate tossing a fair coin by selecting a chip from a bag containing blue chips and red chips. A blue chip represents heads and a red chip represents tails. How many of each color should you put in the bag to make a total of 30 chips?

11. Suppose you plan to use a random digit to represent a toss of a fair coin. Consider the following assignment of digits:

- 1, 2 and 3 represent heads
- 4, 5 and 6 represent tails
- Ignore 0, 7, 8 and 9. If one of these digits occurs, just skip it and move on to the next digit.

Will this assignment of digits produce outcomes (heads or tails) that are like tossing a fair coin? Explain your reasoning.

LESSON 23-1 PRACTICE

Suppose you simulate rolling a fair number cube using the Random Digit Table. Let 1, 2, 3, 4, 5, and 6 represent the numbers on the number cube and ignore the digits 0, 7, 8, and 9.

12. Fill in the chart with numbers that are rolled on the number cube according to the assignment of digits in the instructions above.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome										

Digit	9	2	2	2	7	4	9	2	1	0
Outcome										

13. On a fair number cube, what is the probability of rolling a 5?

14. Reason quantitatively. Give the estimated probabilities for rolling each number on a number cube as simulated in Item 12.

$P(1) =$ $P(2) =$ $P(3) =$

$P(4) =$ $P(5) =$ $P(6) =$

15. Do the answers for $P(5)$ in Items 13 and 14 agree? Explain.

16. Write out a different assignment of digits to represent rolling a fair number cube. Then fill in the chart below with the results. Calculate $P(5)$ for this simulation. Are you surprised by the result? Explain.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome										

Digit	9	2	2	2	7	4	9	2	1	0
Outcome										

My Notes

Learning Targets:

- Design and carry out a simulation.
- Use a simulation to estimate a probability.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

You can use a group of five random digits to represent five tosses of a fair coin.
Your teacher will assign you a row in the Random Digit Table on page 316.

1. Copy your row from the Random Digit Table into the table below in groups of five digits. Then, translate each group into outcomes (such as HHTHT). Use even digits to represent heads and odd digits to represent tails.

Random Digits					
Translated Outcomes					

2. For each group of five digits, determine if the event E occurred, where E = event that two or more heads occur in five tosses. For example, if the five tosses were TTHTH, there are two heads, and the event E occurred.

Digit Group	**1**	**2**	**3**	**4**	**5**
Did *E* Occur? (Yes or No)					

Add your results to a class poster that your teacher will provide by adding five tally marks. For example, if the event E occurred for two of your groups and did not occur for three of your groups, add two tally marks for *yes* and three tally marks for *no*.

Lesson 23-2
Using Random Numbers to Simulate Events

My Notes

3. Use the results from the entire class to estimate the probability of getting two or more heads in five tosses of a fair coin. This is an estimated probability based on a simulation.

4. **Construct viable arguments.** Explain how you would use simulation to estimate the probability of getting four or more heads when a fair coin is tossed six times.

MATH TIP

When you are asked to explain a simulation, you do not have to carry out the simulation—just describe the steps as you would carry them out.

Discuss your answer to Item 4 with a partner. Then answer Items 5–7 based on the simulation described in that item.

5. How many digits do you need to represent an outcome for the probability experiment?

6. Did you use even digits to represent heads and odd digits to represent tails? Why is this an appropriate way to assign digits to heads and tails?

7. Suppose you used the simulation steps you described in Item 4 to simulate 50 outcomes. If 19 of these simulated outcomes included four or more heads, what is the estimated probability of getting four or more heads in six tosses of a fair coin?

My Notes

Now think about a probability experiment that consists of selecting a candy at random from a bag that contains 40% milk chocolate candies and 60% dark chocolate candies.

8. If you are interested in observing whether the selected candy is milk chocolate or dark chocolate, which of the following would be an appropriate way to use random digits to represent this probability experiment? Explain your choice.

Choice 1: Use a random digit to represent a selected candy, with even digits representing milk chocolate and odd digits representing dark chocolate.

Choice 2: Use a random digit to represent a selected candy, with 0, 1, 2 and 3 representing milk chocolate and 4, 5, 6, 7, 8 and 9 representing dark chocolate.

9. Explain how you would use random digits to represent selecting a candy at random from a bag that contained 70% milk chocolate candies and 30% dark chocolate candies.

10. **Reason abstractly and quantitatively.** In Activity 21, you saw a probability experiment for selecting a candy at random from a bag that contained 75% milk chocolate candies and 25% dark chocolate candies. Explain why it is not possible to use a single random digit to represent selecting a candy at random from this bag.

11. Can you think of a way to use a *pair* of random digits (such as 27, 34, and so on) to represent selecting a candy at random from a bag that contains 75% milk chocolate candies and 25% dark chocolate candies? Explain how you would do this.

My Notes

Check Your Understanding

12. A probability experiment consists of selecting a chip at random from a paper bag that contains two red chips and eight green chips. A random digit will be used to represent the selected chip. How would you assign digits to the two possible outcomes?

Outcome	Digits Assigned
Red	
Green	

13. A probability experiment consists of selecting a chip at random from a paper bag that contains 45 red chips and 55 green chips. For each of the following, explain whether the process described would be a correct way to use random digits to simulate this experiment.

a. Use one random digit to represent a selection, with 0, 1, 2, and 3 representing a red chip and 4, 5, 6, 7, 8 and 9 representing a green chip.

b. Use one random digit to represent a selection, with even digits representing a red chip and odd digits representing a green chip.

c. Use two random digits to represent a selection, with even numbers (like 34) representing a red chip and odd numbers (like 35) representing a green chip.

d. Use two random digits to represent a selection, with 00, 01, 02, . . ., 44 representing selecting a red chip and 45, 46, . . ., 99 representing selecting a green chip.

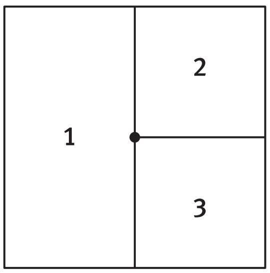
LESSON 23-2 PRACTICE

14. Make sense of problems. The groupings of four numbers represent the results of four rolls of a standard cube numbered 1 through 6. Event *S* represents two numbers that are the same. Based on the simulation, what is the estimated probability for Event *S* to occur?

2235 6225 2526 3523 2211 6214 4463
3314 1366 4345

5525 6316 4264 3165 3516 4153 3366
3262 3664 1526

15. To use random digits to simulate the results from a spinner, what assignment of digits could be made to accurately model the spinner below?



16. A bag contains 10% blue marbles, 40% red marbles and 50% green marbles. The probability experiment involves drawing one marble from the bag, returning the marble to the bag, and then shaking the bag before selecting the next marble. What assignment of digits can accurately simulate probability in this experiment?

17. Groups of four random digits (0 to 9) are listed below. In a two-player game, one player wins when the sum of the four digits is greater than 18, and the other player wins when the sum of the digits is less than 18. Is this a fair game? Explain your reasoning.

5963 2434 6122 6196 3352 6920
5957 2725 5967 9478

7839 0335 3132 5163 7703 1942
7347 3572 7472 2338

18. A bag contains 100 chips with 20 blue chips and 80 red chips. Describe an assignment of random digits that can be used to simulate drawing one chip from the bag.

My Notes

Learning Targets:

- Design and carry out the simulation of a compound event.
- Use a simulation to estimate the probability of a compound event.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Now we would like to introduce you to Sophie—a young dog who loves to play Frisbee. Sophie will be helping with the rest of this activity.

Even though Sophie loves to catch a Frisbee, she isn't very good at actually catching the Frisbee! In the long run, she catches the Frisbee only 10% of the time.

1. If you throw the Frisbee 20 times, about how many of the throws do you think Sophie will catch?

2. You can use a random digit to represent a Frisbee throw. Describe how you would assign the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to the two possible outcomes—Success (Sophie catches the Frisbee) and Failure (Sophie doesn't catch the Frisbee).

My Notes

3. Use your answer to Item 2 to translate the following sequence of 20 random digits into an outcome for the probability experiment that consists of throwing the Frisbee for Sophie 20 times.

8 5 9 4 5 9 9 6 1 5 4 5 8 4 6 1 2 7 0 6

4. How many of the 20 throws in your answer to Item 3 were successes?

5. What is the probability that Sophia will be successful based on the results of the simulation in item 3?

So far, we only have one simulated outcome. To estimate the probability of the event *F*, we would need many more simulated outcomes.

6. If available, use technology to complete this question, or follow the steps below:
- Toss a paper clip onto the page of random digits on page 316.
- The digit on the page that is nearest to the center of the larger loop of the paper clip is the first random number. Write the digit in row 1, Group 1, in the table below.
- Repeat until you have picked and recorded 20 random numbers for row 1, Group 1.
- Continue the activity until you have picked 20 random numbers for groups 2–5.
- Translate each group of 20 random digits into an outcome by indicating which throws were successes and which throws were failures. You will have five groups, each of which has 20 random digits and the results of 20 frisbee throws.

CONNECT TO LANGUAGE ARTS

A ***compound sentence*** consists of two or more independent clauses. In a similar way, a compound event consists of two or more simple events.

TECHNOLOGY TIP

The random number generator on a graphing calculator can be "seeded," which means that every calculator would generate different numbers. For example, you might use your birth date as the starting point (such as January 3, or 0103).

My Notes

7. For each of the five outcomes in Item 6, determine if the event *S*, two or more catches, occurred or not. Write yes or no.

Group 1: ______________

Group 2: ______________

Group 3: ______________

Group 4: ______________

Group 5: ______________

8. For each of the five outcomes in Item 7, determine if the new event *F* (four or more catches) occurred or not.

Add your results to a class poster that your teacher will provide by adding five tally marks. For example, if the event *S* occurred for two of your groups of 20 throws and did not occur for three of your groups, add two tally marks for *yes* and three tally marks for *no*.

9. Use the results from the entire class to estimate the probability that Sophie catches four or more of the 20 Frisbee throws.

Check Your Understanding

10. Suppose that in the long run Sophie could catch a Frisbee 30% of the time instead of 10% of the time. How would you assign digits to the outcomes *success* and *failure* in this situation?

11. Below is a sequence of 10 random digits. Use your assignment of digits from Item 10 to translate this into an outcome for the probability experiment of throwing the Frisbee for Sophie 10 times.

7 6 2 8 4 6 5 0 7 8

12. For the simulated outcome in Item11, how many times out of 10 throws did Sophie catch the Frisbee?

My Notes

LESSON 23-3 PRACTICE

13. Ameda is an 80% free-throw shooter.

a. In preparing to run a simulation, which digits should be assigned to Ameda for making a free throw?

b. Predict how many times Ameda will make at least eight free throws out of 10 in 100 free throws.

c. Using Row 12 of the Random Digits Chart, select 10 digits at a time for 10 events. In how many of these ten simulated events will Ameda make at least eight free throws?

d. What percent of the time did the simulation indicate that Ameda will make at least eight free throws? Was your prediction in part b correct? Explain.

14. Sidney hits the mark 60% of the time when shooting at targets with his bow and arrow.

a. When creating a simulation of his shooting accuracy, which digits should be assigned to Sidney for hitting the target?

b. Predict what percent of the time Sidney will miss the target two times or fewer in five shots, if ten groups of five shots each are taken.

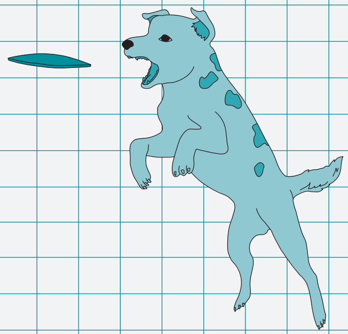
c. Using Row 2 of the Random Digits Chart, select five digits at a time to simulate ten events of five shots each. How many times does the event *missing two or fewer targets* occur?

d. What percent of the time will Sidney miss the target two times or fewer according to the simulation? Was your prediction in part b correct? Explain.

15. **Reason abstractly.** What can be done to increase the accuracy of the results for the events in Items 13 and 14?

16. A machine manufactures good light bulbs with 98% accuracy. How should random digits be assigned to indicate whether the light bulb is good or bad?

My Notes



Learning Targets:

- Design and carry out the simulation of a compound event.
- Use a simulation to estimate the probability of a compound event.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Kirby is a dog that is more talented than Sophie when it comes to catching a Frisbee. Kirby catches the Frisbee 70% of the time.

1. How would you change the assignment of random digits for the outcomes *success* (catches a Frisbee) and *failure* (does not catch a Frisbee) if Kirby is playing instead of Sophie? Be specific about the digits to represent success and the digits to represent failure.

2. Use your assignment of random digits from Item 1 and the 100 random digits shown below to simulate 50 outcomes for the probability experiment that involves throwing the Frisbee for Kirby twice. Write *S* or *F* above each digit.

CONNECT TO COMPUTER SCIENCE

Computer simulations are often used for experiments with a continuum of possible outcomes.

4 0	3 5	2 3	5 9	7 2	1 8	0 7	2 0	6 4	4 3
1 4	1 8	6 2	4 6	9 2	2 1	0 3	0 4	9 8	0 8
8 2	4 4	3 7	6 4	8 8	1 1	8 6	2 8	2 2	5 9
4 1	2 4	6 8	7 9	3 7	6 9	6 8	1 5	9 9	4 5
1 1	3 6	1 3	9 4	6 1	6 2	2 7	9 7	9 4	8 8

My Notes

Use the 50 simulated outcomes to estimate the following probabilities.

3. P(Kirby catches the Frisbee on both throws)

4. P(Kirby catches the Frisbee on exactly one of the two throws)

5. P(Kirby misses on both throws)

6. P(Kirby does not miss on both throws)

7. What do you notice about the sum of the probabilities from Items 5 and 6? Explain why the sum has this value.

Consider throwing the Frisbee to Kirby until he catches it. We want to find the number of throws needed. For example, one possible outcome is that Kirby catches the first throw and then we stop. Another possibility is that Kirby misses the first throw and then catches the second throw, which means that two throws are needed.

We can carry out a simulation of this probability experiment using the following steps:

- Use a random digit to represent each throw.
- The digits 0, 1, 2, 3, 4, 5 and 6 represent a throw that Kirby catches.
- The digits 7, 8 and 9 represent a throw that Kirby misses.
- Use a sequence of random digits to represent an outcome. Start with one digit. If it is a digit that represents a catch, stop and record an outcome of one throw. If the digit is 7, 8 or 9, Kirby did not make the catch and so we "throw" again by looking at the next random digit. We continue until Kirby makes a catch and record the number of throws needed.

For example, using the list of random digits below, we could mark off outcomes using vertical bars as shown.

7 3 | 1 | 6 | 9 4 | 8 6 |

The first outcome used two throws. The second outcome used one throw. The third outcome used one throw. The fourth and fifth outcomes each used two throws.

My Notes

Take a minute to make sure you understand how the sequence of random digits was divided up into outcomes and that the number of throws associated with each outcome is correct.

This gives five simulated outcomes. We have added these outcomes to the tally count shown here.

Number of Throws	Tally Count
1	||
2	|||
3	
4	
5	
6	
7	
8	

8. If available, use a graphing calculator to generate 45 more outcomes. Otherwise, mark the list of random digits below to show 45 more outcomes. Then add the appropriate tally marks to the tally count above. You should have a total of 50 tally marks when you are done.

4 0 3 5 2 3 5 9 7 2 1 8 0 7 2 0 6 4 4 3
1 4 1 8 6 2 4 6 9 2 2 1 0 3 0 4 9 8 0 8
8 2 4 4 3 7 6 4 8 8 1 1 8 6 2 8 2 2 5 9

Use the 50 simulated outcomes to estimate the following probabilities.

TECHNOLOGY TIP

To generate random numbers, enter the following:

1234
math
prob
rand
enter

9. *P*(more than 2 throws are needed)

10. *P*(Only 1 or 2 throws are needed)

11. *P*(Kirby makes a catch on the first throw)

12. **Reason quantitatively.** Is the estimated probability in Item 11 close to 0.70? Explain why this makes sense.

My Notes

13. Explain why it makes sense that the estimated probability in Item 9 is small.

14. **Make sense of problems.** How would you modify the simulation steps (listed after Item 7) if the probability experiment consisted of throwing the Frisbee for Kirby until he makes his second successful catch?

Check Your Understanding

Tracker is a great Frisbee catcher. He makes a catch 90% of the time! Representing a Frisbee throw with a random digit, you decide to use 1, 2, 3, 4, 5, 6, 7, 8 and 9 to represent a success (a catch) and 0 to represent a failure (a miss).

15. Use the random digits below to estimate the probability that Tracker will catch four out of four Frisbee throws.

7710	6867	2696	3995
6243	2793	1007	6597
8536	7831	9432	4241
5157	9197	8914	7169
3587	1083	5976	2246

Number of simulated outcomes:

Estimated probability:

16. Remember Kirby? In the long run, Kirby catches 70% of Frisbee throws. If you were to estimate the probability that Kirby would catch four out of four throws, do you think this probability would be much greater than, about the same as, or much less than the probability you computed for Tracker in Item 9? Explain your reasoning.

17. Use the random digits below with 0, 1, 2, 3, 4, 5 and 6 representing a catch to estimate the probability that Kirby can catch four out of four Frisbee throws.

7710	6867	2696	3995
6243	2793	1007	6597
8536	7831	9432	4241
5157	9197	8914	7169
3587	1083	5976	2246

Number of simulated outcomes:

Estimated probability:

18. How does the estimated probability for Kirby in Item 17 compare to the estimated probability for Tracker in Item 15? Was your answer in Item 16 correct?

My Notes

LESSON 23-4 PRACTICE

Write your answers on notebook paper. Show your work.

Use the information below to answer Items 19–22.

A six-sided number cube contains the numbers 1 through 6. You decide to simulate rolling this number cube using a random digit to represent one roll. The digits 0, 7, 8 and 9 are ignored and the other digits represent the corresponding numbers on the number cube. For example, the sequence of random digits 5, 9, 7, 6, and 4 correspond to three rolls with outcomes 5, 6 and 4 (the 9 and 7 are ignored).

Use the following sequence of random digits to carry out a simulation and then answer Items 19–21.

1	8	3	8	4	8	0	5	0	4
0	0	7	6	6	8	1	1	9	5
8	9	9	4	9	2	5	9	0	4
8	1	4	2	9	7	4	3	2	7
4	4	8	3	1	1	8	5	8	5
1	0	1	8	7	0	3	1	9	9

19. How many rolls of the number cube are represented?

20. How many of the simulated rolls resulted in an even number?

21. Based on this simulation, what is the estimated probability that a number greater than 4 will be rolled?

22. Suppose you plan to roll two number cubes and are interested in the sum of the two numbers rolled. You can simulate an outcome using two random digits—each digit represents an outcome for one of the cubes.
- Using the same assignment of digits described above (ignoring 0, 7, 8 and 9), use the following sequence of random digits to obtain 17 simulated outcomes.
- Then use the simulated outcomes to estimate the probability that the sum will be less than or equal to 5.

1	6	7	1	5	1	2	2	6	4
5	4	5	7	3	6	7	9	0	8
0	1	8	8	1	6	6	8	8	0
3	9	0	3	6	8	0	9	0	7
9	6	6	0	2	8	6	7	2	4
4	1	4	2	4	1	8	4	5	7

My Notes

Use the information below to answer Items 23–26.
A probability experiment consists of selecting a chip at random from a bag that contains 60% green chips and 40% yellow chips.

23. **Construct viable arguments.** Explain how you would use a random digit to represent an outcome for this probability experiment.

24. Suppose the probability experiment is performed three times. Use your assignment of digits from Item 23 to translate the three digits below into outcomes for three repetitions of this experiment.

7 2 6

25. Were all three selections in Item 24 green chips?

26. Use the random digits below to estimate the probability of getting three green chips three times, when a chip is selected at random from a bag that has 60% green chips.

5	9	0	4	3	8	3	9	1
4	3	2	7	9	4	5	1	9
8	5	8	5	3	1	9	5	0

Number of simulated outcomes:

Estimated probability:

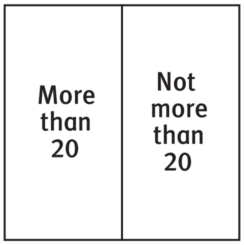
ACTIVITY 23 PRACTICE

**Write your answers on notebook paper.
Show your work.**

Lesson 23-1

1. Thirty percent of the employees at a large company live more than 20 miles away from where they work. One employee of this company will be selected at random and asked if he or she lives more than 20 miles from work. For each of the following artificial processes, state whether or not it could be used to simulate an outcome.

Process 1: Spin the spinner below to generate an outcome.



Process 2: Use a random digit to represent an outcome, with even digits representing a person who lives more than 20 miles away and odd digits representing a person who does not live more than 20 miles away.

Process 3: Use a random digit to represent an outcome, with 0, 1, 2, 3, 4, 5 and 6 representing a person who lives more than 20 miles away and 7, 8 and 9 representing a person who does not live more than 20 miles away.

Process 4: Use a random digit to represent an outcome, with 7, 8 and 9 representing a person who lives more than 20 miles away and 0, 1, 2, 3, 4, 5 and 6 representing a person who does not live more than 20 miles away.

2. For each process described in Item 1 that you said could not be used, explain why you said this.

Lesson 23-2

3. Suppose that 60% of the teachers in a large school district plan to travel over the winter break. A probability experiment consists of selecting a teacher from this district at random and determining if that teacher plans to travel over winter break. If you were going to simulate this probability experiment using a random digit table, how would you assign digits to the two outcomes *plans to travel* and *does not plan to travel*?

4. Suppose that 30% of the adult residents of a large city donate money to a charity during the month of December. You wonder how likely it is that more than half of a group of 10 randomly selected residents would have donated money to a charity during the previous December. You decide to carry out a simulation to estimate this probability. Using a random digit to represent one randomly selected resident, you decide to use 1, 2 and 3 to represent a person who donated and 0, 4, 5, 6, 7, 8 and 9 to represent a person who did not donate. A group of 10 random digits will represent the 10 randomly selected residents.

a. One group of 10 random digits is shown below. Use *D* to stand for *donated* and *N* to stand for *did not donate* to translate these digits into a simulated outcome.

1 2 8 5 1 8 0 2 6 8

b. How many of the 10 in this simulated outcome donated? Did more than half donate?

c. Below are 20 groups of 10 random digits. Translate each into a simulated outcome and indicate how many donated for each group of 10.

7960323437	2765708474
0097926919	5778989670
9568551023	5423785394
1554531421	9458264639
3147891746	0648225841
0488453882	3187471549
0654288233	7436697726
0877147658	0869477704
3612939234	4296869289
4819503479	6594533331

d. What is the estimated probability for more than half of the group to donate?

Lesson 23-3

5. Explain how you might select a person at random from the students in your math class.

6. A brown paper bag contains 60 plastic chips. Each chip has a different number, and the chips are numbered from 1 to 60. Suppose that a chip is selected at random from this bag.

a. What is the probability that the selected number is divisible by 5?

b. Give an example of an event that has probability $\frac{1}{2}$.

c. Give an example of an event that has probability $\frac{1}{3}$.

7. A box contains many colored balls. Some of the balls are yellow. Describe what you would do in order to calculate an *estimated probability* of selecting a yellow ball if a ball is selected at random from this box.

Lesson 23-4

8. Recall the probability experiment of selecting a student at random from Ms. Bailey's seventh grade class. Some events are defined below.

Event	Description
M	Selected student is male
F	Selected student is female
R	Selected student chose read minds as super power
S	Selected student sent more than 60 texts

Use the survey data to find the following probabilities.

a. $P(M) =$

b. $P(R) =$

c. $P(R') =$

d. $P(S') =$

e. $P(M \text{ and } R) =$

f. $P(F \text{ and } R) =$

g. $P(F \text{ and } S) =$

h. $P(M \text{ and } S) =$

9. State the events represented by each complement.

a. R'

b. S'

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

10. A probability experiment consists of tossing five fair coins. You want to estimate the probability that at least four of the coins land heads up. What would be the advantage of using simulation to estimate this probability rather than estimating the probability by observing many outcomes of the actual experiment?

Random Digit Table

Row 1	7 1 1 3 6	4 6 8 8 9	1 9 0 5 4	1 6 7 1 5	1 2 2 6 4
Row 2	6 8 5 7 5	8 8 4 4 9	0 5 6 1 7	5 4 5 7 3	6 7 9 0 8
Row 3	9 3 0 9 8	7 0 5 8 7	5 4 2 3 2	0 1 8 8 1	6 6 8 8 0
Row 4	9 6 7 7 9	6 9 3 1 3	1 9 1 6 8	3 9 0 3 6	8 0 9 0 7
Row 5	1 7 5 4 0	5 9 9 8 2	1 9 7 8 3	9 6 6 0 2	8 6 7 2 4
Row 6	5 5 0 9 4	2 0 6 7 3	6 8 3 3 6	4 1 4 2 4	1 8 4 5 7
Row 7	1 4 5 3 6	1 1 4 1 4	3 9 6 1 1	6 3 0 4 4	6 4 8 8 5
Row 8	1 8 3 8 4	8 0 5 0 4	9 3 8 7 7	7 2 6 3 8	5 4 6 5 2
Row 9	0 0 7 6 6	8 1 1 9 5	6 8 4 1 4	1 3 2 4 2	1 7 3 5 9
Row 10	8 9 9 4 9	2 5 9 0 4	3 8 3 9 1	1 8 0 8 8	2 0 2 9 5
Row 11	8 1 4 2 9	7 4 3 2 7	9 4 5 1 9	9 7 0 7 4	5 7 2 4 3
Row 12	4 4 8 3 1	1 8 5 8 5	3 1 9 5 0	4 4 8 7 9	1 2 9 6 7
Row 13	1 0 1 8 7	0 3 1 9 9	5 5 2 7 5	4 5 3 6 2	6 8 8 8 2
Row 14	2 1 2 5 1	8 1 4 3 0	1 0 8 8 3	2 5 5 3 8	3 5 4 9 1
Row 15	6 4 3 1 0	6 4 3 2 2	7 9 9 7 9	4 1 3 7 6	6 3 2 3 7
Row 16	0 1 0 5 3	0 0 2 1 3	1 1 1 9 8	1 8 4 0 7	7 8 9 7 0
Row 17	8 8 3 6 4	8 5 9 4 5	9 9 6 1 5	4 5 8 4 6	1 2 7 0 6
Row 18	1 0 7 7 7	9 5 8 3 5	8 7 3 5 3	6 4 6 4 9	1 3 2 6 9
Row 19	4 6 4 7 6	7 8 5 7 9	3 2 1 0 3	1 2 8 5 1	8 0 2 6 8
Row 20	9 4 0 3 5	9 4 9 9 6	5 5 9 2 2	9 5 1 1 5	1 2 1 4 7
Row 21	8 8 4 2 7	5 1 9 1 1	1 4 0 9 8	3 1 6 4 0	6 3 0 0 3
Row 22	7 2 6 3 1	3 2 4 2 3	2 2 6 9 9	9 2 1 4 5	8 3 6 7 6
Row 23	7 6 3 8 4	6 5 0 7 8	7 4 1 0 9	3 6 9 5 3	6 1 0 9 6
Row 24	9 9 3 2 4	3 0 4 8 8	0 4 7 1 4	5 3 9 0 4	2 4 9 1 8
Row 25	9 7 9 9 2	6 4 8 4 0	6 0 6 1 5	6 9 9 2 2	3 7 4 0 3
Row 26	7 7 1 0 6	6 8 6 7 2	2 6 9 6 8	3 9 9 5 2	8 8 2 3 8
Row 27	6 2 4 3 2	2 7 9 3 5	1 0 0 7 7	6 5 9 7 2	2 8 8 7 1
Row 28	8 5 3 6 4	7 8 3 1 2	9 4 3 2 7	4 2 4 1 2	8 4 1 9 7
Row 29	5 1 5 7 8	9 1 9 7 5	8 9 1 4 3	7 1 6 9 4	3 6 4 5 5
Row 30	3 5 8 7 0	1 0 8 3 3	5 9 7 6 4	2 2 4 6 0	8 7 1 3 4

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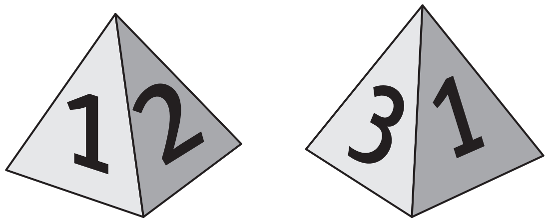
Probability and Simulation
FLIPPING COINS AND RANDOM CHOICES

Write your answers on notebook paper or grid paper. Show your work.

Use the information below to answer Items 1–4.

A probability experiment consists of flipping a fair coin and rolling a four-sided number pyramid that has the numbers 1, 2, 3 and 4 written on its sides. The outcome of a roll of a four-sided number pyramid is the number on the face that lands on the bottom. For example, for the four-sided number pyramid shown below (in two views, so that you can see the three sides that are not on the bottom), the outcome of the throw would be a 4.



Front View **Side View**

One outcome for this probability experiment is (H, 1) which represents a head on the coin and a 1 on the number pyramid.

1. List all of the possible outcomes for this probability experiment. How many different possible outcomes are there?
2. Construct a table with two rows and four columns that shows the different possible outcomes.
3. Construct a tree diagram that shows the possible outcomes for this probability experiment.
4. Explain why the table and the tree diagram are both ways of representing the sample space for this experiment.

Use this information below to answer Items 5–9.

A middle school has four teachers who teach seventh-grade math. Ms. Roy and Mr. Daly each teach two sections of seventh-grade math, and Ms. Kotz and Mr. Teague each teach three sections of seventh-grade math. A probability experiment consists of selecting one of the four teachers at random and then selecting one of that teacher's sections of seventh-grade math at random. One possible outcome for this experiment is (Roy, First Section).

5. Construct a tree diagram that shows all of the outcomes for this probability experiment.
6. How many different outcomes are in the sample space of this probability experiment?
7. Add appropriate probabilities to the branches of your tree diagram.
8. Multiply across the appropriate branches to obtain a probability for each outcome.

9. Are the outcomes equally likely? If not, give an example of two outcomes that do not have the same probability.

10. A probability experiment consists of selecting a chip at random from a box that contains 40 chips. There are 30 red chips in the box and 10 green chips in the box. Allan plans to carry out a simulation of this probability experiment. He will use a random digit to represent a selection and if the digit is a 1, 2 or 3, the simulated outcome is red. If the random number is not 1, 2 or 3, the simulated outcome will be green. Is Allan's plan a good one? Explain why or why not.

Use the information below to answer Items 11–14.

Suppose that 30% of the students at Laguna Middle School participate in school-sponsored after-school activities. A student will be selected at random and asked whether or not he or she participates in after-school activities.

11. If you want to simulate the probability experiment of selecting a Laguna Middle School student at random and you are just interested in whether the selected student participates in after-school activities or not, which of the following would be a correct plan?

Plan 1: Use a random digit to represent a selection. If the selected digit is 1, 2 or 3, the simulated outcome will be a student who participates in after-school activities, and if the digit is 0, 4, 5, 6, 7, 8 or 9, the simulated outcome will be a student who does not participate in after-school activities.

Plan 2: Use a random digit to represent a selection. If the selected digit is even, the simulated outcome will be a student who participates in after-school activities, and if the digit is odd, the simulated outcome will be a student who does not participate in after-school activities.

12. Using the correct plan from Item 11, translate the random digits below into five simulated outcomes for this probability experiment. How many of the five simulated outcomes were students who participated in after-school activities?

4	1	3	7	6

13. Now consider the probability experiment of selecting a student at random five times and counting how many of the five students selected participate in after-school activities. Translate the random digits below into 10 simulated outcomes. (In other words, look at 10 sets of five selections and count the number of students who participate in after-school activities for each set of five. Use the correct plan from Item 11.)

Use the simulated outcomes to estimate the probability that three or more students out of five randomly selected participate in after-school activities.

9	6	7	7	9	6	9	3	1	3	1	9	1	6	8	3	9	0	3	6
1	7	5	4	0	5	9	9	8	2	1	9	7	8	3	9	6	6	0	2
5	5	0	9	4	2	0	6	7	3										

14. Now consider a probability experiment that consists of selecting students at random until you get a student who participates in after-school activities. Using the random digits below and the correct assignment of digits from Item 11, how many students would be selected to complete the *first* simulated outcome?

8	0	2	6	8	1	2	7	0	6

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)	• Clear and accurate understanding of listing outcomes and probabilities of experiments, and of conducting simulations.	• Adequate understanding of listing outcomes and probabilities of experiments, and of conducting simulations.	• Difficulty with listing outcomes and probabilities of experiments, and with conducting simulations.	• Incorrect or incomplete understanding of outcomes and probabilities of experiments and of simulations.
Problem Solving (Items 7, 8, 12, 13, 14)	• Accurately and precisely interpreting an experiment or simulation.	• Interpreting an experiment or simulation.	• Difficulty interpreting an experiment or simulation.	• No understanding of interpreting an experiment or simulation.
Mathematical Modeling / Representations (Items 2, 3, 4, 5, 11, 12, 13, 14)	• Accurately using tables and trees to represent outcomes, and simulations to model experiments.	• Adequate use of tables and trees to represent outcomes, and simulations to model experiments.	• Difficulty using tables and trees to represent outcomes, and simulations to model experiments.	• Inaccurate or incomplete use of tables and trees to represent outcomes, and simulations to model experiments.
Reasoning and Communication (Items 4, 9, 10)	• Clear and accurate explanation of sample spaces and simulations.	• Adequate explanation of sample spaces and simulations.	• Difficulty explaining sample spaces and simulations.	• An inaccurate explanation of sample spaces and simulations.

Statistics

6

Unit Overview

In this unit, you will begin your study of statistics. You will learn how to select a random sample from a population and how to use data from the random sample to learn about the population. You will also use sample data to compare two populations.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- population
- sample
- census

Math Terms

- sampling variability
- sample mean
- random sample
- population mean

ESSENTIAL QUESTIONS

? Why is it important to select at random when choosing a sample from a population?

? How can sample data be used to learn about a population?

? How can sample data be used to compare two populations?

EMBEDDED ASSESSMENTS

These assessments, following activities 25 and 26, will give you an opportunity to demonstrate your understanding of statistics and your ability to use sample data to draw conclusions.

Embedded Assessment 1:

Random Sampling and Sampling Variability p. 357

Embedded Assessment 2:

Comparing Populations p. 391

UNIT 6

Getting Ready

Write your answers on notebook paper. Show your work.

Use the following information to answer Items 1–6.

Each of the 10 students in Mr. Finn's honors math class was asked how many hours per week he or she spent studying in a typical school week. The responses are given here:

6	9	12	4	7
5	6	10	3	8

1. Calculate the mean and median of this data set.
2. Calculate the first and third quartiles for this data set.
3. Calculate the IQR (interquartile range) for this data set.
4. Draw a box plot for this data set.
5. Draw a dot plot for this data set.
6. Calculate the mean absolute deviation (MAD) for this data set. You can use the table below to help organize your work.

Data value	Distance from the mean
6	
9	
12	
4	
7	
5	
6	
10	
3	
8	

Statistics

ACTIVITY 24

Class Data

Lesson 24-1 Class Data

Learning Targets:

- Determine from what population data has been collected.
- Determine if a data collection is a census.
- Display and analyze data in circle graphs, bar charts, and dot plots.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Summarizing Paraphrasing, Interactive Word Wall

In sixth grade, you collected data about your class and used graphs and statistics (such as the mean and the median) to summarize those data. This was a way to learn about your class, and because it was possible to get data from every student in the class, you could make clear statements about characteristics of your class.

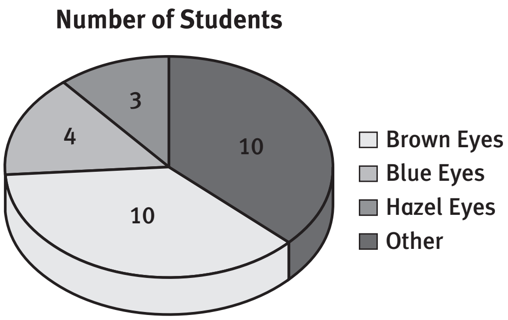
The group that you are interested in learning about is called a ***population.*** When you are able to collect data from every individual in the group you are interested in, you have what is called a ***census*** of that group.

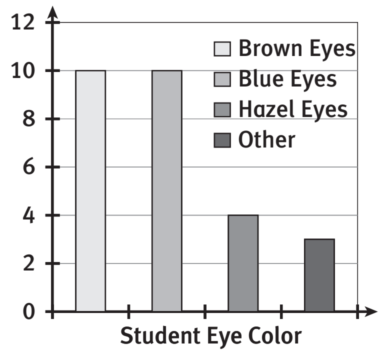
Suppose that you wanted to gather some information about the members of your current class, such as eye color.

1. What is the population in this example?

2. **Reason abstractly.** Explain why choosing five students in your class to survey eye color would not be a census.

Assume that you survey your class and get the following results for eye color: brown eyes, 10; blue eyes, 10; hazel eyes, 4; and other, 3. To analyze this data, you might display it in a circle graph or a bar chart.





3. Look at the circle graph. Calculate the percentage of students with each eye color.

4. Look at the data in the circle graph. Explain whether any of the data is equivalent and why or why not.

My Notes

ACADEMIC VOCABULARY

A ***population*** is the whole group that you are interested in learning about.

A ***census*** is a study where data is collected from everyone in the whole population. The US census takes place every 10 years.

MATH TIP

Equivalent means to have the same value. Fractions, expressions, and ratios are equivalent when they have the same values.

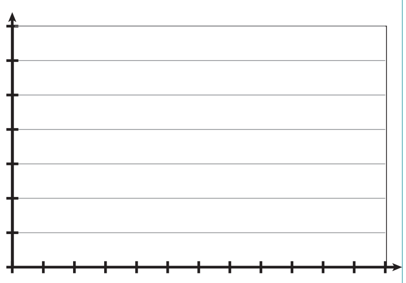
My Notes

MATH TIP

Displaying data visually helps you make **part-to-whole** or **part-to part** comparisons. A part-to-whole comparison is made by comparing one data point to the entire set of data; e.g., by calculating a number as a percentage of the whole. A part-to-part comparison is made by comparing two or more data points.

TECHNOLOGY TIP

If you have access to a computer and programs that create charts, use it to enter your values for birth months and create the pie graph and bar chart.



5. Explain a **part-to-whole** comparison of students with brown eyes to the entire class.

6. Make a **part-to-part** comparison of students with brown eyes and hazel eyes.

7. Now, collect data on your classmates' birth months. Count the number of students in your class born in each month and create a table. Use the My Notes space or notebook paper to record the data for each month.

8. Use the data you collected in Item 6 to determine the number of students born in each month. Create a circle graph, dividing the circle into sectors representing each month. Label each sector with the name of the month and the number of students born in that month.

9. Using the data in your circle graph, make part-to-whole and part-to-part comparisons.

10. Identify whether any data in your circle graph is equivalent and explain why or why not.

11. Suppose that another class in your school had birthdays as follows: January, 3; February, 4; March, 6; April, 3; May, 2; June, 1; July, 2; August, 0; September, 4; October 3; November, 1; and December, 1. Use this data to create a bar chart.

12. Make part-to-part and part-to-whole comparisons of the class data.

13. Look at the data in your bar chart. Is any of the data equivalent? Explain why or why not.

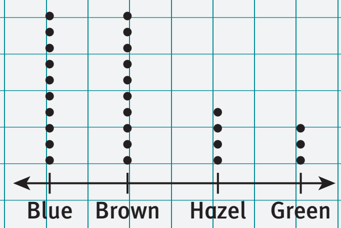
You might also display data in a dot plot. The following dot plot shows the eye colors of a different group of students.

14. Survey your class and list the eye colors of your classmates. Record the number of students with each eye color. Create a dot plot to show this data. Use the My Notes space. Then make part-to-part and part-to-whole comparisons for your data.

15. Look at your class data. Is any of the data equivalent? Explain why or why not.

16. Think of the three methods of displaying data that you have studied. Which method do you think is the most useful, and why? Would your answers change as the amount of your data increases? Explain why.

My Notes



Check Your Understanding

17. Use the following set of data representing the shoe sizes of shoppers who were surveyed to create a bar chart.

$3\frac{1}{2}$	4	$5\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	5	$4\frac{1}{2}$	$3\frac{1}{2}$
$4\frac{1}{2}$	7	5	4	$3\frac{1}{2}$	7	8	$6\frac{1}{2}$

a. What is the most common shoe size?

b. What equivalents are in this set of data?

c. Make part-to-whole and part-to-part comparisons for this bar chart.

18. Survey your class and list their shoe sizes. Make a circle graph and a dot plot showing the sizes.

a. Write a brief summary of what these charts tell you about the shoe sizes of the students in your class.

b. Include part-to-whole and part-to-part comparisons in your summary. Describe any equivalent data.

LESSON 24-1 PRACTICE

19. **Make sense of problems.** Matt decides to collect data from students on the tennis team about how many texts they send in a day. He asks all the players who come to Wednesday's practice to check their phones and records their responses. Is this a census? Explain.

20. Describe part-to-whole and part-to-part comparisons of data and give examples of each.

21. Compare the circle graph, bar chart, and dot plots as methods of displaying data. Explain which you think best displays data and why.

22. Describe a population for which you can perform a census, and explain why the census clearly represents that population.

My Notes

Learning Targets:

- Understand that the way a sample is selected is important.
- Understand that random sampling is a fair method for selecting a sample.
- Use the random-number digit table to select a random sample.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

ACADEMIC VOCABULARY

A ***sample*** is a small part that is representative of a whole. In statistics, the sample is the part of the population that we collect data from.

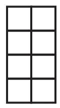
We usually study a sample in order to learn about the population.

Sometimes, instead of collecting data from everyone in the population, we decide to study just a part of the population. For example, instead of collecting data from every student at the school, you might decide to collect data from just 50 students at the school. When we study just a part of the population, the part of the population that we decide to study is called a ***sample.*** The process of choosing a sample from a population is called **sampling**.

You will start your study of sampling by considering a small population. The population consists of 100 middle school students who signed up for a summer reading program. Suppose that we wanted to learn about the average number of books read by these students over the summer.

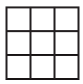
The number of books read by each of the 100 students is represented graphically using rectangles on the Reading Program Data page found at the end of this activity. But don't turn to that page yet! First you need to understand how the data are represented.

Each student is represented by a rectangle that is divided up into small squares. For example, one student is represented by this rectangle:



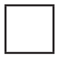
Each small square represents one book, so this rectangle tells us that this student read eight books.

1. What does the following rectangle represent?



My Notes

2. What does the following rectangle represent?



Now, carefully tear out the Reading Program Data page and place it face down on your desk. When your teacher tells you to turn it over, turn it over and look at the population shown on the page. Your teacher will give you a short time to look at the page and think about what the average number of books read by these students is. When you teacher tells you to, turn the page back over and record your guess of the average number of books read for this population in the space provided below.

I think that the average number of books read by the 100 students in the summer reading program is about ________.

Your teacher will provide a poster. Add a dot to the poster to show your guess for the average number of books read. When the dot plot is complete, it will display the guesses for all of the students in the class. Use the class dot plot to answer the following items.

3. Considering all of the guesses from your class, would you say that the guesses are similar or that the guesses showed a lot of variability?

4. After looking at the dot plot, if you had to come up with a single number that would summarize the guesses from the whole class, what number would you use? Is this number close to your original guess?

Rather than guessing at just the population average, you could select a sample from this population and use the data values for the students in the sample to learn about the population. Of course, you would want the students in the sample to be representative of the population.

My Notes

MATH TERMS

The **sample mean** is the average of the data values for a sample.

The **population mean** is the average of the data values for the whole population.

If the sample is selected in a reasonable way, the sample mean can be used as an estimate of the population mean.

Turn over the Reading Program Data page and circle five rectangles that you think are representative of the population. Each of these rectangles represents one of the students in the summer reading program.

5. For each of the five students you selected, count the number of books that each student read (the number of squares that make up the rectangle) and record those data values here:

6. Calculate the mean (average) of the five data values for the students in your sample. This is called the **sample mean**.

Your teacher will provide a poster. Add a dot to the poster to show the mean of the five data values for your sample. When the dot plot is complete, it will display the sample averages for all of the students in the class. Use this dot plot and the earlier dot plot of the class guesses to answer the following items.

7. Considering all of the sample means from your class, would you say that the means were similar or the means showed a lot of variability?

8. **Use appropriate tools strategically.** In what ways are the two dot plots similar? In what ways are they different? Does one of the two dot plots show less variability than the other? If so, which one?

My Notes

9. Reason quantitatively. If you were to do a census of the population, you would find that that actual population mean is 7.5. Based on the two dot plots, does it surprise you that the population mean is 7.5? Explain your thinking.

10. If you really wanted to learn about the average number of books read by students in the summer reading program, do you think that just taking a quick overall look at the population and then making a guess is a good strategy? Did it work well when the students in your class did this?

11. If you wanted to learn about the average number of books read by students in the summer reading program by studying a sample of five students from this population, would picking five students that you think are representative of the population and then calculating the sample mean be a good strategy? Did it work well when the students in your class did this?

12. In general, people are not very good at picking a representative sample! Can you think of another way that you could select five students from this population that you think would be more likely to produce a sample that was representative of the population?

So far, you have seen that first impression estimates and other methods of selecting a sample from a population don't work very well! In this example, even the method of picking students who you thought were representative did not work well, because most people tend to overlook the many small rectangles in the population that correspond to students who read only one or two books. People are just not very good at selecting a sample that is representative of the population—even if they are trying to do a good job!

My Notes

MATH TERMS

A sample that is formed by selecting individuals from the population at random is called a **random sample.**

Instead of letting your own personal ideas or biases influence the way a sample is selected, we need a fair method for selecting a sample. We do this by selecting from the population at random. When individuals in a population are selected at random, the result is a **random sample**.

When individuals are selected at random, every individual in the population has the same chance of being included in the sample. Here are three ways that you could select a random sample of five students from the population of the 100 students in the summer reading program:

Method 1: Write each of the 100 student names on a slip of paper, place all of the slips of paper in a box, mix them well, and then select five slips of paper. The names written on these slips would be the five students you would include in the sample.

Method 2: Number the students in the population from 1 to 100, giving each student a different number. Write each of the numbers from 1 to 100 on a slip of paper, place all of the slips of paper in a box, mix them well, and then select five slips of paper. The students corresponding to the numbers on these slips would be the five students you would include in the sample.

Method 3: Number the students in the population from 00 to 99 (using the two-digit numbers 00, 01, 02, and so on), giving each student a different two-digit number. Using a table of random digits, you could get five two-digit numbers. The students corresponding to these numbers would be the five students you would include in the sample.

13. Which of the three methods described would take the most time to implement? Which would take the least time? Why do you think this?

My Notes

14. Number the rectangles on the Reading Program Data page, starting with the number 00 and ending with the number 99. Make sure that every rectangle gets a number. There are 100 rectangles (corresponding to students) in this population, so you will need to use all of the numbers from 00 to 99.

A table of random digits appears at the end of this activity. This is the same table you used in Activity 23, but it is reproduced here just in case you misplaced that page. Tear out this page.

Toss a paper clip onto the page of random digits. Mark the digit on the page that is closest to the center of the larger loop of the paper clip. Starting with that digit, write 10 digits below, dividing them up into two-digit numbers. (If by chance you get a duplicate two-digit number, just ignore that one and use the next two digits from the table.

My five two-digit numbers:

15. Find the rectangles with these five numbers. For each of these five students, count the number of books that the student read (the number of squares that make up the rectangle) and record those data values here:

16. Calculate the mean of the five data values for the students selected.

17. Is the mean you computed in Item 16 a sample mean or a population mean? Share your response with your group. Incorporate your understanding into your group discussion to confirm your knowledge and use of key mathematical language.

My Notes

Add your sample mean to a class dot plot on a poster that your teacher will provide. This dot plot will display the sample means for the different random samples selected by the students in your class. Use this dot plot and the earlier dot plot of the class means of self-selected samples to answer the following items.

18. Reason abstractly. Considering all of the means from the class random samples, would you say that the means were similar to each other or that the means showed a lot of variability?

19. How does the dot plot of the sample means from random samples compare to the class dot plot of means from the self-selected samples? In what ways are the two dot plots similar? In what ways are they different? Does one of the two dot plots show less variability than the other? If so, which one?

20. If you wanted to learn about the average number of books read by students in the summer reading program by studying a sample of five students from this population, is picking five students at random from the population and then calculating the sample mean a good strategy? Did it work well when the students in your class did this?

21. Suppose that instead of a selecting a random sample of size 5 (five students in the sample), each student in your class had selected a random sample of size 10. If the sample means for these samples were used to make a dot plot, how do you think that this dot plot would be different from the dot plot for samples of size 5? Do you think it would be centered in about the same place? Do you think it would show more or less variability?

22. Why is selecting a random sample a good way to learn about a population?

My Notes

Check Your Understanding

Every year, a large school district holds a spelling bee. This year, 400 students have entered the spelling bee. You want to learn about the mean age of students who enter the spelling bee.

23. What is the population of interest to you?

24. If you wanted to do a census of this population, from how many students would you need to collect data?

25. Describe how you could select a random sample of 20 students from this population.

26. Sarah thinks it will be OK to collect data on just the age from the first 20 students eliminated from the spelling bee and then use the mean age of these students to estimate the mean age of all the students entered in the spelling bee. Explain to Sarah why this is not a good idea.

LESSON 24-2 PRACTICE

This Saturday, the Walk-a-thon to raise money for cancer research will be held. Livia wants to learn about the average amount raised by registered participants in the age range of 11 to 15. For the event, 125 participants have registered for this age group.

27. What is the population of interest to Livia?

28. If Livia wanted to do a census of this population, from how many people would Livia need to collect data?

29. **Make sense of problems.** If all the registered walkers in the age range of 11 to 15 attend, describe how you would select a random sample of 12 from this population.

30. Livia decides to collect data from the last 15 walkers to check in at the walkathon. Explain why this may not be a good idea.

31. After the event, Livia adds up all the donations for the registered participants in the age range 11 to 15 and divides by 125 to find the average amount raised. Is this a census? Explain.

ACTIVITY 24 PRACTICE

1. Explain what it means to say that selecting a random sample from a population is a fair way to select a sample.

2. Write a few sentences describing what you learned in Activity 24 about selecting a sample.

3. You are interested in learning about how many text messages are sent by ninth-grade students at Reseda High School. You select 30 students at random from the ninth graders at Reseda High and ask each one how many text messages they send in a typical day.

a. Are these 30 students the population or a sample?

b. Before your started, a friend suggested that you collect data from the 30 ninth graders in Mr. Rossman's physics class. Why is it better to select ninth-grade students at random?

Use the information below to answer Items 4 and 5.

Forty students belong to the robotics club at Morro Middle School. The ages of these students are shown in the table below.

Student	Age	Student	Age	Student	Age	Student	Age
01	12	11	14	21	14	31	12
02	12	12	13	22	14	32	12
03	11	13	13	23	13	33	14
04	12	14	13	24	12	34	13
05	12	15	14	25	13	35	14
06	13	16	12	26	12	36	13
07	14	17	13	27	12	37	13
08	12	18	13	28	12	38	12
09	15	19	13	29	11	39	14
10	12	20	13	30	13	40	13

Suppose that you did not know the ages of all of these students and that you planned to select a sample of six students and use the sample mean to estimate the mean age of the students in the robotics club.

4. Explain why it would not be a good idea to just pick the first six students on the list of students in the table above.

5. Below is a sequence of random digits, arranged in groups of two. Use this list to select a random sample of six students from the students in the robotics club. Ignore any two-digit numbers in the list that are greater than 40. If you come across a two-digit number you have already used, skip it and go on to the next two-digit number.

71	92	07	53	88	80	28	58	89	29
92	95	96	67	72	34	45	21	38	53
86	30	30	93	07	30	92	88	70	53

a. Which students are in the sample?

b. What are the ages of the students in the sample?

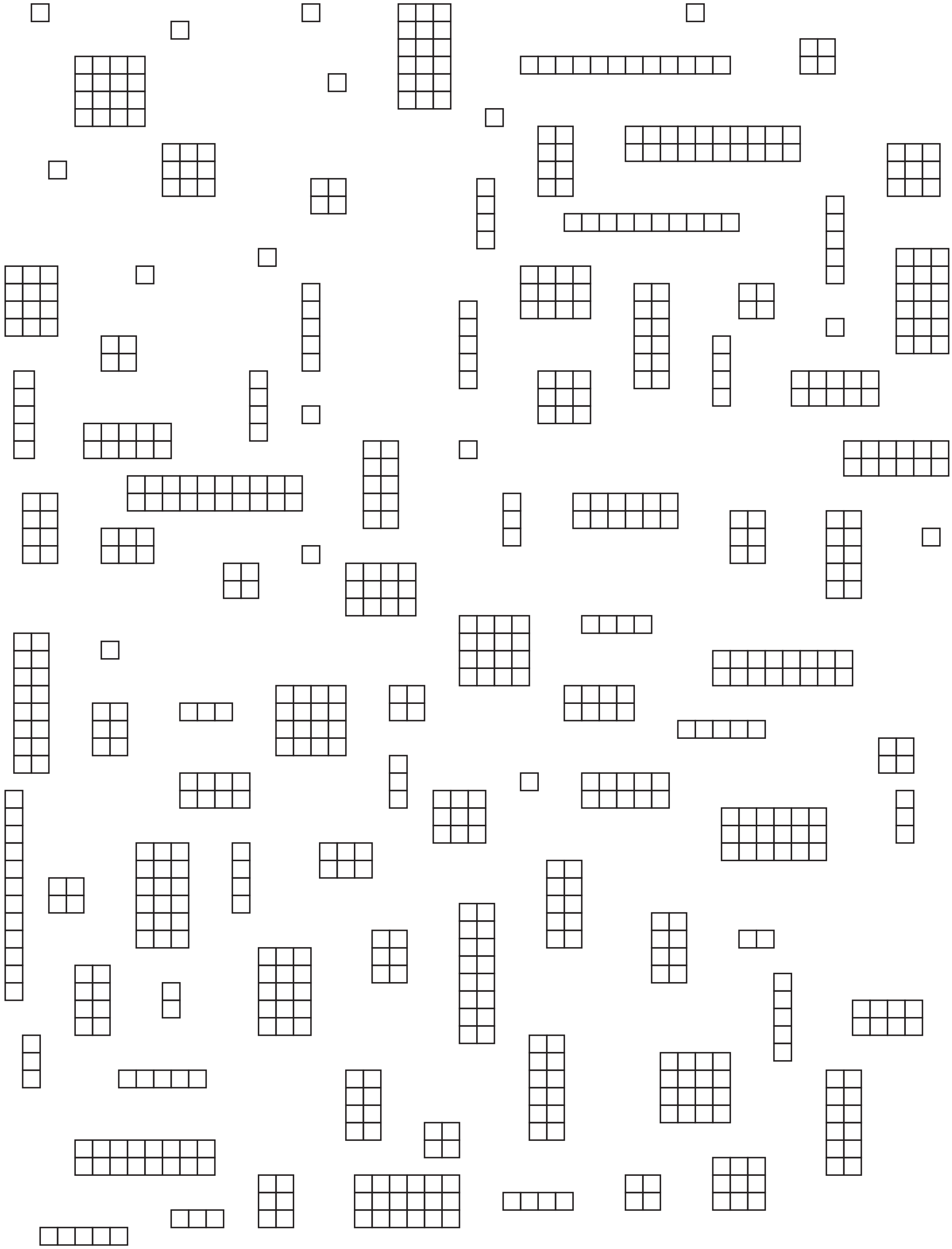
c. What is the mean of these six ages?

d. Is the mean you calculated in part c a population mean or a sample mean?

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

6. Why is it better to select a random sample than to just decide who will be in the sample by picking people that you think will be representative of the population?

Reading Program Data
Number of Books Read for 100 Students in Summer Reading Program



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Random Digit Table

Row 1	7 1 1 3 6	4 6 8 8 9	1 9 0 5 4	1 6 7 1 5	1 2 2 6 4
Row 2	6 8 5 7 5	8 8 4 4 9	0 5 6 1 7	5 4 5 7 3	6 7 9 0 8
Row 3	9 3 0 9 8	7 0 5 8 7	5 4 2 3 2	0 1 8 8 1	6 6 8 8 0
Row 4	9 6 7 7 9	6 9 3 1 3	1 9 1 6 8	3 9 0 3 6	8 0 9 0 7
Row 5	1 7 5 4 0	5 9 9 8 2	1 9 7 8 3	9 6 6 0 2	8 6 7 2 4
Row 6	5 5 0 9 4	2 0 6 7 3	6 8 3 3 6	4 1 4 2 4	1 8 4 5 7
Row 7	1 4 5 3 6	1 1 4 1 4	3 9 6 1 1	6 3 0 4 4	6 4 8 8 5
Row 8	1 8 3 8 4	8 0 5 0 4	9 3 8 7 7	7 2 6 3 8	5 4 6 5 2
Row 9	0 0 7 6 6	8 1 1 9 5	6 8 4 1 4	1 3 2 4 2	1 7 3 5 9
Row 10	8 9 9 4 9	2 5 9 0 4	3 8 3 9 1	1 8 0 8 8	2 0 2 9 5
Row 11	8 1 4 2 9	7 4 3 2 7	9 4 5 1 9	9 7 0 7 4	5 7 2 4 3
Row 12	4 4 8 3 1	1 8 5 8 5	3 1 9 5 0	4 4 8 7 9	1 2 9 6 7
Row 13	1 0 1 8 7	0 3 1 9 9	5 5 2 7 5	4 5 3 6 2	6 8 8 8 2
Row 14	2 1 2 5 1	8 1 4 3 0	1 0 8 8 3	2 5 5 3 8	3 5 4 9 1
Row 15	6 4 3 1 0	6 4 3 2 2	7 9 9 7 9	4 1 3 7 6	6 3 2 3 7
Row 16	0 1 0 5 3	0 0 2 1 3	1 1 1 9 8	1 8 4 0 7	7 8 9 7 0
Row 17	8 8 3 6 4	8 5 9 4 5	9 9 6 1 5	4 5 8 4 6	1 2 7 0 6
Row 18	1 0 7 7 7	9 5 8 3 5	8 7 3 5 3	6 4 6 4 9	1 3 2 6 9
Row 19	4 6 4 7 6	7 8 5 7 9	3 2 1 0 3	1 2 8 5 1	8 0 2 6 8
Row 20	9 4 0 3 5	9 4 9 9 6	5 5 9 2 2	9 5 1 1 5	1 2 1 4 7
Row 21	8 8 4 2 7	5 1 9 1 1	1 4 0 9 8	3 1 6 4 0	6 3 0 0 3
Row 22	7 2 6 3 1	3 2 4 2 3	2 2 6 9 9	9 2 1 4 5	8 3 6 7 6
Row 23	7 6 3 8 4	6 5 0 7 8	7 4 1 0 9	3 6 9 5 3	6 1 0 9 6
Row 24	9 9 3 2 4	3 0 4 8 8	0 4 7 1 4	5 3 9 0 4	2 4 9 1 8
Row 25	9 7 9 9 2	6 4 8 4 0	6 0 6 1 5	6 9 9 2 2	3 7 4 0 3
Row 26	7 7 1 0 6	6 8 6 7 2	2 6 9 6 8	3 9 9 5 2	8 8 2 3 8
Row 27	6 2 4 3 2	2 7 9 3 5	1 0 0 7 7	6 5 9 7 2	2 8 8 7 1
Row 28	8 5 3 6 4	7 8 3 1 2	9 4 3 2 7	4 2 4 1 2	8 4 1 9 7
Row 29	5 1 5 7 8	9 1 9 7 5	8 9 1 4 3	7 1 6 9 4	3 6 4 5 5
Row 30	3 5 8 7 0	1 0 8 3 3	5 9 7 6 4	2 2 4 6 0	8 7 1 3 4

Exploring Sampling Variability

Sample Speak

Lesson 25-1 Sample Statistic and Sampling Variability

My Notes

Learning Targets:

- Understand the difference between variability in a population and sampling variability.
- Know that increasing the sample size decreases sampling variability.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

In Activity 24, each student in your class selected a random sample of size $n = 5$ from the population of the 100 students in a summer reading club. For this population, the mean (average) number of books read was ____.

MATH TIP

The letter n is used to represent the sample size.

1. When you selected a sample of size 5 from this population and calculated the sample mean, was the sample mean you found exactly equal to the population mean?

2. **Reason abstractly.** If you were to take a different random sample of five students from this population, do you think you would get a sample mean exactly equal to the population mean? Do you think the sample mean for this new sample would be the same as the sample mean from the first random sample you selected in Activity 24? Explain why you think this.

3. When the students in your class each selected a random sample from the population of summer reading club students, why did everyone *not* get the same value for their sample means?

My Notes

MATH TERMS

A ***sample statistic*** is a numerical value that is calculated using data from a sample.

MATH TERMS

Sampling variability is the variability in the values of a sample statistic that occurs because different samples include different individuals when random samples are selected from a population.

Suppose that you are going to select a random sample from a population, and you are going to calculate a **sample statistic**, such as the mean or the median, for your sample. The value of your sample statistic will vary depending on the sample that you happen to select, because different samples may include different individuals from the population. This sample-to-sample variability is called **sampling variability**. In this activity, you will explore sampling variability and see why it is important to think about sampling variability when you try to use data from a sample to learn about a population. We will start by revisiting the summer reading club population and then move on to investigating a mock election.

Revisiting the Summer Reading Club Population

To complete this section, you will need the numbered Summer Reading Program Data page and the Random Digit Table that you used in Activity 24.

Before you complete Items 4–7 below, your teacher will assign you a sample size of 10, 15 or 20.

My assigned sample size is ______.

Toss a paper clip onto the page of random digits. Mark the digit on the page that is closest to the center of the larger loop of the paper clip. Starting with that digit and taking two digits at a time, write down enough two-digit numbers to select your sample. (For example, if your assigned sample size is 10, you will need 10 two-digit numbers. Ignore any two-digit numbers that are repeats of previous two-digit numbers.) Write your two-digit numbers below.

4. Find the rectangles corresponding to the numbers you obtained from the random digit table. For each of these selected students, determine the number of books read and record those values below.

5. Calculate the sample mean for your sample.

Mean for random sample 1:

My Notes

6. Repeat this entire process to select another random sample, and calculate the sample mean for this sample.

Mean for random sample 2:

7. Are the two sample means you calculated in Items 5 and 6 equal? If not, explain why they are different.

Add your two sample means to a dot plot on a poster that your teacher will provide. Be sure to add your dots to the poster that corresponds to the sample size you were assigned.

8. Take a look at the three dot plots. How are the dot plots for random samples of size $n = 10$, $n = 15$, and $n = 20$ similar? How are they different?

9. Do sample means tend to differ more from one sample to another for random samples of size 10 or for random samples of size 20?

10. **Make sense of problems.** If you were going to select a random sample from the summer reading club population and use the sample mean as an estimate of the population mean, would you expect the sample mean to be closer to the population mean if the sample size was 10 or if the sample size was 20? Explain how the dot plots of the sample means that your class constructed support your answer.

Check Your Understanding

11. For each of the statements below, decide if it describes variability in a population or if it describes sampling variability.

Statement 1: There is variability in the number of books read, because not every student read the same number of books.

Statement 2: The mean number of books read for the students in one random sample of five summer reading club students may be different from the mean for the students in a different random sample.

12. Does sampling variability increase or decrease if you increase the sample size?

LESSON 25-1 PRACTICE

13. The numbers represent number of books read by 20 students in the summer reading program. Calculate the sample mean for the sample.

1 4 4 9 16 5 10 4 10 12 1 10 3 9 1 4 15 5 12 12

14. The two-digit numbers represent rectangles of books read in the summer reading program for a sample of 15 students. Locate the appropriate rectangle on the sheet and record the books read by each student. Then calculate the sample mean for the sample.

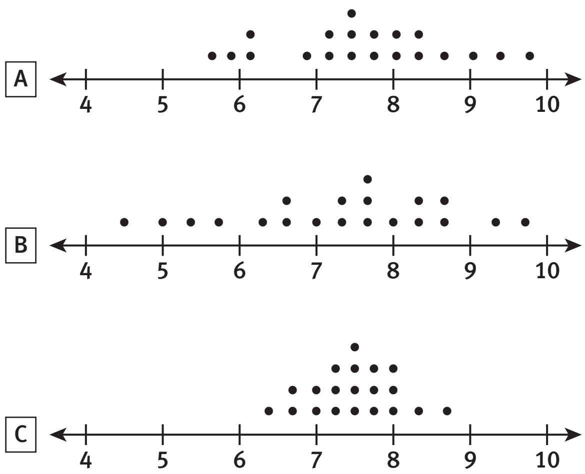
94 03 59 49 96 55 92 29 51 15 12 14 78 84 27

15. Refer to the Random Digit Table to select two-digit random numbers to represent a sample of size 10.

- Record the random numbers in the chart.
- Locate the appropriate rectangle on the sheet to determine how many books were read by each of these students.
- Calculate the sample mean.

My Notes

16. Construct viable arguments. Refer to the three dot plots labeled A, B, and C. If the graphs represent the plots of sample means from samples of size 11, 14, and 18, which dot plot represents which sample size? Explain your reasoning.



17. Think of the population consisting of all of the students who try out for sports teams at your school. For each of the statements below, decide if it describes variability in the population or sampling variability.

Statement 1: If you recorded the number of years each student has been playing sports, there would be variability in these numbers. Not all students have been playing sports for the same number of years.

Statement 2: The mean age of the students in one sample of 10 students would probably be different from the mean age for 10 students in a different sample.

Statement 3: The average height of students who try out for athletic teams will tend to vary from one random sample of 5 students to another.

Statement 4: Not all students weigh the same amount, so there is variability in students' weights.

My Notes

Learning Targets:

- Use data from a random sample to estimate a population characteristic.
- Understand the implications of sampling variability when estimating a population characteristic.
- Use data from a random sample to draw a conclusion about a population.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Predicting Election Results

Suppose that your school has decided to carry out a mock election to see who the students at your school would elect president of the United States. Each student will vote and will choose between the Republican candidate (Candidate 1) and the Democrat candidate (Candidate 2). The school newspaper wants to write an article on the upcoming election and has asked you to predict who will win the election, Candidate 1 or Candidate 2.

1. One way to proceed would be to interview *every* student at your school and ask each one which of the candidates he or she plans to vote for. What is the advantage of this approach? What is one disadvantage of this approach?

Suppose that you decided to select a sample of 20 students from your school. You ask each student which candidate will receive their vote. You calculate the proportion of the students in the sample that plan to vote for Candidate 1 and use this as an estimate of the proportion of all the students at the school who will vote for Candidate 1.

2. The editor of the school paper suggests that you use the 20 students in your first period class for your sample. Explain why this is not a good idea. Share your reasoning with your group members and list any details you may not have considered before. If you do not know the exact words to describe your ideas, use synonyms or request assistance from group members to help you convey your ideas. Use nonverbal clues such as raising your hand to ask for clarification of others' ideas.

My Notes

3. What would be a better way to select the sample?

You plan to select a random sample of 20 students and use the proportion of students in this sample who plan to vote for Candidate 1 to predict the election result. For example, if 12 of the 20 students in the sample say they will vote for Candidate 1, you would estimate that the proportion of students at the school who will vote for Candidate 1 is $\frac{12}{20} = 0.60$, or 60%. Because this is more than 50%, you would predict that Candidate 1 will win the election.

4. Reason quantitatively. If you select a random sample of 20 students and predict the election winner, can you be sure that your prediction will be correct? Explain why or why not.

5. Do you think that the chance of making an incorrect prediction will be greater if the actual percentage of students in the entire population who will vote for Candidate 1 is 20%, or if the actual population percentage is 40%? Explain why you think this.

Your teacher will now assign you to one of five groups. Each group will receive a paper bag containing red and white plastic beads. Each bag has a different proportion of red beads. The proportion of red beads in the bag is written on the bag. Pretend that the beads in the bag represent students at your school. A red bead represents a student who will vote for Candidate 1, and a white bead represents a student who will vote for Candidate 2.

My Notes

Record the percentage who will vote for Candidate 1 for your assigned population below.

Percentage who will vote for Candidate 1: ______

6. For your population, will Candidate 1 win the election?

Work with your group to complete Items 7–14.

7. Select a random sample of 20 "students" from your population and calculate the proportion of students in your sample of 20 who will vote for Candidate 1. (For example, if 7 of the 20 beads in your sample are red, the sample proportion would be $\frac{7}{20} = 0.35$, and you would predict that Candidate 1 would lose the election.) Complete the first row of the table that appears on page 355 at the end of this activity. (Carefully tear this page out of your book).

Place the beads back in the bag and mix up the beads in the bag. Then repeat the process of selecting a random sample 24 more times to complete the rest of the table.

8. How many of your 25 predictions were wrong?

9. Based on the last column of the table you created, was the sample proportion usually close to the actual population proportion?

My Notes

10. For your 25 random samples, answer the following:

a. What was the largest prediction error?

b. What was the smallest prediction error?

c. What was a typical prediction error?

Your teacher will provide each group with a piece of chart paper. Your group should make a poster that provides the following information:

- The actual population proportion who will vote for Candidate 1 for your population
- A dot plot of the sample proportions from the 25 random samples
- The number of samples that led to a prediction that was wrong
- The largest prediction error
- The smallest prediction error
- The value of a typical prediction error

After your group has finished making its poster, ask your teacher where to display it along with the posters from the other groups.

Take a few minutes to look at all of the group posters. Then use them to answer the following items.

11. Use appropriate tools strategically. How are the centers of the dot plots related to the actual population proportions?

12. Is the sample-to-sample variability in the sample proportions about the same in all five of the dot plots?

My Notes

13. Do sample proportions tend to be closer to the actual value of the population proportion when the population proportion is 0.2 or 0.5?

14. Reason abstractly. If the population had consisted entirely of white beads, what would the dot plot of sample proportions for 25 random samples have looked like?

Your teacher will now provide a mystery bag. This bag contains a population of red and white beads and the proportion of red beads in the bag is either 0.2, 0.3, 0.4, 0.5, or 0.6. A student in the class will select a random sample of 20 beads from this population.

For the random sample, record the following:

Sample size:
Number of red beads:
Sample proportion of red beads:

15. Based on the sample proportion, would you rule out any of the proportions 0.2, 0.3, 0.4, 0.5, or 0.6 as possibilities for the population proportion for the mystery bag? Explain your reasoning. (*Hint:* you may want to look at the posters that your class created.)

16. Which of 0.2, 0.3, 0.4, 0.5, or 0.6 do you think is the actual value of the population proportion of red beads for the mystery bag? Explain your reasoning.

My Notes

Check Your Understanding

17. Suppose that you are going to select a random sample of students from your school, measure the heights of the students in your sample, and then find the mean height for your sample. There are many different sets of students who could end up forming your sample. Will the sample mean be the same for all possible samples? If so, explain why. If not, what is the phrase used for the fact the sample means will vary according to the sample selected?

18. Return to the context of the previous item. The sample mean will vary according to what sample you happen to select. If you want to reduce this variation, would you choose to select a sample of size 5 or a sample of size 25?

Use the following to answer Items 19 and 20.

Suppose that 60% of the students at your school favor a proposed school policy that would ban cell phones at school. Also suppose that every student in your class selected a different random sample of 50 students from your school and calculated the proportion of students in the sample who favor the ban.

19. If your class constructed a dot plot of all these sample proportions, where would you expect the dot plot to be centered?

20. If every student had selected a random sample of size 25 instead of a sample of size 50, would you expect a dot plot of the sample proportions to be more spread out or less spread out than in a dot plot of sample proportions for samples of size 50?

My Notes

LESSON 25-2 PRACTICE

Using a deck of standard playing cards, remove the jokers and the red twos, threes, fours, fives, sixes, and sevens. Shuffle (mix) the cards remaining in the deck thoroughly. Select samples without looking at the type of card being selected. Return the cards in the sample to the deck and shuffle thoroughly before selecting a new sample. (If decks of playing cards are not available, you could substitute the random number table using two-digit numbers to represent a selection where selecting 01 to 65 represents choosing a black card and selecting 66 to 00 represents choosing a red card.)

Use this situation to simulate an election in which students are determining whether to travel to an historic site to write about what they learn there or to travel to an amusement park to study the science and mathematics of the rides at the park. Select 20 samples of 10, in which red indicates a vote for the historic site destination and black indicates a trip to the amusement park.

21. Model with mathematics. Create a data chart to collect the results of the twenty samples. Use the following headings:
- Sample Number
- Number of Votes for the Amusement Park (black)
- Proportion of Votes in the Sample for the Amusement Park (black)
- Prediction (Win, Lose, Tie for the Amusement Park Trip)
- Prediction Error (Difference Between the Sample Proportion and the Actual Population Proportion, 0.65)

22. Construct a dot plot of the sample proportions from the 20 random samples.

23. How many samples led to a prediction that was wrong?

24. State the largest prediction error.

25. State the smallest prediction error.

26. What is the value of a typical prediction error?

ACTIVITY 25 PRACTICE

1. In your own words, explain the difference between variability in a population and sampling variability.

2. Think of the population consisting of all of the teachers who work in your school district. For each of the statements below, decide if it describes variability in the population or sampling variability.

   Statement 1: If you recorded the number of years each of the teachers has been teaching, there would be variability in these numbers. Not all teachers have been teaching for the same number of years.

   Statement 2: The mean age of the teachers in one sample of 10 teachers would probably be different from the mean age for 10 teachers in a different sample.

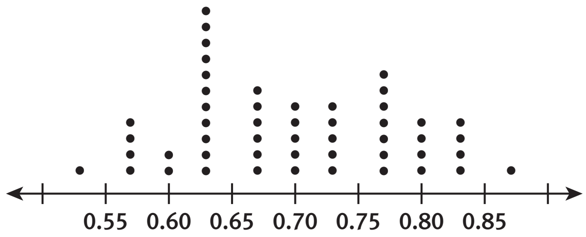
   Statement 3: The proportion of teachers who use public transportation to get to school will tend to vary from one random sample of teachers to another.

   Statement 4: Not all teachers are the same age, so there is variability in teachers' ages.

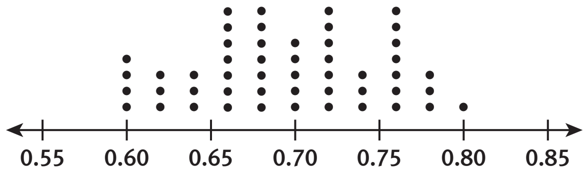
3. Devon selected 50 different random samples of students from her school. For each sample, she determined the proportion of students who prefer rock music to rap music. Frank selected 50 different random samples of students from the same school and determined the proportion who prefer rock music for each of his samples. They produced the dot plots shown below.

   Devon used random samples of size 30. All of Frank's random samples had the same sample size. Do you think that Frank used a sample size that is less than 30, equal to 30, or greater than 30? Explain why you think this.

Devon's sample proportions:

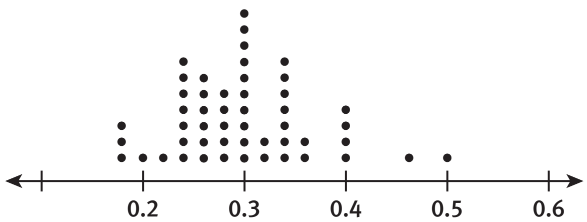


Frank's sample proportions:

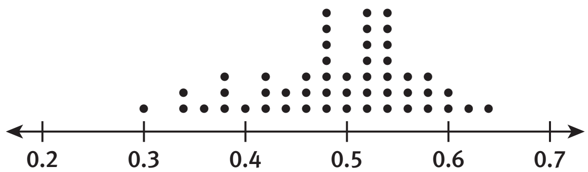


4. Two populations each consist of 1000 beads. In one population, 30% of the beads are red. In the other population, 50% of the beads are red. Many random samples of size 50 were selected from each population, and dot plots were drawn showing the proportions of red beads in the samples. The dot plots are shown below.

   Sample proportions for random samples from a population with 30% red beads:

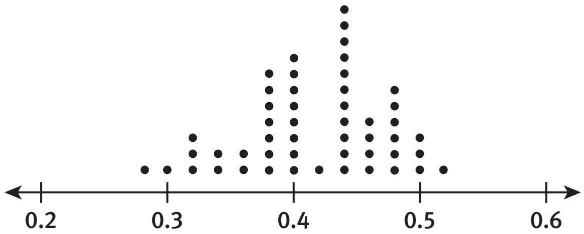


   Sample proportions for random samples from a population with 50% red beads:



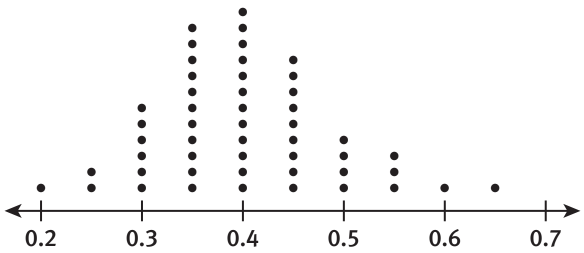
Based on these plots, if you use the sample proportion from a random sample of size 50 as an estimate of the population proportion, do you think your estimate would be closer to the actual value of the population proportion when the population proportion is 0.3 or when the population proportion is 0.5? Explain your choice.

5. Below is a dot plot that shows the sample proportions of red beads for random samples of size 50 from a population of beads that has 40% red beads.

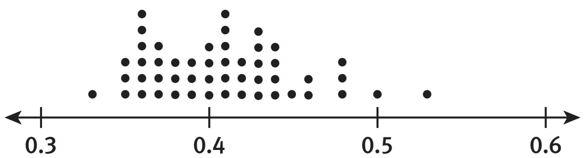


Which of the following do you think is a dot plot of the proportions of red beads for random samples of size 20? Explain your choice.

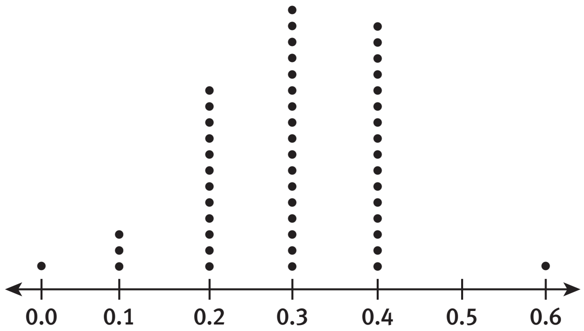
Dot Plot 1



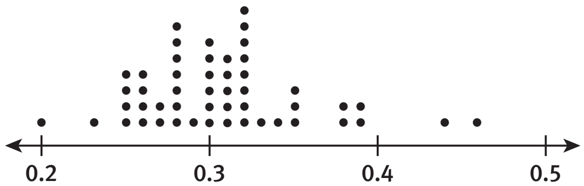
Dot Plot 2



Dot Plot 3



Dot Plot 4



6. Which of the four dot plots from the previous item is most likely to have been generated using sample means from a sample size greater than 50? Explain.

MATHEMATICAL PRACTICES
Model with Mathematics

7. In this activity, you simulated selecting a random sample of 20 students from a population by selecting 20 beads from a bag that contains red beads and white beads. Suppose that 40% of a large population of students would vote for Candidate 1 in a mock election. Can you think of a way to use a random digit table to simulate selecting the random sample that would be used instead of selecting beads from a bag? Describe how you would do this.

Table for Activity 25

Sample	Number in Sample Who Will Vote for Candidate 1	Proportion in Sample Who Will Vote for Candidate 1	Prediction (Candidate 1—Win, Lose or Tie)	Prediction Error Difference Between Sample Proportion and Actual Population Proportion (Sample Proportion Minus Population Proportion)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				

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Random Sampling and Sampling Variability
SCHOOL POPULATIONS

Write your answers on notebook paper or grid paper. Show your work.

1. Each of the following describes a method for selecting a sample of 10 students from the students at your school. For each method described, decide if it would result in a random sample. For purposes of this item, suppose that there are 1000 students at your school and that you have a list of all 1000 student names.

   Method 1: Write all 1000 student names on slips of paper. Put these slips in a box, mix them well, and then draw out 10 names.

   Method 2: Number the students on the list from 000 to 999. Then use the first 10 students on the list.

   Method 3: Number the students on the list from 000 to 999. Then use a table of random digits to obtain 10 different blocks of three digits. Use these 10 three-digit numbers to identify which students to select.

   Method 4: Use the first 10 students that arrive at school tomorrow.

   Method 5: Use the 10 students who make up the girls' volleyball team at your school.

2. Of the methods described in Item 1 that would result in a random sample, which one do you think would take the least time to implement? Explain why you think this method would be the quickest.

Random Sampling and Sampling Variability

SCHOOL POPULATIONS

Write your answers on notebook paper or grid paper. Show your work. Use the following information to answer Items 3–6.

A population consists of the 1000 students enrolled at Morro Bay High School. Some students live very close to the school and others live farther away. Students who live more than three miles from the school ride the bus to school.

3. You ask each student at the school how far away from school they live and record these numbers. Would there be variability in the distances? Is this variability in a population or is it sampling variability?

4. Fran and Zoe each selected a different random sample of students from this population. They each recorded how far from school the students in their sample lived and calculated the sample mean distance from school. Would you expect the two sample averages to be equal or would you expect them to differ? Is this variability in a population or sampling variability?

5. Fran selected a random sample of 20 students and Zoe selected a random sample of 50 students. Do you think that Fran's sample mean or Zoe's sample mean would be closer to the actual mean distance for the whole population? Explain why you think this.

6. Can you be *certain* that the sample mean that you picked in Item 5 will be the one that is closer? Explain.

Random Sampling and Sampling Variability

SCHOOL POPULATIONS

Write your answers on notebook paper or grid paper. Show your work. Use the following information to answer Items 7–10.

A population consists of 600 girls and 400 boys. Jose wanted to investigate what he could expect to happen if he were to take a random sample of 20 people from this population and calculate the proportion of girls in the sample.

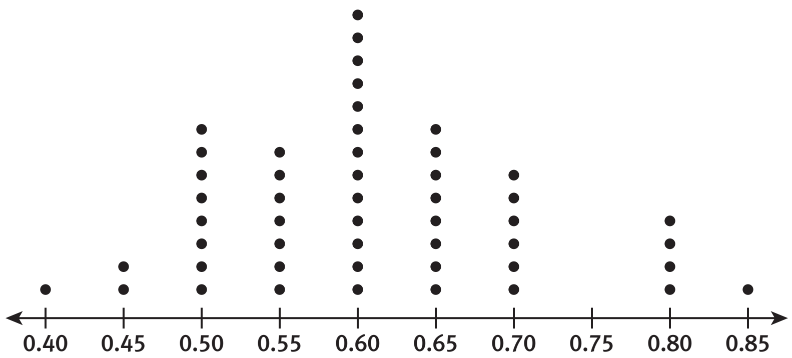
Jose selected 20 students at random and recorded the following data (B = boy and G = girl):

G	G	B	B	G	B	G	B	B	B
G	B	G	G	G	B	G	B	B	G

7. What is the proportion of girls in Jose's sample?

8. Jose's sample proportion was not equal to 0.6, even though 60% of the people in the population are girls. Does this mean that Jose did something wrong when he selected the sample? Explain.

9. Jose decided to take more random samples from this population. He selected 50 different random samples of 20 students. For each of these samples, he calculated the proportion of girls in the sample. A dot plot of Jose's sample proportions is shown below. Did any of Jose's samples result in a sample proportion that was different from the actual population proportion of girls by more than 0.2?



10. There are 1000 students at Jose's school. He selects a random sample of 20 students from his school and six of the students in the sample are girls. Do you think that the proportion of girls at Jose's school is 0.6? Explain why or why not. (*Hint:* Think about the dot plot in Item 9.)

Random Sampling and Sampling Variability

SCHOOL POPULATIONS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 3, 4, 5, 6, 7, 8, 9, 10)	• Clear and accurate understanding of populations, population variability, and sampling variability. • Effective understanding and accuracy in making predictions and drawing conclusions about a population.	• A functional understanding of populations, population variability, and sampling variability. • Making predictions and drawing conclusions about a population that are largely correct.	• Partial understanding of populations, population variability, and sampling variability. • Partially correct predictions and conclusions about a population.	• Inaccurate or incomplete understanding of populations, population variability, and sampling variability. • Inaccurate or incomplete predictions and conclusions about a population.
Problem Solving (Items 7, 9)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1, 3, 4, 5, 6, 8, 9, 10)	• Clear and accurate understanding of sampling a population and obtaining random samples.	• An understanding of population samples and random samples that is largely correct.	• Partial understanding of population samples and random samples.	• Inaccurate or incomplete understanding of population samples and random samples.
Reasoning and Communication (Items 2, 3, 4, 5, 6, 8, 9, 10)	• Precise use of appropriate math terms and language to explain sampling methods, variability, and predictions.	• An adequate explanation of sampling methods, variability, and predictions.	• A misleading or confusing explanation of sampling methods, variability, and predictions.	• An incomplete or inaccurate explanation of sampling methods, variability, and predictions.

Comparative Statistics

Seventh-Grade Students

Lesson 26-1 Two Sample Means

My Notes

Learning Targets:

- Compare the means of two numerical samples.
- Understand that a *meaningful* difference between two sample means is one that is greater than would have been expected due to sampling variability alone.
- Use data from random samples to compare populations.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations, Summarizing, Paraphrasing, Interactive Word Wall

In this activity, you will spend some time learning about a population of seventh-grade students. The population you will be considering is a large group of seventh graders who participated in the Census at School project.

The Census at School project is an international program to help students develop the ability to think statistically. Students from many countries have participated, including students from the United States, Canada, the United Kingdom, Ireland, Japan, Australia, New Zealand, and South Africa. These students have all completed an online survey to provide information about themselves and their homes, schools, and social lives. Some questions on the survey are asked only in a particular country, but there are 13 questions that are included on the survey in all of the countries that participate in Census at School.

In this activity, you will be looking at random samples from the population of Census at School seventh graders. Because not all seventh graders participate in Census at School, keep in mind that this population is *not* the same as the population of *all* seventh graders. This means that any conclusions you make based on sample data will only be about the population of Census at School seventh graders.

Let's start by focusing only on Census at School seventh graders in the United States. One question on the Census at School survey asked students how many minutes it takes them to travel to school.

My Notes

MATH TIP

Remember that the **sample mean** is the average of the data values, and the **sample median** is the middle data value in a list that has been ordered from smallest to largest. (If there is an even number of data values, the median is the average of the middle two values.)

MATH TIP

Remember that the **first quartile** is the median of the lower half of the data, and the **third quartile** is the median of the upper half of the data.

The **interquartile range** is the distance between the first and third quartiles.

The U.S. Census at School website (www.amstat.org/censusatschool) has a random sampler that will select a random sample of students from the U.S. Census at School population. The random sampler was used to select a random sample of 20 seventh graders. The data on time to travel to school for these twenty students are shown here:

Sample 1: Time to travel to school (in minutes)

5	7	10	10	15	15	15	15	20	20
20	20	20	20	30	40	45	60	60	60

The 20 times are arranged in order from shortest to longest.

1. For Sample 1, calculate the following:
a. the sample mean

b. the sample median

2. For Sample 1, find the first and third quartiles and the interquartile range (IQR).

3. Draw a box plot for the data of Sample 1.

My Notes

A second random sample (Sample 2) of 20 students is selected from the population of U.S. Census at School seventh graders. The travel times for these 20 students (arranged in order from shortest to longest) are:

Sample 2: Time to travel to school (in minutes)

5	7	8	10	10	15	15	20	20	20
20	20	30	30	30	40	45	55	60	60

4. For Sample 2, calculate the following:

a. the sample mean

b. the sample median

c. The first and third quartiles

d. The interquartile range

5. Model with mathematics. Draw a box plot for the data of Sample 2.

My Notes

6. Does it surprise you that the box plot for Sample 1 and the box plot for Sample 2 are not identical? Explain why or why not.

7. What term is used to describe differences in sample statistics for different random samples from the same population?

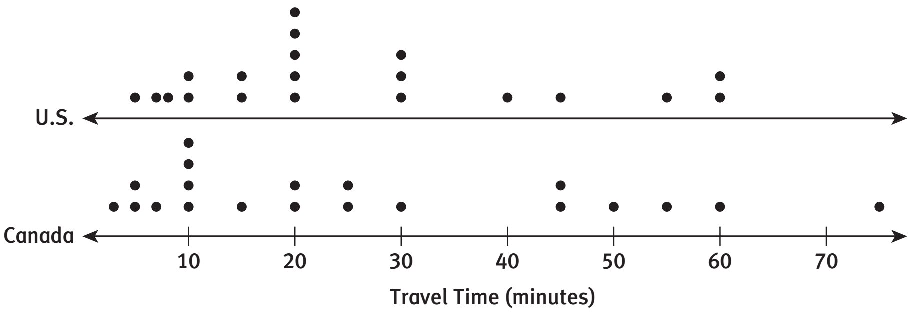
8. Are the mean of Sample 1 and the mean of Sample 2 the same? Is this what you would expect? Explain.

9. Calculate the difference in the two sample means by calculating Sample 1 mean − Sample 2 mean.

Because Sample 1 and Sample 2 were selected from the same population, we know that if there is any difference in the two sample means, it is because of chance differences that occur from one random sample to another. Two random samples from the same population will probably have different sample means. This sampling variability is what makes trying to compare two populations based on sample data tricky! Let's investigate.

My Notes

Suppose that you wanted to know if the average travel time to school for Census at School seventh graders in Canada and the average travel time for Census at School seventh graders in the U.S. are different. A random sample of 20 seventh graders was selected from each of these two populations. Dot plots of the travel times for these samples are shown here:



The sample mean for the U.S. sample is 26.00 minutes, and the sample mean for the Canada sample is 26.25 minutes.

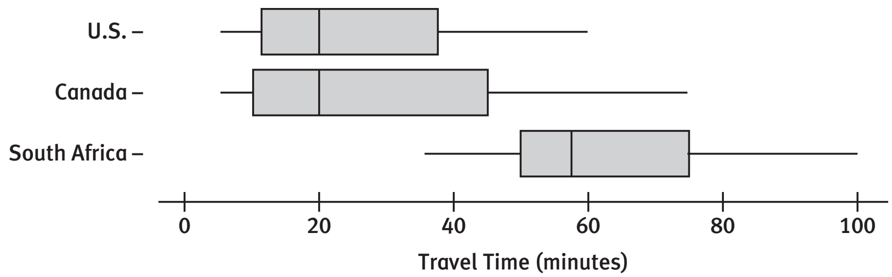
10. Calculate the difference between the U.S. and Canada sample means by calculating U.S. sample mean − Canada sample mean.

11. In Item 9 you calculated the difference in the sample means for Sample 1 and Sample 2, which were random samples from the same population. How does the difference between the U.S. sample mean and the Canada sample mean compare to the difference that you calculated in Item 9?

12. **Reason abstractly.** Based on the U.S. sample mean and the Canada sample mean, do you think it is reasonable to conclude that the U.S. population mean and the Canada population mean are different, or do you think that the two population means might be the same? Explain your reasoning.

My Notes

Take a look at the three box plots shown below. One box plot is from the random sample from the U.S. Census at School population, one is from the random sample from the Canada Census at School population, and one is from a random sample from the South Africa Census at School population.



The sample means are

U.S. sample mean: 26.00 minutes
Canada sample mean: 26.25 minutes
South Africa sample mean: 60.75 minutes

Working with a partner, use the box plots and sample means to answer Items 13–18. Make notes as you listen to your partner. Ask and answer questions clearly to aid comprehension and to ensure understanding of your partner's ideas.

13. How are the box plots for the U.S. sample and the Canada sample similar? How are they different? Are the differences large or small?

My Notes

14. Explain how the box plots and sample means support the following statement:

Based on the U.S. random sample and the Canada random sample, we cannot be sure that the distributions of travel times and the mean travel times for the U.S. population and the Canada population are different.

15. How are the box plots for the U.S. sample and the South Africa sample similar? How are they different? Are the differences large or small?

16. **Construct viable arguments.** Explain how the box plots and sample means support the following statement:

Based on the U.S. random sample and the South Africa random sample, the mean travel times for the U.S. population and the South Africa population are probably different.

17. If there is a lot of overlap in the box plots of random samples from two populations, does this suggest that the difference in the two sample means or medians might be due to sampling variability or does it suggest that the two populations might differ in some important way?

My Notes

18. If there is not a lot of overlap in the box plots of a random sample from each of two populations, does this suggest that the difference in the two sample means or medians might be due to sampling variability or does it suggest that the two populations might differ in some important way?

When comparing two populations based on the means of random samples from the two populations, there are two possibilities to be considered:

Possibility 1: The difference in the sample means is *not meaningful.* The difference is not very big. The two population means might be the same and the sample means may be different only because sample means tend to differ from the population mean (sampling variability).

Possibility 2: The difference in the sample means is *meaningful*. The difference is big enough that we think that it cannot be due to just sampling variability. The two population means are probably different.

At this point, you should be wondering just how big the difference in sample means needs to be for us to choose Possibility 2 and conclude that the population means are different. That is a good question! Completing the rest of this activity will help you to answer this question.

Check Your Understanding

19. Suppose that the mean height for a random sample of 25 U.S. Census at School seventh graders is 163 cm and that the mean height for a random sample of 25 Canada Census at School seventh graders is 165 cm. Explain why this does not tell us that the mean height of *all* U.S. Census at School seventh graders is different from the mean height of *all* Canada Census at School seventh graders.

20. For which of the following cases would it be reasonable to conclude that two population means are different based on the sample means from random samples from the populations?

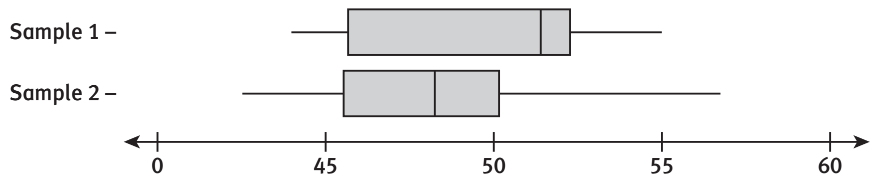
Case 1: The difference in the two sample means is greater than what would be expected due to sampling variability if the population means were equal.

Case 2: The difference in the sample means is consistent with what would be expected due to sampling variability if the population means were equal.

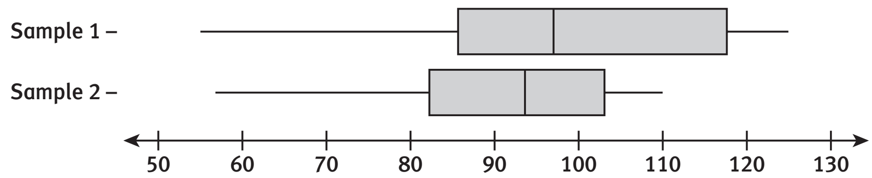
My Notes

21. For which of the following pairs of box plots is it most likely that the two samples were drawn from populations that had different population means? What is it about the box plots that led you to this choice?

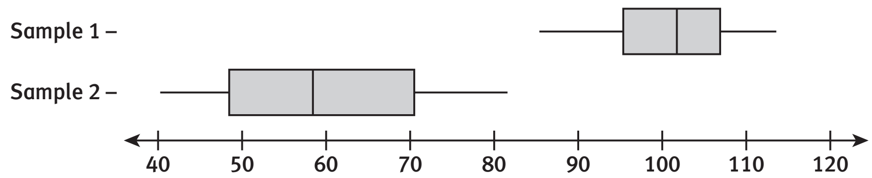
Box Plot Pair 1



Box Plot Pair 2



Box Plot Pair 3



My Notes

LESSON 26-1 PRACTICE

Consider the two samples of data from the McKenzie School. The numbers represent the time in seconds that it took each child to cover a distance of 50 meters.

Girls' Times: 8.3, 8.6, 9.5, 9.5, 9.6, 9.8, 9.9, 9.9, 10.0, 10.0, 10.0, 10.1, 10.3, 10.5

Boys' Times: 7.9, 8.0, 8.2, 8.2, 8.4, 8.6, 8.8, 9.1, 9.3, 9.5, 9.8, 9.8, 10.0, 10.1, 10.3

22. Calculate the sample mean and sample median of each data set.

23. Based on the sample means, do you conclude that the distributions of times from the boys' population and girls' population are different? Explain.

24. Calculate the first quartile and third quartile for each data set.

25. **Model with mathematics.** Draw a box plot for each data set on the same scale.

26. Based on a comparison of the box plots, do you conclude that the population means for the boys' times and girls' times are significantly different? Explain.

My Notes

Learning Targets:

- Compare population means for populations with approximately the same amount of variability.
- Express the difference in the sample means in terms of mean absolute deviation (MAD).
- Draw conclusions about population differences based on sample size and the difference in sample means relative to the MAD.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Create Representations, Summarizing, Paraphrasing, Interactive Word Wall

To decide if a difference in two sample means is meaningful, we need to look at what kinds of differences are typical when the population means are equal. In other words, we need to see what kinds of differences are typical of sampling variability alone.

MATH TIP

A difference in sample means is meaningful if it is bigger than what would be expected from sampling variability alone.

To investigate, we will consider the population of U.S. Census at School seventh-grade boys. Using the Census at School random sampler, a random sample of 10 boys from this population was selected. The number of hours usually spent doing homework each week for each boy in the sample is shown here:

Sample 1 Homework Hours per Week

9	4	2	4	2	2	4	4	3	11

A second random sample of 10 boys from the population resulted in the following 10 homework times:

Sample 2 Homework Hours per Week

9	2	9	4	10	4	1	4	3	7

MATH TIP

Remember that the **MAD (mean absolute deviation)** is the average distance of the observations from the mean of the data set. It is a measure of variability in a data set.

For the entire population of U.S. Census at School seventh-grade boys, the homework time distribution has population mean = 6.0 hours and population MAD = 3.4 hours. These values were calculated using all of the data values in the entire population.

1. Calculate the following:

a. mean of Sample 1:

b. mean of Sample 2:

c. difference in sample means (Sample 1 − Sample 2):

My Notes

MATH TIP

To express a difference in sample means in terms of the MAD, calculate:

$$\frac{\text{difference in sample means}}{\text{MAD}}$$

One way to look at the difference in two sample means is to express this difference in terms of the MAD. Here the population MAD was 3.4. Dividing the difference in sample means by the MAD gives

$$\frac{-0.8}{3.4} = -0.24$$

This tells us that the difference in sample means was 0.24 times the MAD for this population.

Expressing the difference in sample means in terms of the MAD allows us to judge the difference in sample means relative to the variability in the population. This is helpful, because a difference of 1 might be considered small or large depending on the context. For example, a difference of 1 hour is small if we are talking about the amount of time that seventh graders spend at school in a year, but a difference of 1 hour is very large if we are talking about how long seventh graders spend getting ready for school each day!

Here are the times spent on homework per week for two more random samples from the population of U.S. Census at School seventh-grade boys:

Sample 3 Homework Hours per Week

9	11	5	11	12	1	1	6	6	3

Sample 4 Homework Hours per Week

6	9	8	12	11	4	1	7	1	10

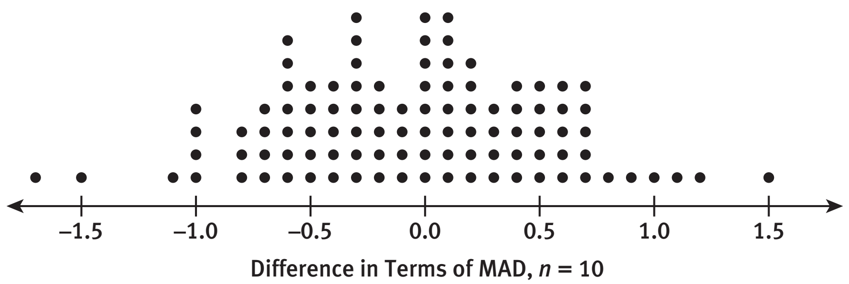
2. Calculate the sample means and the difference in the sample means. Express the difference in sample means in terms of the population MAD.
a. mean of Sample 3:
b. mean of Sample 4:
c. difference in sample means (Sample 3 − Sample 4):
d. difference in sample means in terms of MAD:

3. **Reason quantitatively.** Based on your answer to Item 2, would you be surprised if two different random samples of ten U.S. Census at School seventh-grade boys had sample mean number of hours spent on homework that differed by 0.1 MAD? Explain your thinking.

My Notes

To understand what kind of differences in sample means result from sampling variability alone, we could continue to select pairs of random samples from the population of U.S. Census at School seventh-grade boys and express each difference in sample means in terms of the population MAD. This was done 98 times, so that with the two that you have already looked at (which had differences of − 0.24 MAD and − 0.12 MAD) there were a total of 100 pairs of samples. The first 10 new pairs are shown in the table that follows. All 100 differences were used to make the dot plot that follows the table.

First Sample Mean	Second Sample Mean	Difference	Difference in Terms of MAD
3.80	6.65	−2.85	−0.84
6.07	6.69	−0.62	−0.18
7.59	2.53	5.06	1.49
6.08	6.24	−0.16	−0.05
7.11	4.92	2.19	0.64
4.71	6.66	−1.95	−0.57
4.69	5.29	−0.60	−0.18
4.19	4.34	−0.15	−0.04
7.30	5.10	2.20	0.65
7.38	6.68	0.70	0.21



4. Based on the dot plot above, would you be surprised if two different random samples of ten U.S. Census at School seventh-grade boys had sample means that differed by 0.5 MAD? Explain your thinking.

My Notes

5. Based on the dot plot on the previous page, would you be surprised if two different random samples of ten U.S. Census at School seventh-boys had sample means that differed by 2.0 MAD? Explain your thinking.

Here are two samples of 10 seventh graders. One is a random sample from the population of U.S. Census at School seventh-grade boys. The other sample is a random sample from the population of U.S. Census at School seventh-grade girls. Use these samples to answer Items 6–9.

Boys Homework Hours per Week

10	6	2	3	1	9	11	1	8	3

Girls Homework Hours per Week

3	10	10	13	2	10	13	5	3	5

6. Calculate the following:
a. sample mean for girls:
b. sample mean for boys:
c. difference in sample means (girls − boys):

7. Express the difference in the two sample means in terms of the MAD for the population of seventh-grade boys (the MAD for the population of seventh-grade girls was about the same as the MAD for the population of boys). The MAD for the population of boys was 3.4.

8. Locate the value you just found on the horizontal axis given in the dot plot that appears just before Item 4. Do you think that the difference in the sample mean number of homework hours for girls and boys might be just due to sampling variability? Explain why you think this.

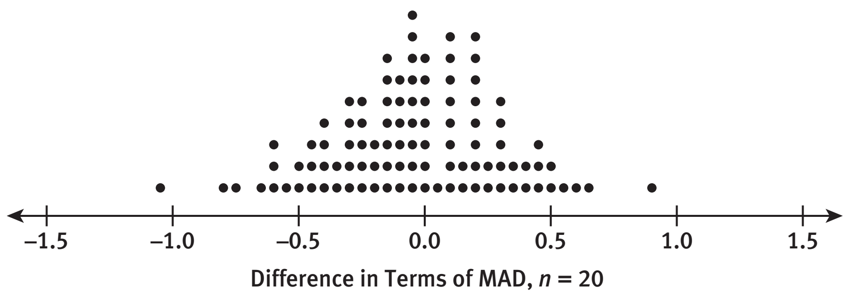
9. Suppose that the difference in sample means for girls and boys had been 2.0 MADs. Would you have answered Item 8 differently? Explain why or why not.

Now take another look at the dot plot of the differences in sample means. Notice that if two random samples of size 10 are taken from the population of U.S. Census at School seventh-grade boys, it is not likely that you would see a difference in sample means of more than 1.5 MADs. This happened in only one out of 100 pairs of random samples.

This suggests that it might be possible to say that if the difference in two sample means is less than 1.5 MADs, the difference might be just sampling variability and the difference is not big enough to conclude that the two samples came from populations with different population means.

While this works for a sample size of 10, will it also work for other sample sizes? Let's see . . .

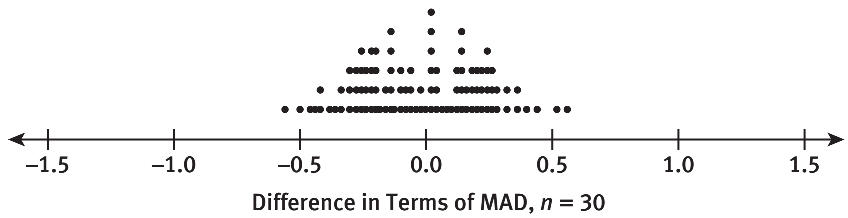
Consider the population that consists of the number of text messages sent in a typical day for the population of U.S. Census at School seventh-grade girls. This population has a MAD of 72. One hundred pairs of random samples of 20 girls were selected from this population, and the differences in the sample means were expressed in terms of the MAD. These values were used to make the dot plot that follows.



My Notes

Compare the dot plot for samples of size 20 to the dot plot for samples of size 10. Notice that for samples of size 20, only one pair of samples had a difference in means that was more than 1 MAD. So for samples of size 20, if the sample means differed by 1.1 MADs we would think this is more than just sampling variability. For samples of size 10, a difference of 1.1 MADs might just be sampling variability.

You have already seen box plots for sample sizes of 10 and 20. Consider the dot plot below for pairs of samples where the sample size was 30.



10. Use appropriate tools strategically. Use the dot plot above to complete the following sentence:

For two random samples of size 30, I would think that the population means were not equal if the sample means differed by more than ______ MADs.

My Notes

11. The MAD for the number of text messages sent in a day for the population of U.S. Census at School seventh-grade girls was 72. The MAD for the population of U.S. Census at School boys was also about 72. Suppose that a random sample of 30 girls had a sample mean number of text messages sent of 68 and a random sample of 30 boys had a sample mean number of text messages sent of 53. Is the difference in sample means large enough to conclude that the mean number of text messages sent for the population of girls and the mean for the population of boys are different? Explain your answer.

CONNECT TO AP STATISTICS

In high school, you will see other methods that allow you to compare two populations.

In AP Statistics, you will learn a formal method for comparing two populations, called hypothesis testing.

The three statements in the box below can be used to informally compare two population means, as long as the population MADs are about the same.

Informal Guidelines for Comparing Two Population Means

If two populations have about the same amount of variability, then

1. **For random samples of size 10, two population means are probably different if the sample means differ by more than 1.5 MADs.**
2. **For random samples of size 20, two population means are probably different if the sample means differ by more than 1.0 MADs.**
3. **For random samples of size 30, two population means are probably different if the sample means differ by more than 0.5 MAD.**

My Notes

Check Your Understanding

For each of the following populations, indicate whether you would conclude that *the population means might be equal*, or *the population means are probably different*. Justify your choice.

12. Population 1: U.S. Census at School seventh graders
Population 2: U.S. Census at School eighth graders
Variable of interest: Hours doing homework per week
The two populations both have a MAD of about: 7 hours
Sample size: 30
Mean of random sample from Population 1: 6.3 hours
Mean of random sample from Population 2: 6.6 hours

a. Conclusion and justification:

b. Does this lead you to think that the mean number of hours spent doing homework for seventh graders and the mean for eighth graders are different?

13. Population 1: U.S. Census at School seventh-grade girls
Population 2: U.S. Census at School seventh-grade boys
Variable of interest: Hours of sleep on a school night.
The two populations both have a MAD of about: 1 hour
Sample size: 10
Mean of random sample from Population 1: 8.2 hours
Mean of random sample from Population 2: 7.1 hours

a. Conclusion and justification:

b. Does this lead us to think that the mean number of hours of sleep on a school night for seventh-grade girls and the mean for seventh-grade boys are different?

My Notes

LESSON 26-2 PRACTICE

For the following items, indicate whether you would conclude that *the population means might be equal* or *the population means are probably different*. Justify your choice.

14. Population 1: U.S. Census at School seventh graders
Population 2: South Africa Census at School seventh graders
Variable of interest: Height (in cm)
The two populations both have a MAD of about 9 cm.
Sample size: 20
Mean of random sample from Population 1: 163 cm
Mean of random sample from Population 2: 151 cm

a. Reason quantitatively. Conclusion and justification:

b. Does this lead us to think that the mean heights for U.S. seventh graders and the mean for South Africa seventh graders are different?

15. Population 1: U.S. Census at School seventh-grade boys
Population 2: U.S. Census at School seventh-grade girls
Variable of interest: Hours spent watching TV per week
The two populations both have a MAD of about 2 hours.
Sample size: 30
Mean of random sample from Population 1: 5.8 hours
Mean of random sample from Population 2: 3.3 hours

a. Reason quantitatively. Conclusion and justification:

b. Does this lead us to think that the mean number of hours spent watching TV for seventh-grade boys and the mean for eighth-grade girls are different?

16. Population 1: McKenzie School fifth-grade girls
Population 2: McKenzie School fifth-grade boys
Variable of interest: Times to cover a 50m distance
The two populations both have a MAD of about 0.3 seconds.
Sample size: 10
Mean of random sample from Population 1: 10.4 seconds
Mean of random sample from Population 2: 10.0 seconds

a. Reason quantitatively. Conclusion and justification:

b. Does this lead us to think that the mean number of seconds to cover a distance of 50m for fifth-grade girls and the mean for fifth-grade boys are different?

My Notes

Learning Targets:

- Calculate the mean absolute deviation (MAD)
- Use two random samples to compare population means.
- Draw conclusions about populations with similar amounts of variability based on the difference of two sample means.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think-Pair-Share, Create Representations, Summarizing, Paraphrasing, Interactive Word Wall

There is one last thing to consider. In all of the examples you have seen so far, the population MAD was provided. But it is not realistic to think that you would know the value of the population MAD. The rest of this activity suggests how you might come up with an estimate of the MAD so that you can still use the guidelines on page 377 to compare two population means.

The steps that you can follow are summarized below.

Using Two Random Samples to Compare Population Means

Steps:

1. Construct a dot plot or box plot of each sample to see if the variability in each of the two samples is about the same.

   If the variability in each of the two samples looks to be about the same, continue on to Steps 2–5 below.
2. Calculate the mean and MAD for Sample 1 and the mean and MAD for sample 2.
3. Get an estimate of the common MAD by averaging the two sample MADs.
4. Express the difference in sample means in terms of the common MAD from Step 3.
5. Use the guidelines for comparing two population means (on page 377) to decide whether the difference in sample means is large enough to conclude that the population means are probably different.

Lesson 26-3
Calculating MAD for a Sample

My Notes

Now let us consider more data from the Census at School seventh-grade students. Suppose that you wanted to know if the average amount of time spent talking on the telephone is different for seventh-grade girls and seventh-grade boys. A random sample of 10 girls from the population of U.S. Census at School seventh-grade girls was selected using the random sampler on the Census at School website. The numbers of hours spent talking on the phone per week for these 10 girls were:

5	12	12	8	14
6	5	14	3	1

1. Calculate the sample mean number of hours spent talking on the phone for these 10 girls.

2. We can calculate the MAD for this sample by finding the distances from the sample mean for each data value and then calculating the average of these distances. The distance from the sample mean can be calculated by subtracting the sample mean from the data value and then ignoring the sign if the difference is negative. (In other words, what we're calculating here is the absolute value of the difference.) Complete the missing entries in the following table.

Data Value	Distance from the Mean
5	3
12	4
12	
8	
14	
6	
5	
14	
3	
1	

3. Calculate the value of the MAD for this sample.

MAD =

My Notes

A random sample of 10 boys from the population of U.S. Census at School seventh-grade boys was also selected. The numbers of hours spent talking on the phone per week for these 10 boys were:

5	3	1	1	0
13	0	3	10	0

4. Model with mathematics. Draw two dot plots of the sample data—one for the sample of girls and one for the sample of boys. Be sure to use the same numerical scale for both dot plots.

5. Does it seem from the box plots that there is about the same amount of variability in the phone data for girls and the phone data for boys?

6. You have already calculated the following sample statistics:

sample mean for girls = 8.0 hours
sample MAD for girls = 4.0 hours

Now calculate the mean and MAD for the data in the sample of boys. (You can use the table below to help organize your work for computing the MAD.)

sample mean for boys =
sample MAD for boys =

Data Value	Distance from the Mean
5	
3	
1	
1	
0	
13	
0	
3	
10	
0	

My Notes

7. Average the two sample MADs to get a common estimate of the population MADs.

common MAD =

8. Express the difference in the two sample means in terms of the common MAD.

9. Based on the difference in sample means, do you think that the mean number of hours spent talking on the phone for seventh-grade girls and the mean for seventh-grade boys are probably different, or do you think that they might be the same? Justify your answer.

10. **Reason quantitatively.** Suppose the two sample sizes had been 20 instead of 10, but that the sample means and MADs were still the same as the ones you calculated. Would you have answered Item 9 differently? Explain why or why not.

My Notes

Check Your Understanding

11. Suppose that the mean number of hours spent playing sports per week for a random sample of 15 boys selected from the students at Los Osos Middle School was 2.3 hours.

a. If we were to select a second random sample of 15 boys from this school, do you think that the mean for this second sample would also be 2.3 hours? Explain why or why not.

b. Suppose a random sample of 15 girls was selected from this school, and that the mean number of hours spent playing sports for the girls in this sample was 2.2 hours. This sample mean is different from the sample mean for the boys. Explain why this does not necessarily mean that the mean number of hours spent playing sports for the population of *all* boys at the school is different than the mean for *all* of the girls at the school.

12. For each of the following, indicate whether you would conclude that the population means might be equal or if you would conclude that the population means are probably different. Justify your choice.

a. Population 1: seventh graders at Los Osos Middle School
Population 2: eighth graders at Los Osos Middle School
Variable of interest: Hours spent on Facebook per week
The two populations both have a MAD of about: 0.5 hours
Sample size: 30
Mean of random sample from population 1: 1.3 hours
Mean of random sample from population 2: 1.6 hours

b. Population 1: seventh graders at Los Osos Middle School
Population 2: ninth graders at Los Osos High School
Variable of interest: Hours spent on homework per week
The two populations both have a MAD of about: 2 hours
Sample size: 20
Mean of random sample from population 1: 4.3 hours
Mean of random sample from population 2: 6.0 hours

My Notes

LESSON 26-3 PRACTICE

Use the following information to answer Items 13–19.

Suppose that you wanted to know if the average amount of time spent playing computer games and video games is different for seventh-grade girls and seventh-grade boys. A random sample of 10 girls from the population of U.S. Census at School seventh-grade girls was selected using the random sampler on the Census at School website. The numbers of hours spent playing computer and video games per week for these 10 girls were as follows:

20	2	0	0	5
1	3	2	14	4

A random sample of 10 boys from the population of U.S. Census at School seventh-grade boys was also selected. The numbers of hours spent playing computer and video games per week for these 10 boys were as follows:

5	3	1	12	3
2	5	4	5	21

13. Model with mathematics. Draw two dot plots of the sample data—one for the sample of girls and one for the sample of boys. Be sure to use the same numerical scale for both dot plots.

14. Based on the dot plots, does it look like there is about the same amount of variability in the phone data for the girls and the phone data for the boys?

My Notes

15. Calculate the sample mean and MAD for each of the two samples.

You can use the tables below to help organize your work for computing the MADs.

Girls	
Data Value	**Distance from the Mean**
20	
2	
0	
0	
5	
1	
3	
2	
14	
4	

Boys	
Data Value	**Distance from the Mean**
5	
3	
1	
12	
3	
2	
5	
4	
5	
21	

16. Average the two sample MADs to get a common estimate of the population MADs.

17. Express the difference in the two sample means in terms of the common MAD.

18. Based on the difference in sample means, do you think that the mean number of hours spent playing computer and video games for seventh-grade girls and the mean for seventh-grade boys are probably different, or do you think that they might be the same? Justify your answer.

19. Suppose the two sample sizes had been 20 instead of 10, but that the sample means and MADs were still the same as the ones you calculated. Would you have answered Item 18 differently? Explain why or why not.

ACTIVITY 26 PRACTICE

Write your answers on notebook paper or grid paper. Show your work.

Students participating in Census at School complete two online activities. In one activity, **reaction time** is measured by having the student click on a stop button as quickly as possible after the screen changes color. The computer measures the time between when the color changes and the student clicks. The second activity is a memory test. In this activity, the student uncovers pairs of pictures. If the pictures match, they stay uncovered. If the pictures don't match, they are covered up again. The computer measures how long it takes to uncover all of the pairs and records this as a **memory test score**. A student with a good memory will be able to complete this activity faster and would have a lower score than a student who is not as good at remembering what pictures they have seen.

If you would like to try out these online activities, you can find them at U.S. Census at School website (www.amstat.org/censusatschool/students.cfm)

1. The sample mean reaction time for a random sample of 15 U.S. Census at School seventh graders was 0.35 seconds. The sample mean for a random sample of 15 Japan Census at School seventh graders was 0.33 seconds. Explain why this does not tell us that the mean reaction time of *all* U.S. Census at School seventh graders is different from the mean reaction time of *all* Japan Census at School seventh graders.
2. Suppose that the MAD for the memory test score for the population of U.S. Census at School seventh graders is about 15. The MAD for Japan Census at School seventh graders is also about 15. For each of the following sample sizes and pairs of sample means, determine if you would conclude that the mean memory test score for U.S. Census at School seventh graders and the mean memory test score for Japan Census at School seventh graders might be the same.

   a. Sample size = 10
   U.S. sample mean = 45
   Japan sample mean = 49

   b. Sample size = 30
   U.S. sample mean = 45
   Japan sample mean = 49

   c. Sample size = 20
   U.S. sample mean = 45
   Japan sample mean = 59

   d. Sample size = 30
   U.S. sample mean = 58
   Japan sample mean = 40

Use the following information to answer Items 3–9.

Suppose that you wanted to know if the mean reaction time is different for U.S. Census at School seventh graders and New Zealand Census at School seventh graders. A random sample of 10 students from the population of U.S. Census at School seventh graders was selected using the random sampler on the Census at School website. The reaction times (in seconds) for these 10 students were:

0.37	0.38	0.36	0.31	0.46
0.27	0.30	0.34	0.37	0.36

A random sample of 10 students from the population of New Zealand Census at School seventh graders was also selected. The reaction times for these 10 students were:

0.38	0.34	0.36	0.37	0.34
0.34	0.31	0.38	0.33	0.45

3. Draw two dot plots of the sample data—one for the sample of U.S. seventh graders and one for the New Zealand sample. Be sure to use the same numerical scale for both dot plots.
4. Do the dot plots suggest that there is about the same amount of variability in the reaction time data for U.S. seventh graders and the reaction time data for New Zealand seventh graders?

5. Calculate the sample mean and MAD for each of the two samples. You can use the tables below to help organize your work for computing the MADs.

United States	
Data Value	**Distance from the Mean**
0.37	
0.38	
0.36	
0.31	
0.46	
0.27	
0.30	
0.34	
0.37	
0.36	

New Zealand	
Data Value	**Distance from the Mean**
0.38	
0.34	
0.36	
0.37	
0.34	
0.34	
0.31	
0.38	
0.33	
0.45	

6. Average the two sample MADs to get a common estimate of the population MADs.

7. Express the difference in the two sample means in terms of the common MAD.

8. Based on the difference in sample means, do you think that the mean reaction time for U.S. seventh graders and the mean for New Zealand seventh graders are probably different, or do you think that they might be the same? Justify your answer.

9. Suppose the two sample sizes had been 20 instead of 10, but that the sample means and MADs were still the same. Would you have answered Item 8 differently? Explain why or why not.

Use the following information to answer Items 10–15.

Suppose that you wanted to know whether the mean memory test score is different for U.S. Census at School seventh graders and Canada Census at School seventh graders. A random sample of 10 students from the population of U.S. Census at School seventh graders was selected using the random sampler on the Census at School website. The memory test scores for these 10 students were as follows:

42	34	48	37	53
40	51	37	45	43

A random sample of 10 students from the population of Canada Census at School seventh graders was also selected. The memory test scores for these 10 students were as follows:

38	32	44	39	39
43	38	28	32	37

10. Draw two dot plots of the sample data—one for the sample of U.S. seventh graders and one for the Canada sample. Be sure to use the same numerical scale for both dot plots.

11. Do the dot plots suggest that there is about the same amount of variability in the memory test score data for U.S. seventh graders and the reaction time data for Canada seventh graders?

12. Calculate the sample mean and MAD for each of the two samples. You can use the tables below to help organize your work for computing the MADs.

United States	
Data Value	**Distance from the Mean**
42	
34	
48	
37	
53	
40	
51	
37	
45	
43	

Canada	
Data Value	**Distance from the Mean**
38	
32	
44	
39	
39	
43	
38	
28	
32	
37	

13. Average the two sample MADs to get a common estimate of the population MADs.

14. Express the difference in the two sample means in terms of the common MAD.

15. Based on the difference in sample means, do you think that the mean memory test score for U.S. seventh graders and the mean for Canada seventh graders are probably different, or do you think that they might be the same? Justify your answer.

16. Suppose the two sample sizes had been 30 instead of 10, but that the sample means and MADs were still the same. Would you have answered Item 15 differently? Explain why or why not.

Use the five step method described on page 380 to answer Items 17 and 18.

17. A random sample of 10 girls and a random sample of 10 boys were selected from the population of U.S. Census at School seventh graders. The data on reaction times for the students in these samples is shown below. Based on these two samples, would you conclude that the population mean reaction time for U.S. Census at School seventh-grade girls is different from the mean for U.S. Census at School seventh-grade boys? Be sure to show all five steps in your answer.

Girls' Reaction Times				
0.36	0.38	0.44	0.42	0.36
0.27	0.32	0.32	0.33	0.33
Boys' Reaction Times				
0.31	0.27	0.28	0.34	0.33
0.22	0.27	0.32	0.31	0.31

18. A random sample of 10 girls and a random sample of 10 boys were selected from the population of U.S. Census at School seventh graders. The data on memory test scores for the students in these samples is shown below. Based on these two samples, would you conclude that the population mean memory test score for U.S. Census at School seventh-grade girls is different from the mean for U.S. Census at School seventh-grade boys? Be sure to show all five steps in your answer.

Girls' Memory Test Scores				
39	38	43	31	27
40	44	32	34	32
Boys' Memory Test Scores				
44	40	34	30	38
39	38	47	39	44

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

19. Explain why you cannot automatically conclude that two populations have different means just because random samples from the two populations have different sample means.

Write your answers on notebook paper or grid paper. Show your work.

1. The sample mean arm span (the distance from the middle finger on one hand to the middle finger on the other hand when the arms are extended, measured in cm) for a random sample of 15 U.S. Census at School seventh-grade girls was 155 cm. The sample mean for a random sample of 15 U.S. Census at School eighth-grade girls was 157 cm. Explain why this does not tell us that the mean arm span of *all* U.S. Census at School seventh-grade girls is different from the mean arm span time of *all* U.S. Census at School eighth-grade girls.

Use the following information to answer Items 2–8.

Suppose that you wanted to know whether the mean arm span is different for U.S. Census at School seventh-grade girls and U.S. Census at School seventh-grade boys. A random sample of 10 students from the population of U.S. Census at School seventh-grade girls was selected using the random sampler on the Census at School website. The arm spans (in cm) for these 10 students were as follows:

171	152	176	147	167
165	152	161	152	147

A random sample of 10 students from the population of U.S. Census at School seventh-grade boys was also selected. The arm spans for these 10 students were as follows:

176	168	176	155	154
160	170	172	179	190

2. Draw two dot plots of the sample data—one for the sample of U.S. seventh-grade girls and one for the sample of seventh-grade boys. Be sure to use the same numerical scale for both dot plots.
3. Do the dot plots suggest that there is about the same amount of variability in the arm span data for U.S. seventh-grade girls and the arm span data for U.S. seventh-grade boys?
4. Calculate the sample mean and MAD for each of the two samples. You can use the tables shown to help organize your work for computing the MADs.

Seventh-Grade Girls	
Data Value	**Distance from the Mean**
171	
152	
176	
147	
167	
165	
152	
161	
152	
147	

Seventh-Grade Boys	
Data Value	**Distance from the Mean**
176	
168	
176	
155	
154	
160	
170	
172	
179	
190	

5. Average the two sample MADs to get a common estimate of the population MADs.

6. Express the difference in the two sample means in terms of the common MAD.

7. Based on the difference in sample means, do you think that the mean arm span for U.S. seventh-grade girls and the mean for U.S. seventh-grade boys are probably different, or do you think that they might be the same? Justify your answer.

8. Suppose the two sample sizes had been 20 instead of 10, but that the sample means and MADs were still the same as the ones you calculated. Would you have answered Item 7 differently? Explain why or why not.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5, 6, 7, 8)	• Clear and accurate understanding of variability and mean absolute deviation (MAD).	• A functional understanding of variability and MAD.	• Partial understanding of variability and MAD.	• Inaccurate or incomplete understanding of variability and MAD.
Problem Solving (Items 7, 8)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 2, 3, 4, 5, 6)	• Clear and accurate understanding of representing a sample with a dotplot, mean, and MAD.	• Correctly representing a sample with a dotplot, mean, and MAD.	• Partial understanding of representing a sample with a dotplot, mean, and MAD.	• Little or no understanding of representing a sample with a dotplot, mean, and MAD.
Reasoning and Communication (Items 1, 3, 7, 8)	• Precise use of appropriate math terms and language to explain variability, MAD, and conclusions drawn from the MAD.	• An adequate explanation of variability, MAD, and conclusions drawn from the MAD.	• A misleading or confusing explanation of variability, MAD, and conclusions drawn from the MAD.	• An incomplete or inaccurate explanation of variability, MAD, and conclusions drawn from the MAD.

Personal Financial Literacy

7

Unit Overview

Being financially literate means taking responsibility for learning how to calculate income taxes on wages and how to create a budget to plan your spending and savings. Using a budget estimator will help you look at monthly earnings and expenses. You will also learn how to look ahead to long-term savings for future needs. Creating a net worth statement will help you see how families evaluate how well they're doing on their financial goals. As you continue through the unit, you will apply what you have learned to solve real-world financial problems.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- income tax
- property tax
- budget
- variable expense
- assets
- compound interest
- coupon
- sales tax
- take-home pay
- fixed expense
- net worth
- liabilities
- monetary incentive
- rebates

ESSENTIAL QUESTIONS

? How does being financially literate help you manage your money?

? How can you plan ahead for future financial goals?

Getting Ready

Write your answers on notebook paper.
Show your work.

1. Write the letter of each pair of fractions that are equal.
   a. $\frac{2}{3}$ and $\frac{4}{5}$
   b. $\frac{5}{8}$ and $\frac{10}{16}$
   c. $\frac{3}{7}$ and $\frac{7}{15}$
   d. $\frac{2}{5}$ and $\frac{5}{10}$
   e. $\frac{3}{5}$ and $\frac{9}{15}$
2. Write an equivalent fraction with a denominator of 100 for each fraction.
   a. $\frac{6}{20}$
   b. $\frac{2}{5}$
   c. $\frac{1}{4}$
   d. $\frac{36}{25}$
   e. $2\frac{3}{5}$
3. Write each percent as a decimal.
   a. 4.7%
   b. 3.2%
   c. 5.8%
   d. 19.6%
4. Multiply. Describe any pattern you notice.
   a. 6.2735×10
   b. 6.2735×100
   c. 6.2735×1000
5. Divide. Describe any pattern you notice.
   a. $87.345 \div 10$
   b. $87.345 \div 100$
   c. $87.345 \div 1000$
6. Divide.
   a. $40.20 ÷ 12
   b. $500.50 ÷ 0.50
   c. $105.3 ÷ 2.7
7. Explain how fractions and decimals are related.

Budgeting and Money Management

How Much Is Too Much?

Lesson 27-1 Understanding Earnings and Budgets

My Notes

Learning Targets:

- Examine taxes on wages earned and on purchases.
- Analyze a family budget and calculate percentages for each part of a budget.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Summarizing, Create Representations

Learning to manage your money is a skill that you will use throughout life. As you get older, you will start earning money and paying taxes. Among the three major taxes that many people pay are ***income taxes***, ***sales taxes***, and ***property taxes***.

1. The following table shows the median weekly earnings for people with different levels of schooling. Complete the table by calculating the increase in weekly income with each additional level of education.

Education Level	Median Weekly Income (2011)	Increase (in dollars)
Less than high school diploma	$451	—
High school graduate	$638	
Some college, no diploma	$719	
Associate degree	$768	
Bachelor's degree	$1053	
Master's degree	$1263	
Doctoral or professional degree	$1551–1665	

2. What is the median increase in weekly income between a high school degree and a college (bachelor's) degree?

3. Apply mathematics to everyday life. Suppose a family earns total wages of $800 a week and pays state income taxes of 5% on annual earnings.

a. How much money will the family pay as state income tax?

b. Suppose the local government taxes income at 1%. How much additional money will the family pay in taxes to the local government?

c. What amount of money is left after taxes are paid?

ACADEMIC VOCABULARY

An ***income tax*** is a percent of earnings paid to federal, state, or local governments. A ***sales tax*** is a percentage of the cost of a purchase. A ***property tax*** is a percentage of the value of the property owned.

MATH TIP

Remember that a median is the middle number in a range of values.

My Notes

CONNECT TO LAW

Some states do not require its residents to pay state income taxes. Currently those states are Texas, Florida, Washington, Tennessee, New Hampshire, Nevada, South Dakota, Wyoming, and Alaska.

Most people are also required to pay federal income taxes. Three types of taxes are paid to the federal government based on earnings:

- Income tax, which is a percentage of total earnings
- Social Security tax, which is currently 6.2% of earnings up to $113,700
- Medicare tax, which is 1.45% on all income.

Tax rates can change as lawmakers pass new laws. The following table shows current income tax rates. Employers withhold income taxes, plus Social Security and Medicare taxes, from employees' paychecks.

Tax Bracket	Married Filing Jointly	Single
10% Bracket	$0–$17,000	$0–$8,500
15% Bracket	$17,001–$69,000	$8,501–$34,500
25% Bracket	$69,001–$139,350	$34,501–$83,600
28% Bracket	$139,351–$212,300	$83,601–$174,400
33% Bracket	$212,301–$379,150	$174,401–$379,150
35% Bracket	Over $379,150	Over $379,150

People who work are paid hourly wages or a salary. The current federal minimum hourly wage is $7.25 per hour. Some states have increased this minimum. People who are paid a salary receive a fixed amount per year rather than an hourly wage.

4. If you earned $54,000 a year, what tax bracket would you be in?

5. If you earned $9.80 an hour, how much would you make for a 40-hour week? A year? What federal income taxes would be withheld?

6. The money that is left after taxes are withheld is called ***take-home pay***. Calculate the income taxes, Social Security taxes, and Medicare taxes you would pay on earnings of $54,000. What is the take-home pay?

ACADEMIC VOCABULARY

Take-home pay is the amount of money an employee receives in a paycheck. In addition to taxes, other amounts may be withheld for health insurance and contributions to retirement savings.

A ***budget*** is an estimate of expected income and expenses.

Many people create a ***budget*** to manage current income and expenses, while also planning ahead for long-term financial goals. By writing down income and expenses, people can avoid spending more money than they earn each month. Look at the sample budget on the next page. It is made up of income, expenses, and planned savings. This monthly budget is based on take-home earnings for two wage earners.

Monthly Budget

Income (annual take home of $64,646):

Salary/wages (take-home pay)	5,388	
Other (bank account interest)	60	
Total monthly income	5,448	
Expenses:		
Housing (mortgage/rent)	1080	24%
Property tax (if not included in mortgage)	385	
Insurance (home, car, life—medical not withheld by employer)	200	
Food (groceries)	970	
Utilities (electricity, water, gas, trash collection)	230	
Cell Phones	128	
Cable/internet/land line phone bundle	145	
Gasoline for car(s)	210	
Child care expenses (day care, tuition, etc.)	400	
Pet expenses (food, grooming, and veterinarian care)	0	
Credit card charges/payments (average monthly)	410	
Entertainment costs (movies, meals, hobbies, etc.)	300	
Gifts/Charitable donations	100	
Other:	0	
Total expenses	4558	
Savings:		
Emergency fund	100	
Retirement savings	340	
College savings	450	
Total savings	890	

7. What is the amount of total monthly income?

8. What is the amount of total expenses?

9. For each expense, calculate its percentage of total expenses. For example, the housing cost of $1080 is 24% of total expenses (rounded to the nearest whole percent).

10. Which expenses take up the largest share of the budget?

11. What proportion of total take-home pay is saved by this family?

My Notes

ACADEMIC VOCABULARY

A ***fixed expense*** is one that does not change over a period of time. A ***variable expense***, varies—or changes—over time.

Looking at the budget on the preceding page, you see several different types of expenses. Some expenses, such as a mortgage or rent, are ***fixed***. Others, called ***variable expenses***, can change each month. For example, money spent on entertainment is an example of a variable expense.

In the previous budget, the following expenses would be fixed for a period of time:

- Mortgage/rent
- Property tax
- Insurance
- Cable/internet/phone
- Cell phone costs
- Child care

These expenses would be considered variable because a consumer can decide to spend more or less on them each month. For example, by conserving electricity, utility costs could be lowered.

- Food
- Utilities
- Gasoline
- Pet expenses
- Credit card charges
- Entertainment expenses
- Gifts/charitable donations

12. Calculate the percentage of fixed expenses and variable expenses in the budget on the preceding page.

13. **Communicate reasoning.** If a family wants to reduce its overall expenses, which costs should it look at? Explain why.

Housing, food, and other costs vary in different parts of the country, and within different areas of a state. An online budget estimator is helpful in calculating the costs where you live. An example is at http://www.pine-grove.com/online-calculators/budget-calculator.htm. Many others are available.

14. Research the household costs for your area. Use an online budget estimator to find the minimum household budget (total take-home income) needed for a family in your city. Identify the hourly wage needed to meet this budget. Remember to consider the taxes to be paid on income earned.

Check Your Understanding

15. What taxes are deducted from earnings by most employers?

16. If you earn $700 per week, and you have total taxes of 22.65% taken out by your employer, what is your take-home pay?

17. How does creating a budget help people manage money?

LESSON 27-1 PRACTICE

18. Give examples of the types of taxes that wage earners pay to federal, state, or local governments.

19. If you earn $68,000 per year and you pay total taxes of 32.45%, what is your monthly take-home pay?

20. Calculate total expenses in this budget. Then find the percentage of total expenses for each individual expense. Round percents to the next whole number.

Expenses:

Housing (mortgage/rent)	1280
Property tax (if not included in mortgage)	485
Insurance (home, car, life—medical not withheld by employer)	340
Food (groceries)	1120
Utilities (electricity, water, gas, trash collection)	260
Cell Phones	130
Cable/internet/land line phone bundle	165
Gasoline for car(s)	230
Child care expenses (day care, tuition, etc.)	500
Pet expenses (food, grooming, and veterinarian care)	120
Credit card charges/payments (average monthly)	530
Entertainment costs (movies, meals, hobbies, etc.)	350
Gifts/Charitable donations	100
Total expenses	

21. Using the total expenses in Item 18, calculate the expenses as a percent of take-home pay of $6780.

22. **Communicate mathematical ideas.** With total expenses of $6,390 and take-home pay of $7,450, how much money is left for savings? What percentage of total income is savings?

23. Identify the fixed and variable expenses in Item 20. Calculate the percentage of the total for each.

24. Use a budget estimator to calculate the hourly wage needed to have family take-home income of $3600 per month. Assume total federal taxes of 22.45% and no state income tax.

My Notes

ACADEMIC VOCABULARY

Net worth is the difference between what is owned and what is owed. ***Assets*** are items owned, while ***liabilities*** are amounts owed.

Learning Targets:

- Construct a statement of financial net worth.
- Calculate and compare simple and compound interest earnings.
- Analyze and compare sales taxes and various ways to save money on purchases.

Suggested Learning Strategies: Close Reading, Marking the Text, Create Representations

Many families measure their financial progress toward meeting goals by calculating their ***net worth***. To do this, they collect information on everything owned and its current value. This might include a house, a car, savings, and investments in stock or bonds.These items are ***assets***. Most people also owe money to creditors, such as for a mortgage or a car payment. These amounts are ***liabilities***. To create a net worth statement, you first list the value of all the assets, or items owned, and calculate a total. Next, list all the amounts owed, the liabilities, and calculate a total. Subtract the total liabilities from the total assets to find the net worth.

Example A

Assets:

House	$238,000
General savings	48,000
College fund	28,000
Retirement fund	72,000
Total Assets	386,000

Liabilities:

Mortgage owed	110,000
Credit card debt	1,800
Balance on student loans	23,000
Equity loan for home improvement	25,000
Total Liabilities	159,800

Net Worth: 226,200

Assets − Liabilities = Net Worth
$386,000 − $159,800 = $226,200

Try These A

a. Calculate the net worth of someone with assets of $198,000 and liabilities of $154,000.

b. What is the value of assets if someone has a net worth of $142,500 and liabilities of 87,400?

c. What is the value of liabilities if total assets are $204,800 and net worth is $128,900?

My Notes

1. Organize the following information, and use it to create a statement of net worth. Use the My Notes space.

   House, $231,160
   Credit card debt, $2,680
   Savings account, $22,500
   Car loan, $14,790
   Retirement savings, $87,600
   Balance in checking account, $12,368
   Mortgage loan, $142,760
   Value of second car (no loan), $4,700
   Student loans, $32,650

A statement of net worth is useful to see how well you are meeting long-term financial goals. For example, you may want to save a certain amount of money for eventual retirement. Checking net worth regularly shows how much of the goal has been met. When planning for long-term savings, it's a good idea to check that you are earning compound interest on your savings instead of just simple interest.

If you have savings that earns simple interest, the interest earnings are calculated once a year.

2. Calculate the simple interest on savings of $18,470. Use an annual interest rate of 2.7%.

3. Calculate the simple interest on savings of $9,028 invested for six months at a rate of 3.4%.

Savings that earn ***compound interest*** will earn more money over a period of time than with simple interest.

MATH TIP

Remember that the formula for simple interest is $I = R \times T$ (Interest = Rate times Time).

ACADEMIC VOCABULARY

A ***compound interest*** on savings is paid more than once a year, thus earning interest on the interest paid each compounding period.

Example B

You have $10,000 to invest for 2 years, and you want to calculate and compare simple and compound interest of 4%. Interest is compounded annually.

Step 1:	Calculate simple interest.	$10{,}000 \times 0.04 = 400$
Step 2:	Multiply the interest times 2.	$400 \times 2 = 800$
Step 3:	Calculate compound interest.	$10{,}000 \times 0.04 = 400$
Step 4:	Add the principal and interest.	$10{,}000 + 400 = 10{,}400$
Step 5:	For year 2, multiply by 4%.	$10{,}400 \times 0.04 = 416$
Step 6:	Add the interest earned for 2 years.	$400 + 416 = 816$

With compounding, you would earn $16 more than with simple interest. If the interest is compounded more often, such as semiannually, quarterly, or monthly, the earnings would be higher.

My Notes

Try These B

a. Use simple interest and compound interest of 3% to compare the earnings on savings of $20,000 invested for two years.
b. Explain how compound interest results in higher earnings on savings over a period of years.

One way to increase savings is to find ways to reduce monthly costs. There are several ways to reduce costs. Turning out lights or using less water each month can reduce utility costs. Waiting to buy items when they're on sale can reduce costs for many household needs, such as furniture, clothing, and food.

4. Your local store has a sale with 20% off your favorite jeans. If the jeans are regularly priced at $39.98, what is your sale price? How much would you save?

ACADEMIC VOCABULARY

A ***monetary incentive*** is a special offer that reduces the total cost of one or more items, such as "buy one, get one free." A ***coupon*** is a document offering a reduction in price on a specific item, such as a box of cereal.

Another method that stores use to get customers to buy items is to offer a ***monetary incentive***.

5. Suppose your family's favorite restaurant is offering an incentive of one meal at the regular price and the second one at half price. If both meals regularly cost $8.50, how much would you pay? What is the savings?

Stores may use ***coupons*** to get customers into their stores. Grocery stores often use coupons that give a discount of a specific amount of money, such as 50 cents, on the price of an item.

6. Your family shops at a local grocery that offers coupons on many different items. On one shopping trip, your family redeems coupons on 5 items for the following amounts: $0.35, $0.50, $0.75, $1.00, and $0.60. What is the total savings?

7. Some coupons offer "two for one" deals where you get two items for the price of one. If you buy one sweater for $24.99, what is your total cost for a second sweater?

ACADEMIC VOCABULARY

A ***rebate*** is a sales promotion used as an incentive to get buyers to purchase a specific product.

Some manufacturers use ***rebates*** to get customers to buy their products. For example, a local store offers a television set for $699 after a rebate of $100. The manufacturer of the television will send the buyer $100 for purchasing that television. As the buyer, you would pay the full price of the television ($799 in this example) and then send the rebate request to the manufacturer to receive the $100.

My Notes

Most rebates are mail-in rebates. You will need to mail in the rebate coupon and a receipt for the purchase to obtain the amount of the rebate. It can take a few months to receive the money on a rebate. Rebates often have time limits on them, so it is important to mail in coupons and receipts soon after a purchase.

Some stores offer "instant rebates," in which case the amount of the rebate is taken off the price of the item at the time of purchase.

8. Suppose you buy a bicycle that has a manufacturer's rebate of $50. The bicycle costs $179.97 before the rebate. How much will you pay for the bicycle at the store? After the rebate?

9. The local hardware store is offering an instant rebate of $25 on all purchases over $150. If you bought items for $45, $78, and $12, what would be the amount of your rebate?

MATH TIP

It's important to remember that buying items you do not need—even on sale or with a coupon or rebate—does not save money.

Check Your Understanding

10. Explain what a statement of net worth is and the information on it.

11. Explain simple and compound interest and compare the benefits of one over the other.

12. Describe and compare incentives offered to customers to get them to buy products.

LESSON 27-2 PRACTICE

13. Use the following information to create a statement of net worth.
House (net value), $122,600
Credit card debt, $1,675
Savings account, $34,500
Car loan, $12,680
Retirement savings, $78,700
Balance in checking account, $13,686
Mortgage loan, $153,440
Value of second car (no loan), $5,400
Student loans, $28,520

14. **Communicate reasoning.** Why might a statement of net worth be useful?

15. Calculate simple interest on $7,000 at 3.2% for a year.

16. Calculate compound interest on $4,500 for 2 years at 4%, compounded semiannually.

17. Calculate and compare simple interest and compound interest of 5% on the amount of $12,000 for 5 years.

18. Calculate savings on a purchase of $59.98 with 15% off.

19. Using coupons worth $0.40, $0.60, $1.25, $0.75, and $0.50, how much would you save if you bought the related products?

20. Describe the difference between a rebate and an incentive to "buy one, get one free."

ACTIVITY 27 PRACTICE

1. List the federal taxes that are usually withheld from paychecks.
2. If you earn $49,400 a year, what amounts of federal taxes would be withheld?
3. Your local city taxes income at 1.5%. If you earn $42,680, how much would you pay in taxes to your city?
4. Research the typical household expenses for your area. Then create a budget to meet those expenses? How much money would you need to earn, either as a salary or an hourly wage, to pay these expenses? Remember to include taxes on the earned income.
5. Create a budget using the following information:

Housing (mortgage/rent)	1170
Interest earned on savings (monthly)	78
Salary (take-home pay)	5,120
Insurance (home, car, life—medical not withheld by employer)	280
Food (groceries)	910
Utilities (electricity, water, gas, trash collection)	245
Cell Phones	110
Cable/internet/land line phone bundle	155
Gasoline for car(s)	190
Child care expenses (day care, tuition, etc.)	550
Savings for retirement	350
Credit card charges/payments (average monthly)	430
Entertainment costs (movies, meals, hobbies, etc.)	250
Gifts/Charitable donations	150
Savings for college	400

6. Explain the difference between fixed and variable expenses. How can they be used to change monthly costs?
7. Use the following information to create a statement of net worth:

House	$293,000
General savings	68,000
College fund	39,000
Retirement fund	92,000
Car	12,500
Mortgage owed	134,600
Credit card debt	1,940
Balance on student loans	18,670
Equity loan for home improvement	30,000

8. Calculate and compare simple and compound interest of 4.2% on savings of $15,000 for 10 years.
9. Calculate 25% savings on a purchase of $68.98.

MATHEMATICAL PRACTICES
Communicate Mathematical Ideas

10. Explain how using a monthly budget and calculating net worth are used to help plan for and meet long-term financial goals.

Symbols

$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$\lvert a \rvert$	absolute value: $\lvert 3 \rvert = 3$; $\lvert -3 \rvert = 3$
$\sqrt{\ }$	square root
%	percent
$\perp$	perpendicular
$\parallel$	parallel
(x, y)	ordered pair
$\overset{\frown}{AB}$	arc
$\overleftrightarrow{AB}$	line AB
$\overrightarrow{AB}$	ray AB
$\overline{AB}$	line segment AB
$\angle A$	angle A
$m\angle A$	measure of angle A
$\triangle ABC$	triangle ABC
π	pi; $\pi \approx 3.14$; $\pi \approx \frac{22}{7}$

Formulas

Perimeter	
P	= sum of the lengths of the sides
Rectangle	$P = 2l + 2w$
Square	$P = 4s$
Circumference	$C = 2\pi r$

Area	
Circle	$A = \pi r^2$
Parallelogram	$A = bh$
Rectangle	$A = lw$
Square	$A = s^2$
Triangle	$A = \frac{1}{2}bh$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$

Surface Area	
Cube	$SA = 6e^2$
Rectangular Prism	$SA = 2lw + 2lh + 2wh$
Cylinder	$SA = 2\pi r^2 + 2\pi rh$
Cone	$SA = \pi r^2 + \pi rl$
Regular Pyramid	$SA = B + \frac{1}{2}pl$
Sphere	$SA = 4\pi r^2$

Volume	
Cylinder	$V = Bh, B = \pi r^2$
Rectangular Prism	$V = lwh$
Triangular Prism	$V = Bh, B = \frac{1}{2}bh$
Pyramid	$V = \frac{1}{3}Bh$
Cone	$V = \frac{1}{3}\pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$

Linear function	
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-intercept form	$y = mx + b$
Point-slope form	$y - y_1 = m(x - x_1)$
Standard form	$Ax + By = C$

Quadratic Equations	
Standard Form	$ax^2 + bx + c = 0$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Other Formulas	
Pythagorean Theorem	$a^2 + b^2 = c^2$, where c is the hypotenuse of a right triangle
Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Direct variation	$y = kx$
Inverse variation	$y = \frac{k}{x}$

Temperature	
Celsius	$C = \frac{5}{9}(F - 32)$
Fahrenheit	$F = \frac{9}{5}C + 32$

Properties of Real Numbers

Reflexive Property of Equality	For all real numbers a, $a = a$.
Symmetric Property of Equality	For all real numbers a and b, if $a = b$, then $b = a$.
Transitive Property of Equality	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	For all real numbers a and b, if $a = b$, then a may be replaced by b.
Additive Identity	For all real numbers a, $a + 0 = 0 + a = a$.
Multiplicative Identity	For all real numbers a, $a \cdot 1 = 1 \cdot a = a$.
Commutative Property of Addition	For all real numbers a and b, $a + b = b + a$.
Commutative Property of Multiplication	For all real numbers a and b, $a \cdot b = b \cdot a$.
Associative Property of Addition	For all real numbers a, b, and c, $(a + b) + c = a + (b + c)$.
Associative Property of Multiplication	For all real numbers a, b, and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
Distributive Property of Multiplication over Addition	For all real numbers a, b, and c, $a(b + c) = a \cdot b + a \cdot c$.
Additive Inverse	For all real numbers a, there is exactly one real number $-a$ such that $a + (-a) = 0$ and $(-a) + a = 0$.
Multiplicative Inverse	For all real numbers a and b where $a \neq 0$, $b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{b}{a} \cdot \frac{a}{b} = 1$ and $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Multiplication Property of Zero	For all real numbers a, $a \cdot 0 = 0$ and $0 \cdot a = 0$.
Addition Property of Equality	For all real numbers a, b, and c, if $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	For all real numbers a, b, and c, if $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	For all real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	For all real numbers a, b, and c, $c \neq 0$ if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
Zero Product Property of Equality	For all real numbers a and b, if $a \cdot b = 0$ then $a = 0$ or $b = 0$ or both a and b equal 0.
Addition Property of Inequality*	For all real numbers a, b, and c, if $a > b$, then $a + c > b + c$.
Subtraction Property of Inequality*	For all real numbers a, b, and c, if $a > b$, then $a - c > b - c$.
Multiplication Property of Inequality *	For all real numbers a, b, and c, $c > 0$, if $a > b$, then $a \cdot c > b \cdot c$. For all real numbers a, b, and c, $c < 0$, if $a > b$, then $a \cdot c < b \cdot c$.
Division Property of Inequality*	For all real numbers a, b, and c, $c > 0$ if $a > b$, then $\frac{a}{c} > \frac{b}{c}$. For all real numbers a, b, and c, $c < 0$ if $a > b$, then $\frac{a}{c} < \frac{b}{c}$.

*These properties are also true for $<$, $\leq$, $\geq$.

Table of Measures

Customary	Metric
Distance/Length	
1 foot (ft) = 12 inches (in.)	1 centimeter (cm) = 10 millimeters (mm)
1 yard (yd) = 3 feet (ft) = 36 inches (in.)	1 meter (m) = 100 centimeters (cm)
1 mile (mi) = 5280 feet (ft)	1 kilometer (km) = 1000 meters (m)
Volume	
1 cup (c) = 8 fluid ounces (fl oz)	1 liter (L) = 1000 milliliters (mL)
1 pint (pt) = 2 cups (c)	
1 quart (qt) = 2 pints (pt)	
1 gallon (gal) = 4 quarts (qt)	
Weight/Mass	
1 pound (lb) = 16 ounces (oz)	1 gram (g) = 1000 milligrams (mg)
	1 kilogram (kg) = 1000 grams (g)
Time	
1 minute (min) = 60 seconds (sec)	1 year (yr) = 365 days (d)
1 hour (hr) = 60 minutes (min)	1 year (yr) = 52 weeks (wk)
1 day (d) = 24 hours (hr)	1 year (yr) = 12 months (mo)
1 week (wk) = 7 days (d)	

SpringBoard Learning Strategies

READING STRATEGIES

STRATEGY	DEFINITION	PURPOSE
Activating Prior Knowledge	Recalling what is known about a concept and using that information to make a connection to a new concept	Helps students establish connections between what they already know and how that knowledge is related to new learning
Chunking the Activity	Grouping a set of items/questions for specific purposes	Provides an opportunity to relate concepts and assess student understanding before moving on to a new concept or grouping
Close Reading	Reading text word for word, sentence by sentence, and line by line to make a detailed analysis of meaning	Assists in developing a comprehensive understanding of the text
Graphic Organizer	Arranging information into maps and charts	Builds comprehension and facilitates discussion by representing information in visual form
Interactive Word Wall	Visually displaying vocabulary words to serve as a classroom reference of words and groups of words as they are introduced, used, and mastered over the course of a year	Provides a visual reference for new concepts, aids understanding for reading and writing, and builds word knowledge and awareness
KWL Chart (Know, Want to Know, Learn)	Activating prior knowledge by identifying what students know, determining what they want to learn, and having them reflect on what they learned	Assists in organizing information and reflecting on learning to build content knowledge and increase comprehension
Marking the Text	Highlighting, underlining, and /or annotating text to focus on key information to help understand the text or solve the problem	Helps the reader identify important information in the text and make notes about the interpretation of tasks required and concepts to apply to reach a solution
Predict and Confirm	Making conjectures about what results will develop in an activity; confirming or modifying the conjectures based on outcomes	Stimulates thinking by making, checking, and correcting predictions based on evidence from the outcome
Levels of Questions	Developing literal, interpretive, and universal questions about the text while reading the text	Focuses reading, helps in gaining insight into the text by seeking answers, and prepares one for group and class discussions
Paraphrasing	Restating in your own words the essential information in a text or problem description	Assists with comprehension, recall of information, and problem solving
Role Play	Assuming the role of a character in a scenario	Helps interpret and visualize information in a problem
Shared Reading	Reading the text aloud (usually by the teacher) as students follow along silently, or reading a text aloud by the teacher and students	Helps auditory learners do decode, interpret, and analyze challenging text
Summarizing	Giving a brief statement of the main points in a text	Assists with comprehension and provides practice with identifying and restating key information
Think Aloud	Talking through a difficult text or problem by describing what the text means	Helps in comprehending the text, understanding the components of a problem, and thinking about possible paths to a solution
Visualization	Picturing (mentally and/or literally) what is read in the text	Increases reading comprehension and promotes active engagement with the text
Vocabulary Organizer	Using a graphic organizer to keep an ongoing record of vocabulary words with definitions, pictures, notes, and connections between words	Supports a systematic process of learning vocabulary

SpringBoard Learning Strategies

COLLABORATIVE STRATEGIES

STRATEGY	DEFINITION	PURPOSE
Critique Reasoning	Through collaborative discussion, respond to the arguments of others; question the use of mathematical terminology, assumptions, and conjectures to improve understanding and to justify and communicate conclusions	Helps students learn from each other as they make connections between mathematical concepts and learn to verbalize their understanding and support their arguments with reasoning and data that make sense to peers
Debriefing T	Discussing the understanding of a concept to lead to consensus on its meaning	Helps clarify misconceptions and deepen understanding of content
Discussion Groups	Working within groups to discuss content, to create problem solutions, and to explain and justify a solution	Aids understanding through the sharing of ideas, interpretation of concepts, and analysis of problem scenarios
Group Presentation	Presenting information as a collaborative group	Allows opportunities to present collaborative solutions and to share responsibility for delivering information to an audience
Jigsaw	Reading different texts or passages, students become "experts" and then move to a new group to share their information; after sharing, students go back to the original group to share new knowledge	Provides opportunities to summarize and present information to others in a way that facilitates understanding of a text or passage (or multiple texts or passages) without having each student read all texts
Sharing and Responding	Communicating with another person or a small group of peers who respond to a piece of writing or proposed problem solution	Gives students the opportunity to discuss their work with peers, to make suggestions for improvement to the work of others, and/or to receive appropriate and relevant feedback on their own work
Think-Pair-Share	Thinking through a problem alone, pairing with a partner to share ideas, and concluding by sharing results with the class	Enables the development of initial ideas that are then tested with a partner in preparation for revising ideas and sharing them with a larger group

WRITING STRATEGIES

Drafting	Writing a text in an initial form	Assists in getting first thoughts in written form and ready for revising and refining
Note Taking	Creating a record of information while reading a text or listening to a speaker	Helps in organizing ideas and processing information
Prewriting	Brainstorming, either alone or in groups, and refining thoughts and organizing ideas prior to writing	Provides a tool for beginning the writing process and determining the focus of the writing
Quickwrite	Writing for a short, specific amount of time about a designated topic	Helps generate ideas in a short time
RAFT (Role of Writer, Audience, Format, and Topic)	Writing a text by consciously choosing a viewpoint (role of the writer), identifying an audience, choosing a format for the writing, and choosing a topic	Provides a framework for communicating in writing and helps focus the writer's ideas for specific points of communication
Self Revision / Peer Revision	Working alone or with a partner to examine a piece of writing for accuracy and clarity	Provides an opportunity to review work and to edit it for clarity of the ideas presented as well as accuracy of grammar, punctuation, and spelling

SpringBoard Learning Strategies

PROBLEM-SOLVING STRATEGIES

Construct an Argument	Use mathematical reasoning to present assumptions about mathematical situations, support conjectures with mathematically relevant and accurate data, and provide a logical progression of ideas leading to a conclusion that makes sense	Helps develop the process of evaluating mathematical information, developing reasoning skills, and enhancing communication skills in supporting conjectures and conclusions
Create a Plan	Analyzing the tasks in a problem and creating a process for completing the tasks by finding information needed for the tasks, interpreting data, choosing how to solve a problem, communicating the results, and verifying accuracy	Assists in breaking tasks into smaller parts and identifying the steps needed to complete the entire task
Create Representations	Creating pictures, tables, graphs, lists, equations, models, and /or verbal expressions to interpret text or data	Helps organize information using multiple ways to present data and to answer a question or show a problem solution
Guess and Check	Guessing the solution to a problem, and then checking that the guess fits the information in the problem and is an accurate solution	Allows exploration of different ways to solve a problem; guess and check may be used when other strategies for solving are not obvious
Identify a Subtask	Breaking a problem into smaller pieces whose outcomes lead to a solution	Helps to organize the pieces of a complex problem and reach a complete solution
Look for a Pattern	Observing information or creating visual representations to find a trend	Helps to identify patterns that may be used to make predictions
Simplify the Problem	Using "friendlier" numbers to solve a problem	Provides insight into the problem or the strategies needed to solve the problem
Work Backward	Tracing a possible answer back through the solution process to the starting point	Provides another way to check possible answers for accuracy
Use Manipulatives	Using objects to examine relationships between the information given	Provides a visual representation of data that supports comprehension of information in a problem

Glossary
Glosario

A

absolute value (p. 16, 85) The distance of a number from zero on a number line. Distance or absolute value is always positive. For example, the absolute value of both -6 and 6 is 6.

valor absoluto (págs. 16, 85) Distancia entre un número y el cero en una recta numérica. La distancia o valor absoluto es siempre positivo. Por ejemplo, el valor absoluto de -6 y de 6 es 6.

adjacent angles (p. 139) Angles that have a common side, but no common interior.

ángulos adyacentes (pág. 139) Ángulos que tienen un lado en común, pero no un interior común.

algebraic expression (p. 52) A mathematical phrase that contains one or more numbers, one or more variables, and one or more arithmetic operations; for example, $4a + 7$.

expresión algebraica (pág. 52) Una frase matemática que contiene uno o más números, una o más variables, y una o más operaciones aritméticas; por ejemplo: $4a + 7$.

algebraic statement (p. 52) An equation or inequality that contains variables, such as $(a + b) + c = a + (b + c)$.

ecuación algebraica (pág. 52) Una ecuación o desigualdad que contiene variables, tales como $(a + b) + c = a + (b + c)$.

angle (p. 137) The union of two rays with a common endpoint.

ángulo (pág. 137) Unión de dos rayos con un extremo en común.

ascend (p. 25) To move upward.

ascender (pág. 25) Subir a un lugar más alto.

assets (p. 398) Items that one owns.

bienes (pág. 398) Artículos que uno posee.

B

bar chart (p. 253) A way of displaying categorical data (for example, colors, types, qualities). Each bar represents a different category. The vertical scale for the bar heights is labeled with the count or percent for each category. Also known as a **bar graph.**

gráfica de columnas (pág. 253) Gráfica que usa barras para mostrar datos por categorías (por ejemplo, colores, tipos, cualidades). Cada barra representa una categoría diferente. La escala vertical para las alturas de las barras se rotula con el conteo o porcentaje para cada categoría. También se la conoce como **gráfica de barras.**

budget (p. 394) An estimate of expected income and expenses, with each type of income and expense identified by amount.

presupuesto (pág. 394) Una estimación de ingresos y gastos esperados, identificando la cantidad de cada tipo de ingreso y gasto.

C

census (p. 325) A study that gains information about every member of a population.

censo (pág. 325) Estudio que obtiene información acerca de cada miembro de una población.

center (p. 169) In a circle, a given point from which the set of points in the same plane are an equal distance.

centro (pág. 169) En un círculo, un punto dado desde el cual un conjunto de puntos en un mismo plano están a la misma distancia.

circle (p. 169) The set of points in a plane that are an equal distance from a given point, called the center.

círculo (pág. 169) Conjunto de puntos en un plano que están a igual distancia de un punto dado, llamado centro.

circumference (p. 169) The distance around a circle.

circunferencia (pág. 169) Distancia alrededor de un círculo.

coefficient (p. 59) A number by which a variable is multiplied. For example, in the term $6x$, 6 is the coefficient.

coeficiente (pág. 59) Número por el cual se multiplica una variable. Por ejemplo, en el término $6x$, 6 es el coeficiente.

common denominator (p. 36) A number into which all the denominators of a set of fractions may be divided without a remainder.

común denominador (pág. 36) Un número entre el cual todos los denominadores de un conjunto de fracciones se pueden dividir sin que quede un residuo.

complement of an event (p. 254) The complement of an event includes all possible outcomes of a probability experiment that are not outcomes of the event.

complemento de un suceso (pág. 254) El complemento de un suceso incluye todos los resultados posibles de un experimento de probabilidad que no son resultados del suceso.

complementary angles (p. 137) Two angles whose measures have a sum of 90°.

ángulos complementarios (pág. 137) Dos ángulos cuyas medidas suman 90°.

complex solid (p. 219) In three-dimensional geometry, the result of putting two or more solids together.

cuerpo geométrico complejo (pág. 219) En geometría tridimensional, el resultado de juntar dos o más cuerpos geométricos.

composite figure (p. 180) A figure made up of two or more figures.

figura compuesta (pág. 180) Figura formada por dos o más figuras.

compound interest (p. 399) Interest calculated on the total principal plus the interest earned or owed during the previous time period.

interés compuesto (pág. 399) Interés calculado sobre el capital total más el interés devengado o adeudado durante el período anterior.

congruent (p. 161) When two or more figures are the same shape and size, they are congruent.

congruentes (pág. 161) Cuando dos o más figuras tienen la misma forma y el mismo tamaño, son congruentes.

conjecture (p. 142) An unproved statement that seems to be true.

conjetura (pág. 142) Enunciado no demostrado que parece ser verdadero.

constant (p. 59) A term in an expression that does not change in value because it does not contain a variable. For example, the constant term in the expression $3n + 6$ is 6.

constante (pág. 59) Término de una expresión que no cambia de valor, debido a que no contiene variables. Por ejemplo, el término constante en la expresión $3n + 6$ es 6.

constant rate of change (p. 95) If the rate of change remains the same in a problem situation, it is a constant rate of change.
tasa de cambio constante (pág. 95) Si la tasa de cambio permanece igual en un problema, es una tasa de cambio constante.

constant ratio (p. 94) A constant ratio occurs when one quantity is directly proportional to another; that is, the ratio between the two variables is constant (does not change).
razón constante (pág. 94) Una razón constante ocurre cuando una cantidad es directamente proporcional a otra; es decir, la razón entre las dos variables es constante (no cambia).

conversion factor (p. 85) A ratio relating a number of units to a unit measure. For example, the ratio 12 to 1 is the conversion factor relating inches to feet, and 1 to 12 is the conversion factor relating feet to inches.
factor de conversión (pág. 85) Razón que relaciona un número de unidades con una medida unitaria. Por ejemplo, la razón 12 a 1 es el factor de conversión de pulgadas a pies y 1 a 12 es el factor de conversión de pies a pulgadas.

corresponding parts (p. 161) The sides and angles of similar figures that are in the same relative positions in the figures.
partes correspondientes (pág. 161) Los lados y ángulos de figuras similares que están en la misma posición relativa en las figuras.

coupon (p. 400) A document offering a reduction in price on a specific item, such as a box of cereal.
cupón (pág. 400) Un documento que ofrece una reducción en el precio de un artículo específico, como una caja de cereal.

critique (p. 4) To critique is to analyze and discuss the details of something.
criticar (pág. 4) Criticar es analizar y discutir los detalles de algo.

cross section (p. 197) The intersection of a solid figure and a plane.
sección transversal (pág. 197) Intersección de un cuerpo geométrico con un plano.

D

decompose the figure (p. 180) In geometry, when you divide a composite figure into smaller figures, the phrase "decompose the figure" is sometimes used.
descomponer la figura (pág. 180) En geometría, cuando divides una figura compuesta en figuras más pequeñas, algunas veces se utiliza la frase "descomponer la figur."

descend (p. 25) To move downward.
descender (pág. 25) Bajar; ir desde un lugar a otro más bajo.

diameter (p. 169) Any line segment through the center of a circle with endpoints on the circle.
diámetro (pág. 169) Cualquier segmento de recta que pasa por el centro de un círculo y tiene sus extremos sobre el círculo

discount (p. 125) A reduction in price.
descuento (pág. 125) Una reducción en el precio.

E

equally likely outcome (p. 251) An outcome of a probability experiment in which every different outcome has the same chance of occurring.
resultados igualmente probables (pág. 251) Un resultado de un experimento de probabilidad en el que cada resultado diferente tiene las mismas posibilidades de ocurrir.

equation (p. 60) A statement showing that two numbers or expressions are equal, such as $4 + 3 = 7$ or $x + 5 = 9$.
ecuación (pág. 60) Enunciado que muestra que dos números o expresiones son iguales, como $4 + 3 = 7$ o $x + 5 = 9$.

equivalent expression (p. 55) Two or more expressions that may look different, but represent the same quantity or have equal values when evaluated. The expression $2x + 4x$ and the expression $6x$ are equivalent expressions.
expresión equivalente (pág. 55) Dos o más expresiones que se pueden ver diferente, pero representan la misma cantidad o tienen valores iguales cuando e evalúan. La expresión $2x + 4x$ y la expresión $6x$ son expresiones equivalentes.

estimated probability (p. 261) An estimated probability is one that is calculated by observing the outcome of a probability experiment many times. Estimated probabilities are sometimes also called empirical probabilities.
probabilidad estimada (pág. 261) Una probabilidad estimada es aquella que se calcula observando muchas veces el resultado de un experimento de probabilidad. Las probabilidades estimadas a veces se llaman probabilidades empíricas.

event (p. 253, 280) Any outcome or group of outcomes from a probability experiment.
suceso (págs. 253, 280) Cualquier resultado o grupo de resultados de un experimento de probabilidad.

F

factor (p. 54) Any of the numbers or symbols that when multiplied together form a product; or the process of finding the factors that form a product.
factor (pág. 54) Cualquiera de los números o símbolos que al multiplicarse entre sí forman un producto; o el proceso de hallar los factores que forman un producto.

factor (p. 54) The process of finding the factors that form a product.
factorizar (pág. 54) Proceso de hallar los factores que forman un producto.

fixed expense (p. 396) An expense that does not change over a period of time.
gasto fijo; coste fijo (pág. 396) Un gasto que no cambia en un periodo de tiempo.

I

included angle (p. 152) An angle formed by two sides of a triangle.
ángulo incluído (pág. 152) Un ángulo formado por dos lados de un triángulo.

income tax (p. 393) A percent of earnings paid to federal, state, or local governments as a tax.
impuesto sobre la renta (pág. 393) Un porcentaje de las ganancias que se pagan como impuestos al gobierno federal, estatal o local.

inscribed figure (p. 184) A figure drawn inside another figure so that it fits tightly and touches as many places as posible; for example, when a circle is inscribed in a square, the diameter of the circle and any side of the square have the same measure.
figura inscrita (pág. 184) Una figura dibujada dentro de otra figura de modo que cabe en el interior perfectamente y toca todos los puntos posibles de la figura que la contiene; por ejemplo cuando un círculo está inscrito en un cuadrado, el diámetro del círculo mide lo mismo que cualquier lado del cuadrado.

interest (p. 127) The percentage of a loan amount charged by the lender; interest rates are stated as a percentage.

interés (pág. 127) El porcentaje que un prestamista cobra por un préstamo; las tasas de interés se establecen como un porcentaje.

L

lateral area (p. 199) The sum of the areas of the lateral faces of a solid.

área lateral (pág. 199) Suma de las áreas de las caras laterales de un cuerpo geométrico.

lateral face (p. 199) A face of a solid that is not one of the bases.

cara lateral (pág. 199) Cara de un cuerpo geométrico que no es una de las bases.

liabilities (p. 398) Amounts owed; debts.

obligaciones financieras; pasivos (pág. 398) Cantidades que se deben; deudas.

M

markup (p. 125) An amount added to the original cost of an item to find the selling price.

aumento de precio (pág. 125) Una cantidad agregada al costo original de un artículo para encontrar el precio de venta.

media (p. 75) The various ways by which news and information are communicated to the public; for example, through television, radio, and newspapers.

medios de comunicación (pág. 75) Las varias maneras por las que las noticias y la información se comunican al público; por ejemplo: a través de la televisión, la radio y los periódicos.

monetary incentive (p. 400) A special offer that reduces the total cost of one or more items, such as "buy one, get one free."

incentivo monetario (pág. 400) Una oferta especial que reduce el costo total de uno o más artículos, como por ejemplo: "compras al dos por uno."

N

net (p. 192) A two-dimensional drawing used to represent or form a three-dimensional object or solid.

red (pág. 192) Dibujo bidimensional que se usa para representar o formar un objeto tridimensional o cuerpo geométrico.

net worth (p. 398) The difference between what is owned and what is owed.

patrimonio neto (pág. 398) La diferencia de lo que se tiene menos lo que se debe.

numeric expression (p. 51) An expression that contains numbers and operations. For example, $12 + 0$ and $(10 \times 5) \times 3$ are expressions that contain numbers and operations.

expresión numérica (pág. 51) Una expresión que contiene números y operaciones. Por ejemplo, $12 + 0$ y $(10 \times 5) \times 3$ son expresiones que contienen números y operaciones.

numerical statement (p. 51) An equation that sets two expressions equal; for example, $20 \times 4 = 4 \times 20$.

declaración numérica (pág. 51) Una ecuación que establece que dos expresiones son iguales; por ejemplo: $20 \times 4 = 4 \times 20$.

O

order of operations (p. 56) A set of rules for evaluating expressions with more than one operation. The order is: do calculations inside parentheses; evaluate expressions with exponents; multiply or divide from left to right; add or subtract from left to right.

orden de las operaciones (pág. 56) Conjunto de reglas para evaluar expresiones que contienen más de una operación. El orden es: hacer los cálculos que estén en paréntesis; evaluar las expresiones que tienen exponentes; multiplicar o dividir de izquierda a derecha; sumar o restar de izquierda a derecha.

orientation (p. 163) The way in which a figure is positioned.

orientación (pág. 163) La manera en la cual está colocada una figura.

P

palindrome (p. 51) A word, phrase, or sequence that reads the same backward or forward; for example, the words tot and racecar are palindromes.

palíndromo (pág. 51) Una palabra, frase o secuencia que se lee igual alrevés y al derecho; por ejemplo, "amad a la dama" y "no deseo yo ese don" son palíndromos.

percent (p. 115) A ratio that compares a number to 100 and uses the % symbol.

porcentaje (pág. 115) Razón que compara un número con 100 y usa el símbolo %.

percent equation (p. 116) An equation showing the relationship between the whole and a part (the percent). The percent equation is written as $whole \bullet percent = part$.

ecuación de porcentaje (pág. 116) Una ecuación que muestra la relación entre el entero y una parte (el porcentaje). La ecuación de porcentaje se escribe como $entero \times porcentaje = parte$.

percent error (p. 129) The amount of error between the assumed value and the actual value.

porcentaje de error (pág. 129) La cantidad de error entre un valor estimado y el valor real.

percent increase/decrease (pp. 123) The percent by which an original amount is increased or decreased. This is determined by dividing the change (increase or decrease) by the original amount.

porcentaje de aumento/disminución (págs. 123) Porcentaje en que aumenta o disminuye una cantidad original. Se determina dividiendo el cambio (aumento o disminución) por la cantidad original.

plane (p. 169) A flat surface that extends infinitely in all directions. A parallelogram is usually used to model a plane in two dimensions.

plano (pág. 169) Superficie plana que se extiende infinitamente en todas direcciones. Normalmente se usa un paralelogramo para modelar un plano en dos dimensiones.

population (p. 325) A set of members for which data are to be collected and analyzed with the purpose of drawing some conclusions about some feature of the population.

población (pág. 325) Conjunto de miembros para los cuales se recopilan y analizan datos con el propósito de sacar conclusiones acerca de alguna característica de la población.

population mean (p. 330) The average of the data values for the whole population. If the sample is selected in a reasonable way, the sample mean can be used as an estimate of the population mean.

media poblacional (pág. 330) El promedio de los valores de los datos para la población total. Si la muestra se selecciona de manera razonable, el promedio de la muestra puede ser usado como una estimación de la media poblacional.

predict (p. 228) To make a reasonable guess about something that will happen.

predecir (pág. 228) Hacer una estimación razonable acerca de algo que ocurrirá.

prism (p. 192) A solid with parallel congruent bases that are both polygons. The sides (faces) of a prism are all parallelograms or rectangles. A prism is named according to the shape of its bases.

prisma (pág. 192) Cuerpo geométrico que tiene como bases dos polígonos paralelos y congruentes. Los lados (caras) de un prisma son todos paralelogramos o rectángulos. Un prisma recibe su nombre según la forma de sus bases.

probability (p. 234) The measurement of the likelihood that an event will occur.

probabilidad (pág. 234) Medida de la posibilidad de que un suceso ocurra.

probability experiment (p. 233) The process of observing an outcome when there is chance involved, which means that the outcome is not known prior to doing the experiment.

prueba de probabilidad (pág. 233) El proceso de observar un resultado cuando está implicada la probabilidad, lo cual significa que el resultado no se conoce antes de realizar la prueba.

property (p. 51) In mathematics, a property is a rule or statement that is always true.

propiedad (pág. 51) En matemáticas, una propiedad es una regla o enunciado que es siempre verdadero.

proportion (p. 82, 164) An equation stating that two ratios are equal.

proporción (págs. 82, 164) Ecuación que establece que dos razones son iguales.

R

radius (p. 169) A line segment with one endpoint at the center of a circle and the other endpoint on the circle.

radio (pág. 169) Segmento de recta que tiene un extremo en el centro de un círculo y el otro extremo sobre el círculo.

random digits (p. 297) Digits (0, 1, 2, . . . , 8, 9) arranged in a random order.

números aleatorios (pág. 297) Dígitos (0, 1, 2,..., 8, 9) ordenados en un orden aleatorio.

random sample (p. 332) A sample that is formed by selecting individuals from the population at random.

muestra aleatoria (pág. 332) Una muestra que se forma seleccionando aleatoriamente individuos de entre la población.

rate (p. 79) A comparison of two different units, such as distance and time, or two different things measured with the same unit.

tasa (pág. 79) Comparación entre dos unidades diferentes, como distancia y tiempo, o dos cosas diferentes medidas con la misma unidad.

ratio (p. 79) A comparison of two quantities. Ratios can be written as fractions (indicating a quotient), or using the word "to," or using a colon (:).

razón (pág. 79) Comparación entre dos cantidades. Las razones pueden escribirse como fracciones (que indican un cociente) o usando la palabra "a" o usando dos puntos (:).

rational number (p. 33) Any number that can be written as the ratio of two integers where the divisor is not zero; for example, 8, 1.5, $\frac{2}{5}$, and -3.

número racional (pág. 33) Cualquier número que puede escribirse como razón entre dos enteros, donde el divisor no es cero; por ejemplo, 8, 1.5, $\frac{2}{5}$, y -3.

rebate (p. 400) A sales promotion used as an incentive to get buyers to purchase a specific product.

descuento (pág. 400) Una promoción de ventas como incentivo para conseguir que los compradores compren un producto específico.

regular polygon (p. 207) A polygon with all sides congruent and all angles congruent.

polígono regular (pág. 203) Polígono que tiene todos los lados congruentes y todos los ángulos congruentes.

regular pyramid (p. 207) A pyramid with a base that is a regular polygon.

pirámide regular (pág. 207) Pirámide cuya base es un polígono regular.

relative size (p. 103) The relationship between two items showing how the size of one item is larger or smaller than the other item.

tamaño relativo (pág. 103) La relación entre dos objetos que muestra cómo el tamaño de uno de los objetos es más grande que el otro.

repeating decimal (p. 12) A decimal that has one or more digits following the decimal point that repeat endlessly.

decimal periódico (pág. 12) Decimal que tiene uno o más dígitos que se repiten sin fin después del punto decimal.

right prism (p. 199) A prism on which the bases are directly above each other, making the lateral faces perpendicular to the bases.

prisma rectangular (pág. 199) Un prisma en el cual las bases están directamente arriba una de otra, haciendo que las caras laterales sean perpendiculares a las bases.

S

sales tax (p. 393) A percentage of the cost of a purchase paid as a tax.

impuesto sobre las ventas (pág. 393) Un porcentaje del costo de una compra pagado como impuesto.

sample (p. 328) A small representative group chosen from a population. A good sample can be used to make predictions about the larger population.

muestra (pág. 328) Pequeño grupo representativo escogido de entre una población. Una buena muestra puede usarse para hacer predicciones acerca de la población más grande.

sample mean (p. 330) The average of the data values for a sample.

media de la muestra (pág. 330) El promedio de los valores de los datos de una muestra.

sample space (p. 279) The set of all possible outcomes of an experiment.
espacio muestral (pág. 279) Conjunto de todos los resultados posibles de un experimento.

sample statistic (p. 342) A numerical value that is calculated using data from a sample.
Estadística del muestreo (pág. 342) Un valor numérico que se calcula usando datos de una muestra.

sampling (p. 328) Choosing a portion of the population to study which resembles the entire population.
muestreo (pág. 328) Escoger una porción de la población a estudiar que se asemeje a la población completa.

sampling variability (p. 342) The variability in the values of a sample statistic that occurs when random samples are selected from a population because different samples include different individuals.
variabilidad muestral (pág. 342) La variabilidad en los valores de una estadística del muestreo que ocurre cuando muestras aleatorias se seleccionan de entre una población, debido a que diferentes muestras incluyen individuos diferentes.

scale drawing (p. 104) A drawing that represents an object as an enlargement or reduction of the size of the actual object. The scale factor defines the amount of enlargement or reduction.
dibujo a escala (pág. 104) Dibujo que representa un objeto como una ampliación o reducción del tamaño del objeto real. El factor de escala define la cantidad de ampliación o reducción.

selected at random (p. 252) Selected at random means choosing in a way that gives each member of the group the same chance of being chosen.
escogido aleatoriamente (pág. 252) Escogido aleatoriamente significa que es elegido en una manera que da a cada miembro de un grupo la misma posibilidad de ser elegido.

semicircle (p. 183) An arc whose measure is half of a circle. The area of a semicircle is half of the area of a circle with the same radius.
semicírculo (pág. 183) Un arco cuya medida es la mitad de un círculo. El área de un semicírculo es la mitad del área de un círculo con el mismo radio.

similar figures (p. 161) Figures in which the lengths of the corresponding sides are in proportion and the corresponding angles are congruent.
figuras semejantes (pág. 161) Figuras en las que las longitudes de los lados correspondientes están en proporción y los ángulos correspondientes son congruentes.

simulation (p. 298) An artificial process for generating an outcome instead of using a probability experiment.
simulación (pág. 298) Un proceso artificial para generar un resultado en lugar de usar un experimento de probabilidad.

slant height of the pyramid (p. 205) The height of a triangular face of a pyramid.
altura inclinada de la pirámide (pág. 205) Altura de una de las caras triangulares de una pirámide.

subset (p. 33) A set whose elements are all in another set. Every set is a subset of itself.
subconjunto (pág. 33) Conjunto cuyos elementos están todos en otro conjunto. Todo conjunto es un subconjunto de sí mismo.

supplementary angles (p. 137) Two angles whose measures have a sum of 180°.
ángulos suplementarios (pág. 137) Dos ángulos cuyas medidas suman 180°.

surface area (p. 201) The sum of the areas of the faces of a solid figure.
área superficial (pág. 201) Suma de las áreas de las caras de un cuerpo geométrico.

T

take-home pay (p. 394) The amount of money an employee receives in a paycheck after amounts are withheld for taxes, health insurance, or contributions to retirement savings.
ingreso neto (pág. 394) La cantidad de dinero que recibe un empleado en su salario después de descontar las cantidades para impuestos, seguros médicos y contribuciones para el retiro.

terminating decimal (p. 11) A decimal that has a finite or limited number of digits following the decimal point.
decimal exacto (pág. 11) Decimal que tiene un número finito o limitado de dígitos después del punto decimal.

theoretical probability (p. 259) The ratio of the number of outcomes in which the event can occur to the total number of outcomes in the probability experiment.
probabilidad teórica (pág. 259) Razón del número de maneras en que puede ocurrir un suceso al número de resultados posibles de un experimento de probabilidad.

tip (p. 119) An optional payment—in addition to the cost of the bill—to someone who provides good service.
propina (pág. 119) Un pago opcional, agregado al cobro, para alguien que provee un buen servicio.

tree diagram (p. 286) A graphic organizer for listing the possible outcomes of an experiment.
diagrama de árbol (pág. 286) Organizador gráfico para registrar los resultados posibles de un experimento.

U

unit rate (p. 80) A rate in which one of the quantities represented has a value of one. For example, a rate of 25 miles per gallon is a unit rate that can be written as $\frac{25\text{ miles}}{1\text{ gallon}}$.
tasa unitaria (pág. 80) Tasa en la que una de las cantidades representadas tiene un valor de uno. Por ejemplo, una tasa de 25 millas por galón es una tasa unitaria que puede escribirse como $\frac{25\text{ millas}}{1\text{ galón}}$.

unique (p. 149) "Only" or "single;" in geometry, a triangle that can be drawn in only one way.
único (pág. 149) "Solo" o "sencillo"; en geometría, un triángulo que solo se puede dibujar de una manera.

V

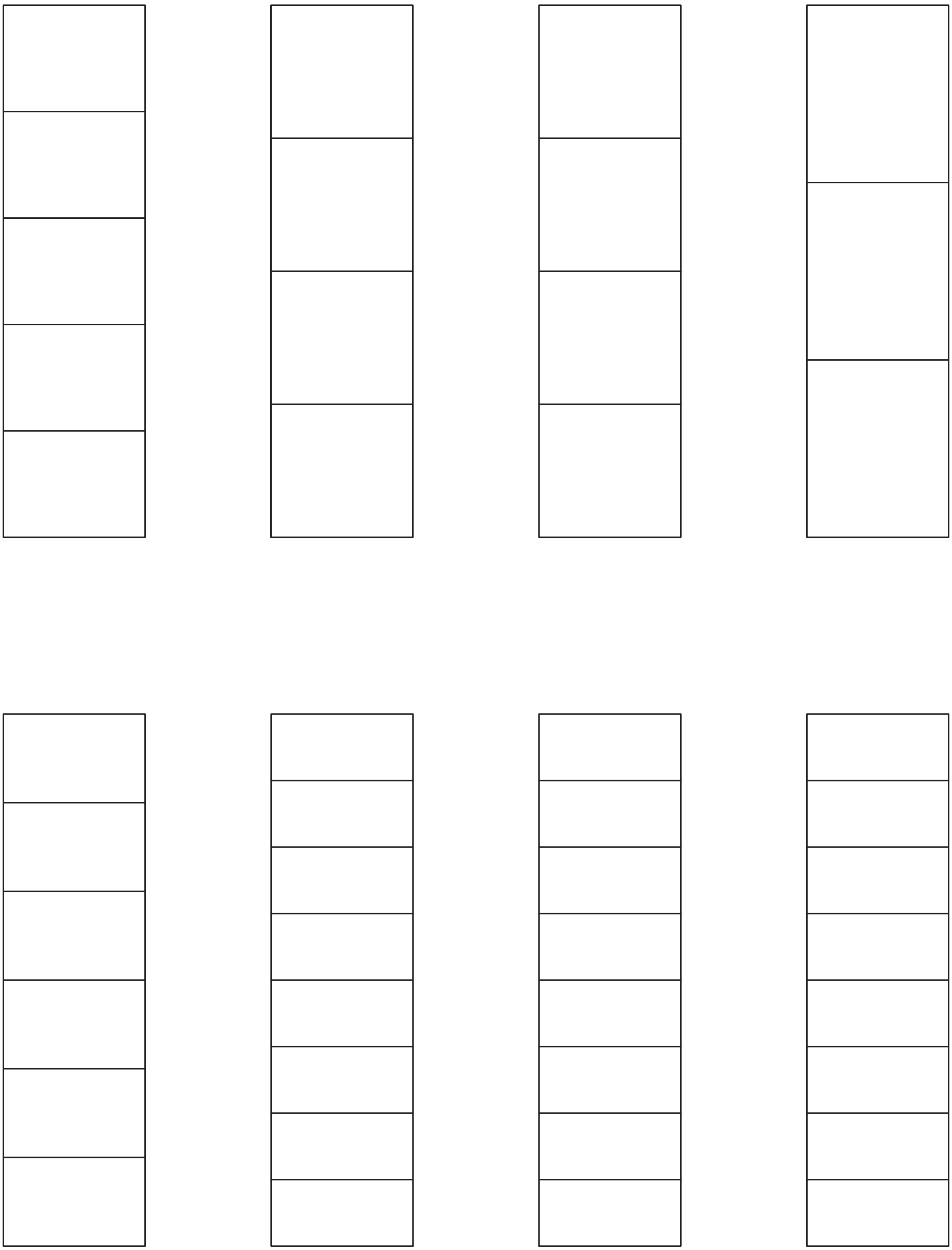
variable (p. 52) A letter or symbol used to represent a number in expressions or equations.
variable (pág. 52) Letra o símbolo que se usa para representar un número en expresiones o ecuaciones.

variable expense (p. 396) An expense that changes—or varies—over time.
costo variable/coste variable (pág. 396) Un gasto que cambia, o varía, con el tiempo.

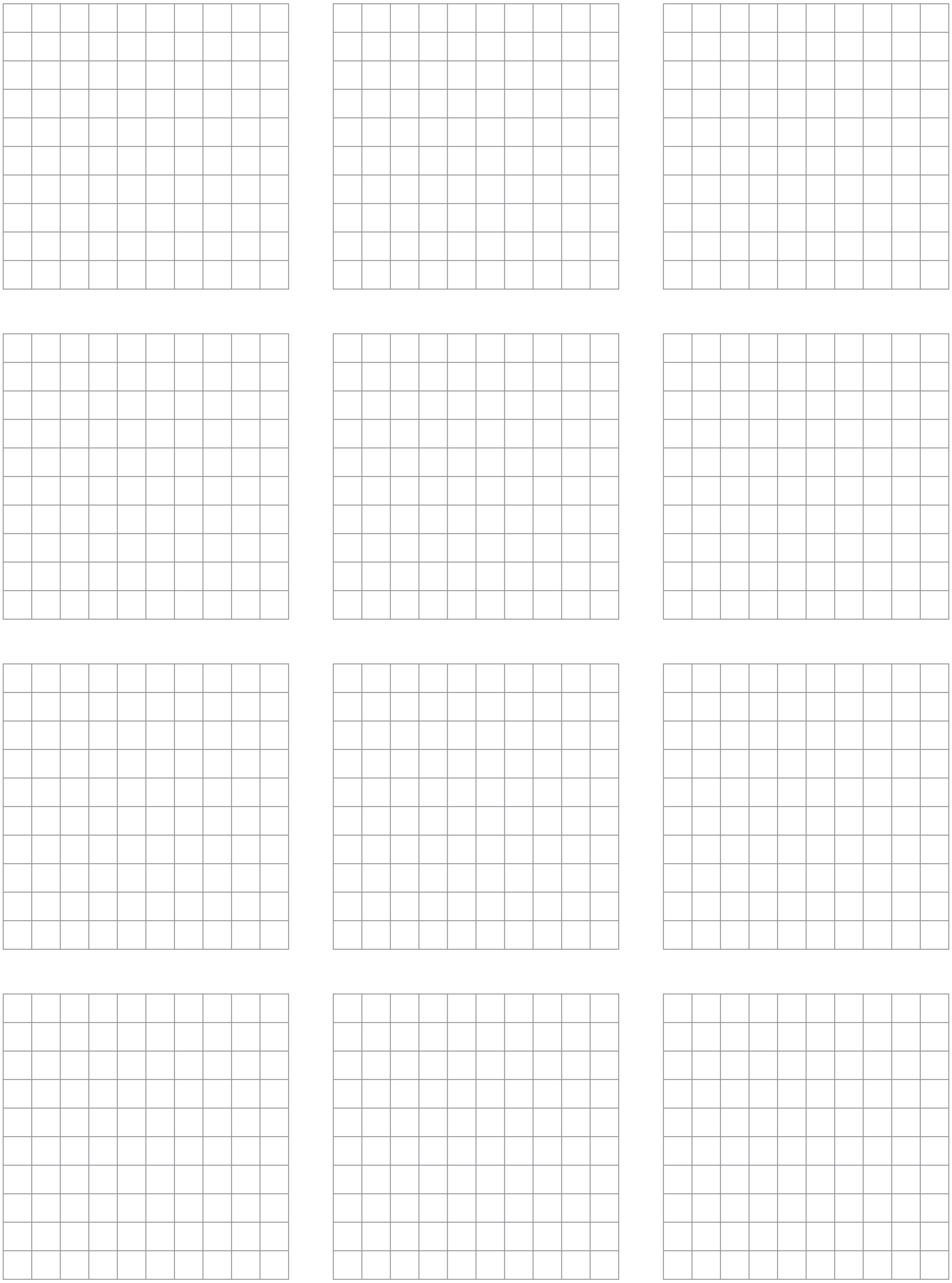
vertex (p. 137) The common endpoint of two rays that form an angle.
vértice (pág. 137) Extremo común de dos rayos que forman un ángulo.

volume (p. 213) The measure of the space occupied by a three-dimensional figure. Volume is measured in cubic units, such as cubic inches ($in.^3$).
volumen (pág. 213) Medida del espacio que ocupa una figura tridimensional. El volumen se mide en unidades cúbicas, como pulgadas cúbicas ($in.^3$).

Fraction Strips



Percent Grids



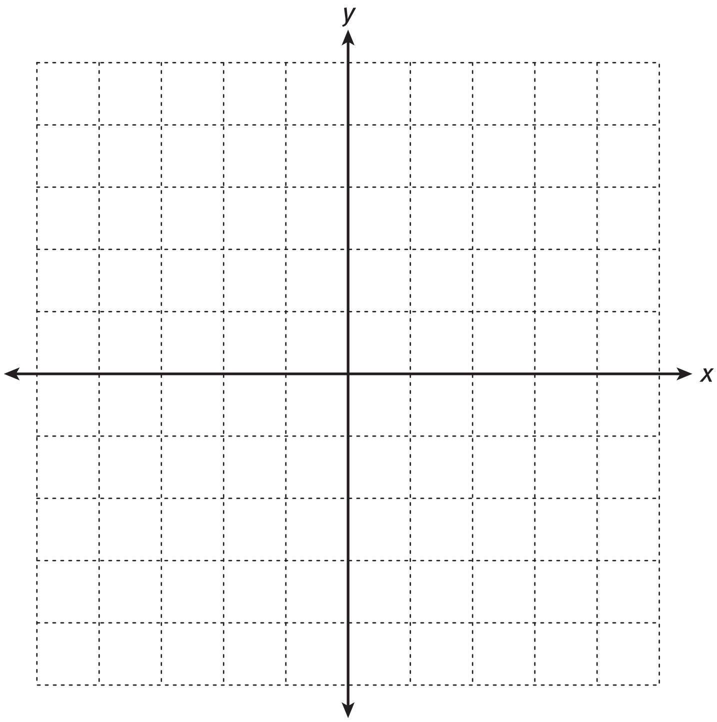
Name: ______________________ Date: __________ Class: __________

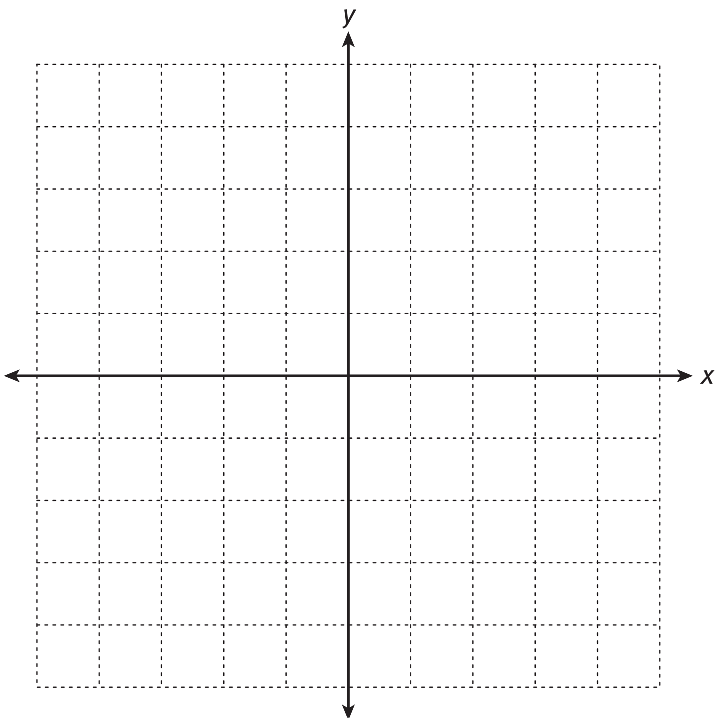
Quarter Inch Grid Paper

Name: ____________________ Date: __________ Class: __________

Centimeter Grid Paper

Tables and Coordinate Grids





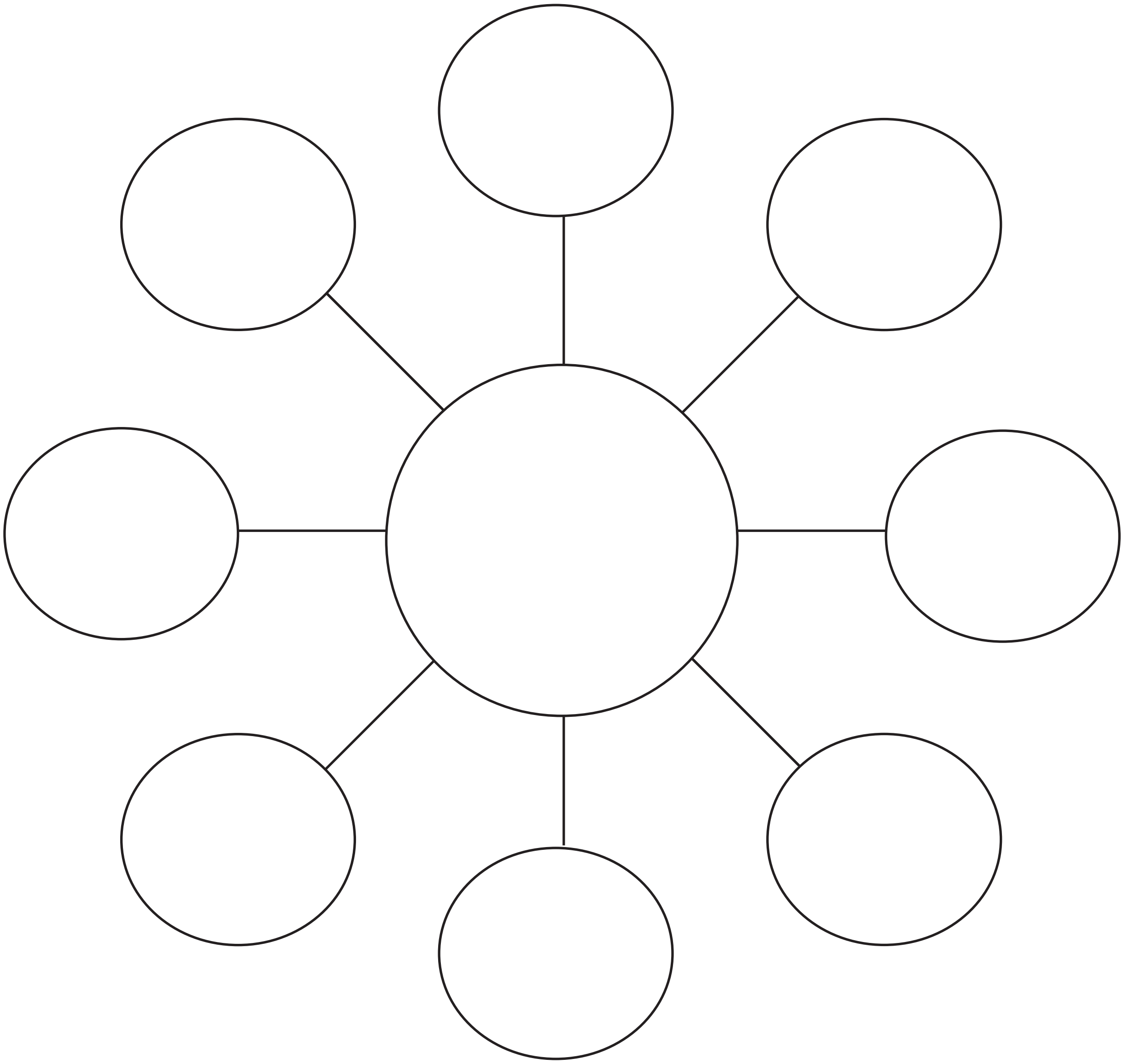
Verbal & Visual Word Association

Definition in Your Own Words	Important Elements

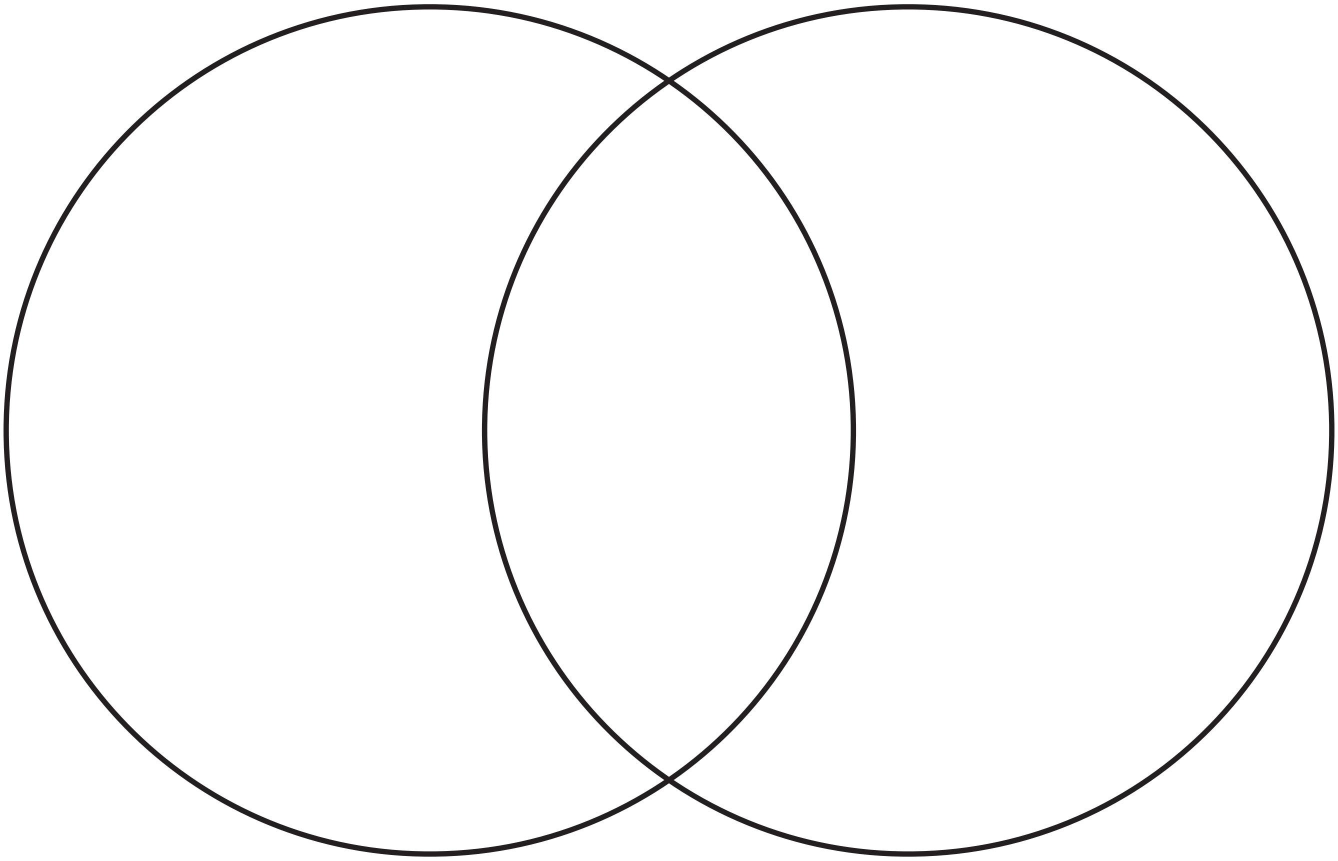
Academic Vocabulary Word

Visual Representation	Personal Association

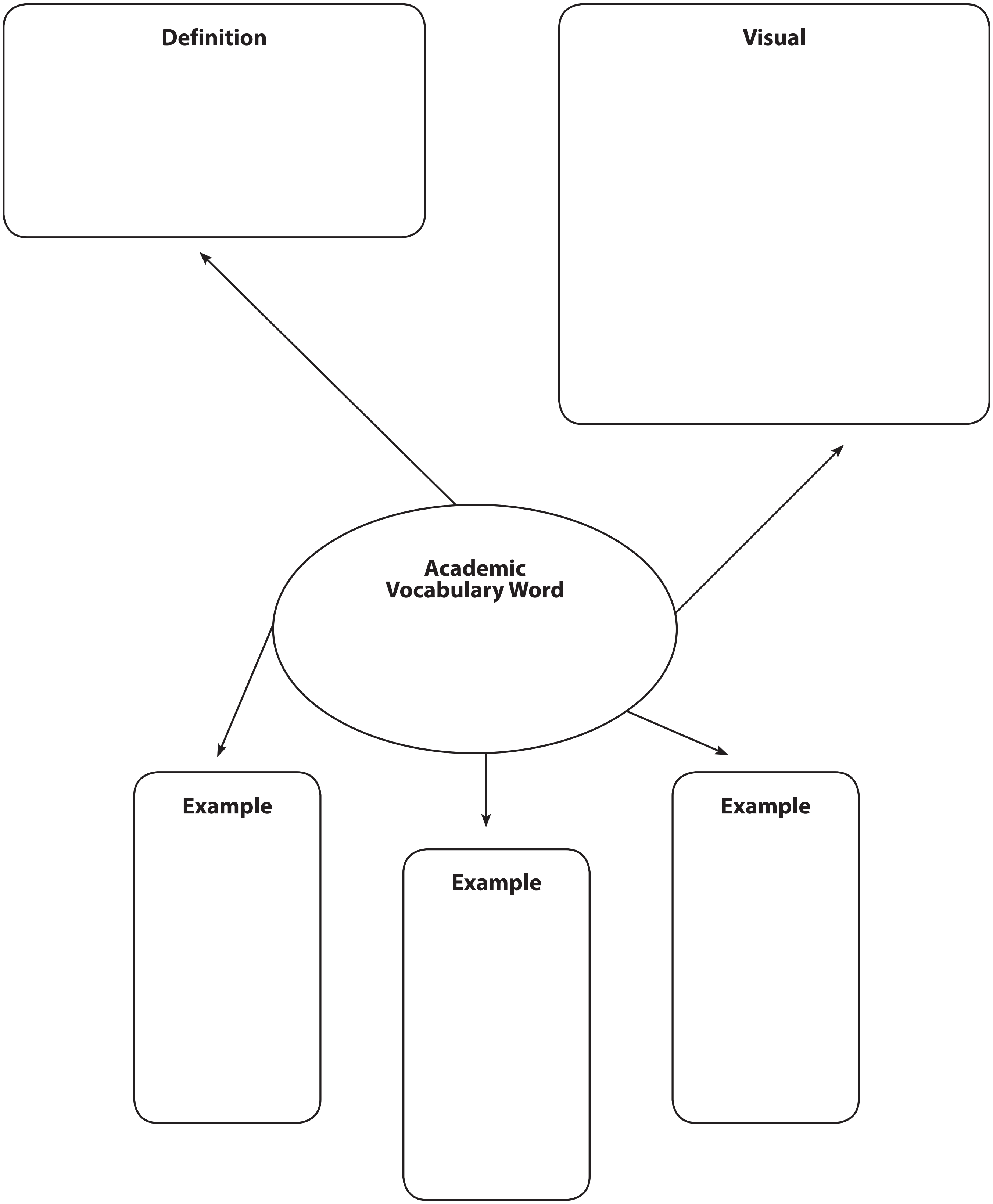
Eight Circle Spider



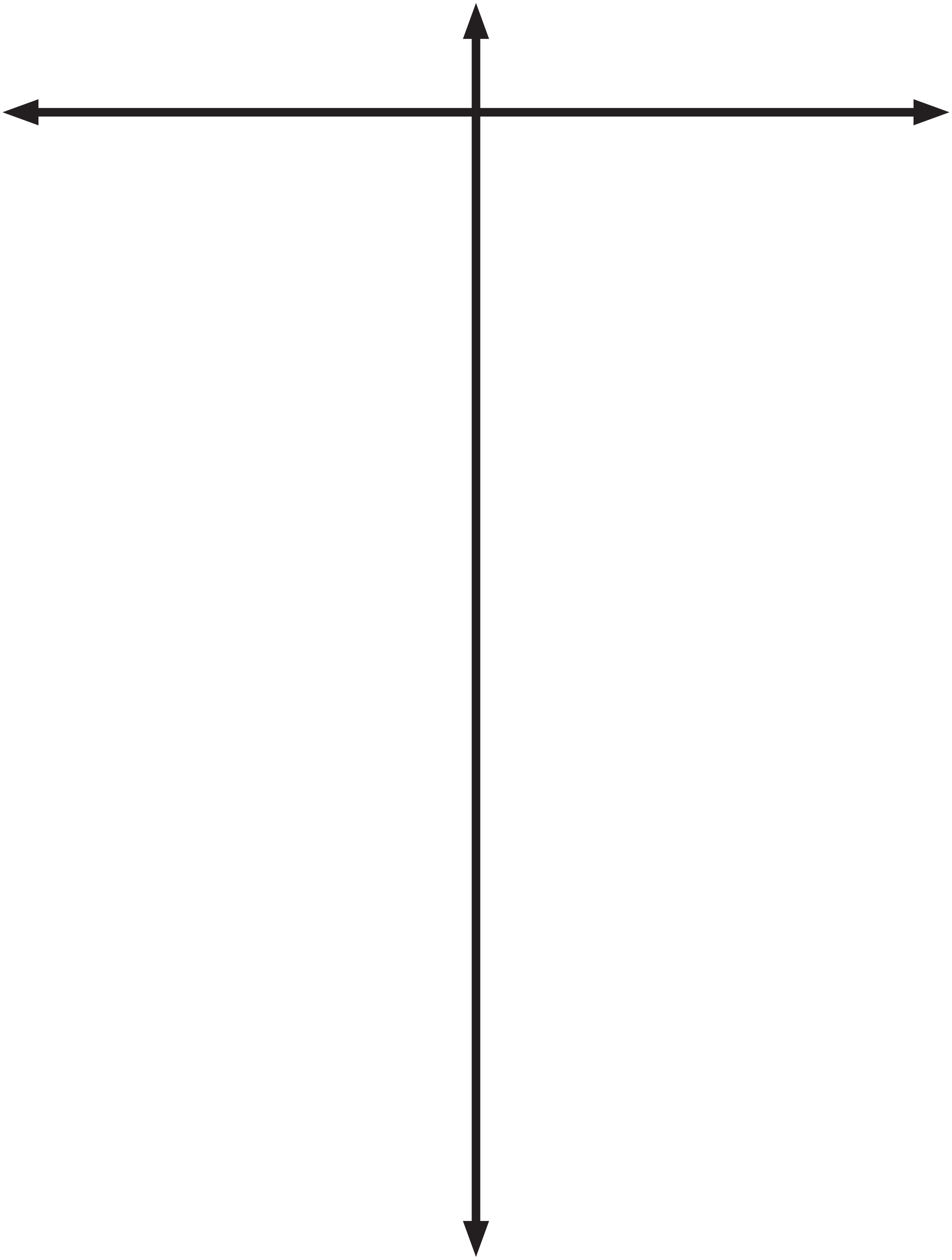
Venn Diagram



Word Map


Definition
Visual
Academic Vocabulary Word
Example
Example
Example

Vertical/Horizontal T-Table



Circle Graph Template